Physics 222 Exam 3
August 6, 2003

Constants:

$c = 3.0 \times 10^{8}$ m/s

1eV = 1.60×10^{-19} J

$h = 6.63 \times 10^{-34}$ J s

$m_e = 9.31 \times 10^{-31}$ kg = 511 keV/c^2 = 0.000549 u

1 u = 931.5 MeV/c^2

hc = 1.24 keV nm

Problem 1 9 Points

(a) A person can barely read a book from a distance of 75 cm. What power of reading glasses would be needed to correct the farsightedness so that the person can read the book at the normal near point of 25 cm? Neglect the distance between the eye and the lens. (3 points)

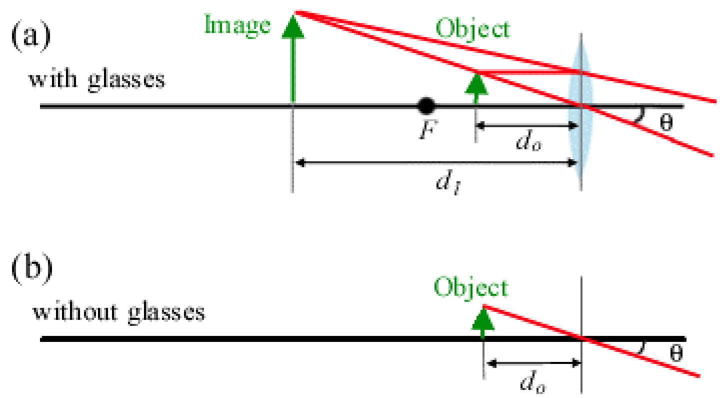
The distance to the object, where the book will be placed, is d_o = 25 cm. The distance to the image is where the person's near point is located, but is negative, since it is on the same side of the lens that light comes from, so d_i = 75 cm. The lens equation gives

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25\text{ cm}} - \frac{1}{75\text{ cm}} = 2.7\text{ m}^{-1} = \mathbf{2.7\ diopters}.$$

(b) Reading glasses and magnifiers are both converging lenses. What is the maximum power of these reading glasses, if they are thought of as magnifying lenses? (3 points)

The lens allows an object which would have had to been viewed at 75 cm to be viewed instead at 25 cm. If the object has height h, with h much less than the distance, the angle subtended is then increased from $\theta = h$ / 75 cm to $\theta' = h$ / 25 cm. The magnification is then $M = \theta'/\theta$ = **3.0x**. Note that if you attempt to use the magnification equation $M = N/f + 1$ to obtain this result, the near point of the uncorrected eye, N = 75 cm, must be used to obtain the correct result.

(c) If the person reads a book 25 cm away and then removes the glasses while looking at the book, will the apparent size of the book change, and if so, by how much? Draw a ray diagram to illustrate your answer. (3 points)



The apparent size of the book will **stay the same**, since it still subtends the same angle θ whether or not the person is wearing glasses. The only difference is that the book is no longer in focus once the glasses are removed, since the object is closer than the near point. It is true that the image will be larger with glasses, but it will also be further away, so its apparent size (angle θ) stays the same. The fact that the angles θ subtended by the image or object are the same with or without glasses can be seen by comparing the ray diagrams (a) and (b) shown.

Problem 2 9 Points

A 35-75 mm zoom lens is rated at *f*/3.5 in the wide-angle setting.

(a) What is the *f*-stop of the lens in its telephoto setting? (2 points)

The *f*-stop is defined as the ratio of the focal length to the lens aperture, f/D. Since only the focal length changes, the ratio of *f*-stops in the two settings is the same as the ratio of focal lengths. Therefore, the *f*-stop in the telephoto setting is

$$3.5 \times (75 \text{ mm} / 35 \text{ mm}) = \mathbf{7.5}.$$

(b) What is the maximum diameter of the lens aperture? (2 points)

Since $f/D = 3.5$ in the wide-angle setting, with $f = 35$ mm, the lens diameter must be

$$D = 35 \text{ mm} / 3.5 = 10 \text{ mm} = \mathbf{1.0\ cm}.$$

(c) If a $^1/_{250}$ second exposure is needed for a wide angle exposure, what exposure is needed in the telephoto setting, if the available choices are $^1/_8$, $^1/_{15}$, $^1/_{30}$, $^1/_{60}$, $^1/_{90}$, $^1/_{120}$, $^1/_{250}$, $^1/_{500}$, and $^1/_{1000}$ seconds? (3 points)

The amount of light reaching the lens per unit time is inversely proportional to the square of the *f*-stop. Therefore, for a fixed intensity, the exposure time is proportional to the square of the *f*-stop. The *f*-stop is 2.14 times as great in the telephoto setting as it is in the wide angle setting, so an exposure time 4.59 times as long is needed. This gives an exposure of $^{1}/_{54.4}$ seconds. The closest setting for the shutter is then $\mathbf{^{1}/_{60}}$ **seconds**.

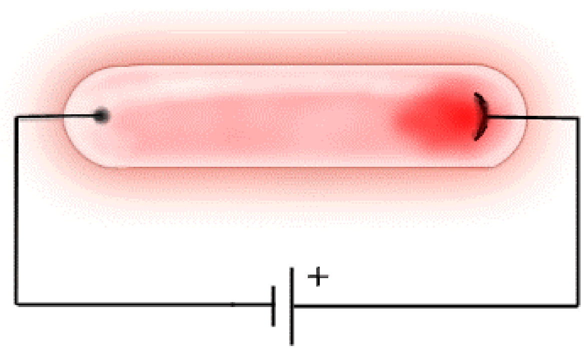
(d) What is the diffraction limit on the angular resolution of the lens at its maximum aperture? (2 points)

Assuming the peak sensitivity of the eye is around 550 nm, the diffraction limit is given by the Rayleigh criterion to be

$\theta = 1.22\ \lambda / D = 1.22 \times 550\text{ nm} / 1.0\text{ cm} = 6.7 \times 10^{-5}$ radians = $\mathbf{3.8 \times 10^{-3}}$ **degrees**, since 180 degrees is π radians. Note that there are 3600 arc seconds in a degree, so this result can be expressed as **14 arc seconds**.

Problem 3 — 10 Points

Low-density hydrogen gas is sealed in a glass tubes containing an anode and cathode as shown. An adjustable voltage source is attached to the electrodes in the tube. The voltage is adjusted until the tube begins to glow red, and a single visible spectral line is seen in a spectrograph. There is no magnetic field. Note: The ground state of hydrogen is –13.6 eV.



(a) The spectral line is due to a transition between two atomic energy levels. What are their principal quantum numbers? (2 points)

The first visible line to appear is the one with lowest energy, or the longest wavelength Balmer line. This transition is from $\boldsymbol{n} = \mathbf{3}$ to $\boldsymbol{n} = \mathbf{2}$.

(b) What is the wavelength of the single visible line? (3 points)

The energy of the photons in this spectral line is given by

$$E = 13.6 \text{ eV} \times (1/2^2 - 1/3^2) = 1.89 \text{ eV}.$$

The wavelength of this photon is found using $E = hc/\lambda$, which gives

$$\lambda = hc/E = 1.24 \text{ keV nm} / 1.89 \text{ eV} = \mathbf{656\ nm}.$$

(c) What is the minimum voltage setting where this spectral line can be seen? (3 points)

Enough energy is needed to excite the $n = 3$ level of hydrogen, whose energy above the $n = 1$ level ground state is

$$E = 13.6 \text{ eV} \times (1 - 1/3^2) = 12.1 \text{ eV}.$$

This is the amount of energy an electron gains after a potential change of 12.1 V, so the voltage setting must be at least **12.1 volts**.

(d) If a magnetic field is turned on around the gas tube, what happens to the visible line in the spectrum? (2 points)

The magnetic field causes the energy levels of hydrogen to be split according to their magnetic quantum number m_l, so the spectral line becomes split into components corresponding to transitions between each of the possible values of m_l for each original level.

Problem 4 12 Points

In this problem, use relativistic kinematics whenever necessary.

(a) A proton has a diameter of about 10^{-15} m. How much momentum must an electron have, at minimum, to probe the internal structure of a proton? Give the result in units of GeV/*c*. This is called deep inelastic scattering, and it is a way the quarks making up the proton can be seen. Hint: Think in terms of the deBroglie wavelength of the electron when answering this question. (3 points)

To probe a size of length L, a wavelength λ less than L is required. In this case, the relevant wavelength is the deBroglie wavelength $\lambda = h/p$ of the electron. Then the minimum electron momentum is $p = h/L = hc/Lc = 1.24$ keV nm / 10^{-6} nm = **1.24 GeV/*c***, where 1 GeV = 10^9 eV.

(b) What is the velocity of the electron in part (a)? (3 points)

Since the electron mass is $m_e = 511$ keV/c^2, the non-relativistic expression $p = mv$ would lead to a velocity much greater than the speed of light. This means relativity must be used. If we know the energy E, then the velocity can be obtained from the factor γ in $E = mc^2 = \gamma\, m_e\, c^2$. The energy is $E = (p^2\, c^2 + m_e^2\, c^4)^{1/2} = pc$, since the electron mass is much smaller than 1.24 GeV/c^2. Then

$$\gamma = E\,/\,m_e\, c^2 = p\,/\,m_e\, c = 1.24\text{GeV} / 511 \times 10^{-6}\ \text{GeV} = 2.43 \times 10^3.$$

Since $\gamma^2 = 1/(1 - v^2/c)$, if follows that

$$v = c\,(1 - \gamma^{-2})^{1/2} = c = \mathbf{3.0 \times 10^8\ m/s}$$

to within one part in 10^7, so that the speed of the electron is very **nearly the speed of light**. Modern particle accelerators can achieve electron energies of over 100 GeV.

(c) ${}^{12}_{5}$B can decay to ${}^{12}_{6}$C via beta decay. If the maximum kinetic energy of the electron emitted is observed to be 13.4 MeV, what is the mass of ${}^{12}_{5}$B in atomic mass units? The atomic mass of ${}^{12}_{6}$C is exactly 12.000000 u. (3 points)

The maximum kinetic energy of the electron emitted in beta decay is equal to the mass difference between the parent and daughter nuclei (times c^2). In terms of atomic mass units, the energy of the electron is

$$13.4\ \text{MeV}\ c^2 / (931.5\ \text{MeV/u}) = (0.0144\ \text{u})\ c^2.$$

Therefore, the mass of ${}^{12}_{5}$B is 12.0000 u + 0.0144 u = **12.0144 u**.

(d) What is the maximum velocity of the electron emitted in part (c)? (3 points)

The kinetic energy is large compared to the rest energy of the electron, so relativistic expressions must be used. The γ factor is given by $\gamma = E/m_e c^2$, where $E = \text{KE} + mc^2$ = 13.4 MeV + 0.511 MeV = 13.9 MeV. This gives γ = 13.9 MeV / 0.511 MeV = 27.2. Then

$$v = c\,(1 - \gamma^{-2})^{1/2} = 0.999\, c = \mathbf{3.0 \times 10^8\ m/s}$$

Again, the electron is in the ultra-relativistic limit, traveling **very close to the speed of light**, but not nearly as close as in part (b).