Physics 222 Exam 2
July 16, 2003

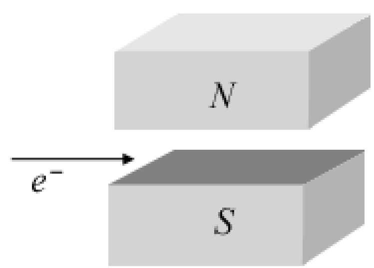
Constants:

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 $\mu_0 = 4\pi \times 10^{-7}$ Tm/A $c = 3.0 \times 10^8$ m/s

1 gauss = 10^{-4} T $e = 1.60 \times 10^{-19}$ C $m_e = 511$ keV/c^2

Problem 1 12 Points

An electron approaches a square magnet with poles 20 cm by 20 cm, separated by a 4.0 cm space. The magnetic field is 17.5 gauss between the poles, and negligible outside the poles. The electron enters in the direct center of the magnet, midway between the poles, with a momentum of 12.0 keV/*c*.



(a) What is the magnetic force acting on the electron when it first enters the poles of the magnet? Give both the magnitude and direction (up, down, left right, out of the page, or into the page). (3 points)

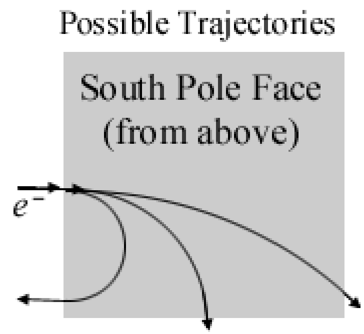
The magnetic force is given by $F = qvB = -epB/m_e$, where p is the momentum of the electron. The magnitude of the force is then

$$F = (1.60 \times 10^{-19}\ \text{C})\ (12.0\ \text{keV}/c)\ (17.5 \times 10^{-4}\ \text{T})\ /\ (511\ \text{keV}/c^2)$$
$$= (6.58 \times 10^{-24}\text{C N} / (\text{mC/s}))\ (3.0 \times 10^{8}\ \text{m/s}) = \mathbf{1.97 \times 10^{-15}\ N}$$

where the unit conversion 1 T = 1 N/(mA) = 1 N/(mC/s) has been used. The magnetic field points downward, from the north to the south pole, and the electron has a negative charge, so the right hand rule shows that the force is directed **out of the page**.

(b) Describe the motion of the electron in detail after it enters the magnet. In particular, which way does it go, what is the shape of its path, and does it become trapped by the magnet, does it leave the magnet, or does it strike a pole? What happens to the velocity? Describe the motion as completely as you can without doing a calculation. A picture may help. (3 points)

Since the magnetic force is always perpendicular to the velocity, the electron moves in a circular arc, curving toward its right as it enters the magnet. It will continue on a circular arc until it encounters an edge of the magnet, and then leave the magnet and continue traveling in a straight line. The speed remains constant throughout the trajectory, since the magnetic force does no work on the charge. Exactly where the electron leaves the magnet depends on the radius of its trajectory, but it must leave somewhere. It cannot remain in the magnet, because the center of its circular arc is on the edge of the magnet (or possibly outside the magnet).



(c) What is the total length of the electron's trajectory while it is between the poles of the magnet? (3 points)

We must determine where the electron leaves the magnet. Since the magnetic force gives the centripetal acceleration, $evB = mv^2/r$, so the circular trajectory will have radius

$$r = mv/(eB) = p/(eB) = (12.0 \times 10^3 \text{ eV}/c)/(e \times 17.5 \times 10^{-4} \text{ T})$$
$$= (6.86 \times 10^6 \text{ V/T})/(3.0 \times 10^8 \text{ m/s}) = 2.29 \text{ cm}.$$

The unit conversion used here is 1 V/T = 1 (Nm/C) / (N/mA) = 1 m^2/s. The electron then makes a semi-circular path through the magnet and exits the magnet a distance of $2r$ = 4.57 cm toward the viewer from where it entered the magnet. The length of the trajectory is the length of this semicircle, πr = **7.19 cm**.

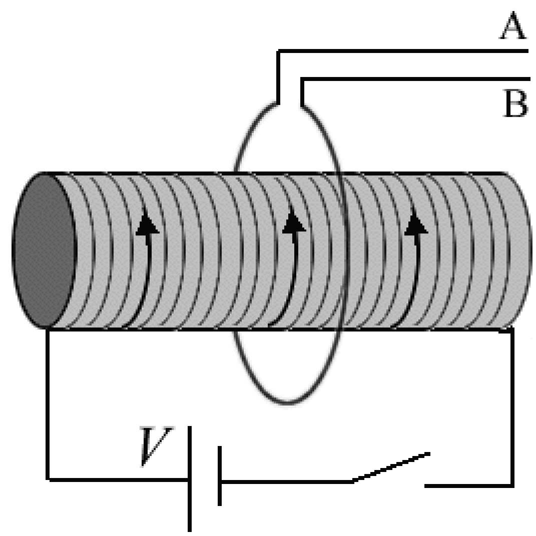
(d) What is the final velocity of the electron, in speed and direction? (3 points)

The speed must be the same as when the electron enters the magnet, since the field does no work on the electron. The speed is therefore given by

$$v = p/m_e = (12.0 \text{ keV}/c) / (0.511 \text{ MeV}/c^2) = 0.234\text{ c} = \mathbf{7.05 \times 10^6 \text{ m/s}}.$$

The direction is to the left, since the electron exits the same face it enters, after a semi-circular trajectory through the magnet.

Problem 2 16 Points



An 11.8 m length of thin copper wire is wound around a nonmagnetic tube of diameter 1.5 cm as shown, to make a coil 5.0 cm in length. A single loop of aluminum wire encloses the middle of the coil, with leads to two terminals A and B. A 1.50 V battery is connected to the coil and switched on. A second or two later, a steady current of 4.25 A flows through the coil from the battery.

(a) What is the magnitude and direction of the magnetic field generated inside the coil if the steady 4.25 A current flows in the direction around the coil shown by the arrows? (3 points)

The magnetic field inside the solenoid is given by $B = \mu_0 N I / l$, where l is the length of the coil, and the number of turns is the length of wire divided by the circumference of the coil,

$$N = 11.8 \text{ m} / (\pi \times 0.015 \text{ m}) = 250.$$

The magnetic field is then

$$B = (4\pi \times 10^{-7} \text{ Tm/A}) \times 250 \times 4.25 \text{ A} / 0.050 \text{ m} = 2.67 \times 10^{-2} \text{ T} = \mathbf{267 \text{ gauss}}.$$

The direction is given by the right-hand rule: B points to the **left**.

(b) What is the self-inductance of the copper coil? (3 points)

The self-inductance L is defined by $\mathcal{E} = -L\, \Delta I/\Delta t$, where $\mathcal{E}$ is the emf generated by a changing current I in the coil. Faraday's Law shows that $\mathcal{E} = -NA\Delta B/\Delta t$, where A is the cross sectional area of the coil. Then $L = NA\, \Delta B/\Delta I = NAB/I$, and the result for B in part

$$L = 250\, \pi\, (0.015\text{m}/2)^2\ (2.67 \times 10^{-2} \text{ T}) / 4.25 \text{ A} = 2.78 \times 10^{-4} \text{ H} = \mathbf{278\ \mu H}.$$

You could also use the solenoid expression $L = \mu_0 N^2 A / l$ to get the same result.

(c) What is the total resistance of the circuit? (2 points)

When a steady current flows, Ohm's Law gives the total resistance:

$$R = V / I = 1.50 \text{ V} / 4.25 \text{ A} = 0.0353\ \Omega = \mathbf{35.3\ m\Omega}.$$

(d) What is the inductive time-constant of the circuit? Is this consistent with the observation that a steady current is observed after a couple of seconds? (3 points)

The inductive time constant is given by

$$\tau = L / R = 2.78 \times 10^{-4} \text{ H} / 0.0353\ \Omega = 7.88 \times 10^{-4} \text{ s} = \mathbf{788\ \mu s}.$$

Since the time constant is much less than a second, this is consistent with the fact that a steady current is observed after a second or two. In fact, a steady current would be seen in just a fraction of a second.

(e) When the switch is first turned on, the rate of change of the current is given by $\frac{\Delta I}{\Delta t} = I_{max} / \tau$, where I_{max} is the final steady current and τ is the time constant found in part (c). What is the initial voltage generated between the terminals A and B, and which is at the higher potential? (3 points)

The voltage between the terminals A and B will be the emf generated by Faraday's Law in the single aluminum coil. Lenz's Law says that the emf must cause a current to flow to oppose the change in flux. Since the flux is increasing to the left, the current must flow from terminal B toward terminal A to oppose it. Therefore, **terminal B is at a higher potential**.

This can be thought of as a transformer with a 250 winding primary and a single winding secondary. When the switch is first closed, no current flows, so the voltage across the primary is initially V = 1.50 V. Since the voltage is changing, a voltage of magnitude V/N will be induced in the secondary. The voltage between the terminals B and A is therefore

$$V_{BA} = V / N = 1.50 \text{ V} / 250 = 6.0 \textbf{ mV}$$

Alternatively, this can be solved using mutual inductance. If M is the mutual inductance of the coil and aluminum wire loop, then $V_{AB} = -M\,\Delta I / \Delta t$. Since the magnetic field is mostly confined to the interior of the large copper coil, $V_{AB} = -A\,\Delta B/\Delta t$, where A is the cross-sectional area of the coil of copper wire. Then, comparing with the derivation of L in part (b), we find that $M = A\,\Delta B/\Delta I = L/N$. Using $\tau = L/R$ and $I_{max} R = V$, we find that

$$V_{BA} = -V_{AB} = M\,(\Delta I / \Delta t) = M\, I_{max} / \tau = M\, I_{max} R / L = I_{max} R / N = V / N.$$

This is the same as the transformer result.

(f) What is the voltage between terminals A and B after the switch has been closed a couple of seconds? (2 points)

Since the current is steady, there is no change in flux, and therefore no magnetic induction. The voltage between the terminals is $V_{AB} = \mathbf{0}$.

Problem 3 12 Points

(a) If a radio receiver can detect a signal with an rms electric field of 1.2 mV/cm, how far away from a 50 kW transmitter can it receive the signal? Assume the signal is transmitted isotropically. (3 points)

The average power per unit area received is $S = P/(4\pi r^2)$ where r is the distance from the transmitter. This is related to the rms value of the electric field, E_{rms}, by $S = \varepsilon_0 c E_{rms}^2$.

$$r^2 = \frac{P}{4\pi\varepsilon_0 c E_{rms}^2} = \frac{\mu_0 cP}{4\pi E_{rms}^2} = \frac{(4\pi \times 10^{-7}\ \text{Tm/A})(3 \times 10^8\ \text{m/s})(5 \times 10^4\ \text{W})}{4\pi (0.12\ \text{V/m})^2}$$

$$= 1.04 \times 10^8\ \text{m}^2.$$

The maximum distance to receive the signal is then r = **10.2 km**.

(b) The Hydrogen α and δ lines have wavelengths 656 nm and 410 nm, respectively. If these fall on a diffraction grating with 6000 lines per cm, what is the angular separation of these two lines in the first order spectrum? (3 points)

The angles for the first order lines satisfy $d \sin \theta_\alpha = \lambda_\alpha$, $d \sin \theta_\delta = \lambda_\delta$, with grating spacing $d = 1/(6000\ \text{cm}^{-1}) = 1.67 \times 10^{-6}$ m. Then a simple calculation gives $\theta_\alpha = 23.2$ degrees, and $\theta_\delta = 14.2$ degrees. The angular separation is then $\theta_\alpha - \theta_\delta$ = **9.0 degrees**.

(c) A tall, narrow siren horn emits a loud 440 Hz tone. What is the maximum width that the horn can have if there are to be no diffraction minima in any horizontal direction? Assume the speed of sound is 343 m/s. (3 points)

This is a single-slit diffraction problem, since the horn is tall and narrow.

The first minima for single slit diffraction occur when $d \sin \theta = \lambda$, where d is the width of the slit, so the condition that no diffraction minima occur is that d is less than λ. The wavelength is

$$\lambda = 343\ \text{m/s} / 440\ \text{Hz} = 78.0\ \text{cm}.$$

Therefore, the maximum width of the horn should be **78.0 cm**.

(d) A thin film of alcohol, with index of refraction 1.36, lies on a flat glass plate of glass with index of refraction 1.51. The reflected light from the film, at normal incidence, has a minimum intensity when the wavelength is 512 nm, and a maximum intensity when the wavelength is 640 nm. What is the thickness of the film, if it is known to be thinner than 2 μm? (3 points)

Since the index of refraction of the alcohol is less than the index of refraction of the glass, 180 degree phase shifts occur for both the wave reflected from the top of the alcohol layer and the bottom. Therefore, the condition for constructive interference is that the path length through the alcohol is an even multiple of the wavelength in the alcohol:

$$2t = m_1 \times 640 \text{ nm}/n_{Alc},$$

where $n_{Alc} = 1.36$ is the index of refraction of the alcohol. The minimum intensity must occur half way between the maxima:

$$2t = (m_2 + ½) \times 512 \text{ nm}/n_{Alc}.$$

The values of m_1 may be 1, 2, 3, …, and the values of m_2 may be 0, 1, 2, …, but we don't know which value. We do know that they are related, since they must give the same thickness. Setting the right-hand sides equal gives

$$640\, m_1 = 512\, m_2 + 256.$$

Dividing both sides of the equation by 128 gives

$$5\, m_1 = 4\, m_2 + 2.$$

The first value of n_2 which makes the right hand side a multiple of 5 is $m_2 = 2$. Then $m_1 = 2$ also. The least possible thickness of the film is then

$$t = 640 \text{ nm} / 1.36 = \mathbf{470\ nm}.$$

There are other solutions giving thicker films. The general solution would be for $m_1 = 4k + 2$, $m_2 = 5k + 2$ with $k = 0, 1, 2, \ldots$ All of these give films at least 6 times as thick, which is more than 2 μm.