Physics 222 Exam 3

July 31, 2002

Constants:

$c = 3.0 \times 10^8$ m/s
1eV = 1.60×10^{-19} J
$h = 6.63 \times 10^{-34}$ J s
$m_e = 9.31 \times 10^{-31}$ kg = 511 keV/c^2
1 light-year = 9.46×10^{15}m
hc = 1.24 keV nm

Problem 1 10 Points

(a) What is the diameter of the lens aperture of a 50 mm camera lens at *f*-stop *f*/8? (2 points)

The *f*-stop is *f* /*D*, where *D* is the lens diameter and *f* is the focal length. Then 8 = 50mm/*D*, giving *D* = **6.25 mm**.

(b) Assume the exposure time was set at 1/240 second in part **(a)**. If the *f*-stop is changed to *f*/16, what exposure time is needed to obtain the same image brightness on the film (the same energy deposited on the film)? (2 points)

The total energy deposited on the film is the intensity of light (power per unit area) times the area of the aperture times the exposure time. The area of the aperture is proportional to D^2, and doubling the *f*-stop reduces *D* by a factor of 2, so changing the aperture to *f*/16 allows ¼ the power to strike the film. This must be compensated by quadrupling the exposure time. So the exposure must be set at **1/60 second** at *f*/16.

(c) A nearsighted person can focus on things only if they are closer than 30 cm. What lens power, in diopters, is needed to correct the vision, if the lens is placed 2 cm in front of the eye? (2 points)

Objects at infinity must be focused at the far point, which is 30 cm from the eye, or 28 cm from the lens. A diverging lens is needed to correct near-sightedness, so the focal length must be –0.28 m. Diopters are the reciprocal of the focal length in meters, so the strength of the lens is **–3.57 diopters**.

(d) What is the primary function of the lens in the human eye? (2 points)

The primary function of the lens is to **make adjustments to focus on objects at various distances**. It does not provide the main focusing, which is done by the cornea.

(e) Would a near-sighted or far-sighted person have clearer vision under water? Why? (2 points)

Because the index of refraction of light is close to that of the eye, the refractive power of the cornea is much less under water. **Near-sighted people** have too much refraction by the cornea, giving a short focal length. Under water, their focal length is longer, so they can see better.

Problem 2 10 Points

(a) A converging lens with focal length 30 cm is placed 25 cm from a diverging lens. Parallel rays of light enter the converging lens, and leave the diverging lens in parallel as well. What is the focal length of the diverging lens? (3 points)

Since parallel rays come in and go out, the focal points of the two lenses must coincide. The converging lens focuses its rays 5 cm behind the diverging lens, so the focal length of the diverging lens must be **–5 cm**.

(b) If this combination of lenses is used as a telescope, what is its magnification? Include the correct sign. (2 points)

The magnification is the product of ratio of the objective lens focal length to the eyepiece focal length. In this case, the converging lens is the objective lens, so the magnification is

$$M = -f_o/f_e = -30\text{cm} /(-5\text{cm}) = \mathbf{6x}.$$

(c) What magnification would the telescope have if you looked through it backward (using the converging lens as the eyepiece)? (2 points)

If the eyepiece and objective lens are interchanged, the magnification becomes

$$M = -f_o/f_e = -(-5\text{cm})/(30\text{cm}) = \mathbf{1/6x = 0.167\ x}.$$

(d) If the lens diameter is 2 cm, what is the smallest detail that can be resolved at a distance of 100 m, assuming a wavelength of 500 nm for visible light? (3 points)

The half-angle to the diffraction minimum is $\theta = 1.22\lambda/D = 3.05 \times 10^{-5}$ radians. The Rayleigh criterion says that the telescope can resolve two objects separated by this angle. At a distance of 100m, that corresponds to a size of $100\text{m} \times 3.05 \times 10^{-5}$ = **3.05 mm**.

Problem 3 10 Points

A rocket is traveling at a speed of 97.4 percent of the speed of light to the nearest star, Proxima Centauri, which is 4.3 light-years from Earth.

(a) How far is it to the star from the point of view of the astronauts while they are traveling? (2 points)

The relativistic length contraction factor is $(1 - 0.974^2)^{1/2} = 0.2265$. So the distance to the star in the rest frame of the rocket is 0.2265 x 4.3 light-years = **0.974 light-years**.

(b) How long will the astronauts take to get to the star, from their point of view? (2 points)

Their speed is 0.974 c, so the time it takes is 0.974 light-years/0.974 c = **1.0 year**.

(c) How long will it take the astronauts to get to the star from the point of view of an Earth-based observer? (2 points)

The time is the distance divided by the velocity: 4.3 light-years/0.974c = **4.42 years**. Alternatively, the relativistic time dilation factor is 1/0.2265, the reciprocal of the length contraction factor. Since the time is 1.0 year in the frame of the rocket, this corresponds to 1.0 yeas/0.2265 = **4.42 years.** This can also be expressed as **4 years and 5 months**.

(d) If the rocket has mass 5.0×10^6 kg, how much work must be done to accelerate it to its final speed of 97.4% of the speed of light, starting from rest? (3 points)

The total relativistic energy is $E = mc^2 = m_0c^2/0.2265$, if $m_0 = 5.0 \times 10^6$ kg is the rest mass. The energy at rest is just m_0c^2, so the change in energy is the work that must be done:

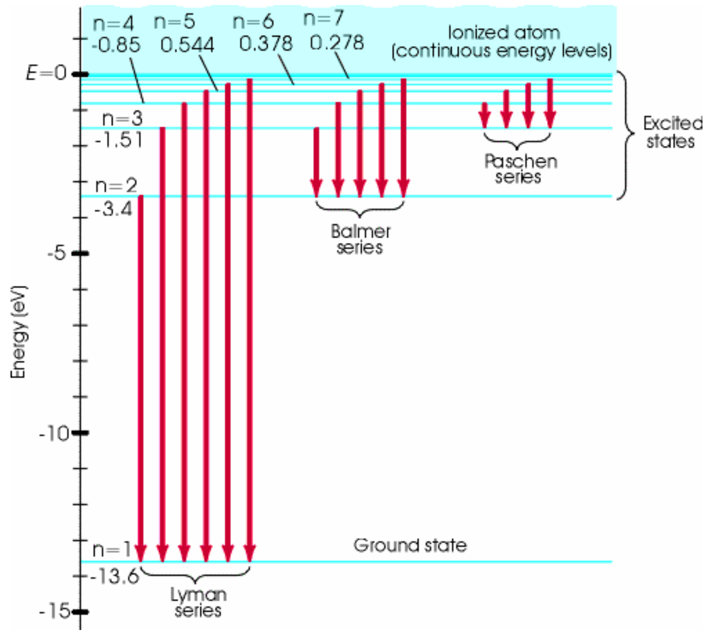
$$W = E - m_0c^2 = m_0c^2\,(1/0.2265 - 1) = (5.0 \times 10^6\text{ kg})\,(3\times10^8\text{ m/s})^2\,(3.415)$$
$$= \mathbf{1.54 \times 10^{24}\,J}$$

(e) Would the astronauts notice a change in their heart rate while in transit? (1 point)

No. In their rest frame, physics is exactly as in any other rest frame, so they can't see any difference for normal processes inside the spacecraft. Their heart rate would be normal.

Problem 4 10 Points

(a) What wavelength of light is needed to ionize a hydrogen atom in its ground state and give the ejected electron a kinetic energy of 10 eV? (3 points)



The energy needed is 13.6 eV to overcome the binding energy, and 10 eV of kinetic energy, giving a total photon energy of 23.6 eV.

$E = hf = hc/\lambda$, so the wavelength is

$\lambda = hc/E$ = 1.24 keV nm/23.6 eV
= **52.5 nm**

(b) What is the deBroglie Wavelength of the ejected electron in part **(a)**? (3 points)

This is a non-relativistic electron, so the kinetic energy is KE = $p^2/2m_e$, where p is the momentum. The kinetic energy is 0.010 keV, so the momentum is

$$p = (2m_e \, \text{KE})^{1/2} = (2 \times 511 \text{ keV}/c^2 \times 0.010 \text{ keV})^{1/2} = 3.20 \text{ keV}/c.$$

The deBroglie wavelength is then $\lambda = hc/pc$ = 1.24 keV nm / 3.20 keV = **0.388 nm**.

(c) What is the wavelength of the photon emitted in the $n = 3$ to $n = 2$ transition in the Balmer series? (2 points)

The difference in energy levels is E = –1.51 eV – (-3.4 eV) = 1.89 eV. This is the energy of the photon emitted. Since $E = hf = hc/\lambda$, the wavelength is

λ = 1.24 keV nm / 1.89 eV = **656 nm**.

(d) At room temperature, when sunlight passes through hydrogen gas, only the Lyman absorption lines are seen. Why? (2 points)

At room temperature, hydrogen gas is almost entirely **in its ground state**, since thermal excitations are not sufficient to populate the excited states at room temperature. Light passing through hydrogen can induce transitions between the ground state and higher energy levels, which causes Lyman absorption lines. (See the diagram.)