Review for Exam 3: Chapters 10 – 13

Physics 1425, Section 1 (Dr. Yost)

Exam 3 will cover chapters 10 through 13. You may use any calculator for this exam, but not notes. You will be given any constants you need, as well as any required moments of inertia, unless the point of the problem is to derive it. You should remember basic algebraic and geometric relationships and trigonometric identities. Physics is cumulative, so you may need concepts from earlier chapters as well

Chapter 10: Rotational Motion about a Fixed Axis

Concepts: Rotational motion, angular quantities, rolling, torque, right hand rules, moments of inertia, angular momentum, conservation of angular momentum

Equations:

Angular velocity and acceleration:

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt} \tag{1}$$

If R is the perpendicular distance from the rotational axis,

$$v = R\omega, \qquad a_{\text{tan}} = R\alpha, \qquad a_{\text{rad}} = R\omega^2. \tag{2}$$

The frequency f, angular velocity ω, and period T are related by

$$f = \frac{1}{T} = \frac{\omega}{2\pi}. \tag{3}$$

If α is constant, all of the analogous formulas to one-dimensional motion hold for the corresponding rotational quantities:

$$\begin{aligned} \omega &= \omega_0 + \alpha t, & \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta, & \overline{\omega} &= \frac{1}{2}(\omega + \omega_0). \end{aligned} \tag{4}$$

If a force F acts a distance R from an axis, with an angle θ between $\vec{R}$ and $\vec{F}$, then the torque is defined to be

$$\tau = R_\perp F = RF_\perp = RF\sin\theta, \tag{5}$$

where $R_\perp$ is the perpendicular projection of R onto F, and $F_\perp$ is the perpendicular projection of F onto R. (The projection forms are often easier to use when the vectors are resolved into cartesian coordinates.) The direction of the torque is perpendicular to both R and F, and given by a right hand rule. If place your fingers along R and curl them toward F, your thumb will point along the direction of τ.

The total torque is related to the angular acceleration by

$$\tau = I\alpha \tag{6}$$

with I the moment of inertia, defined by

$$I = \sum_i m_i R_i^2 \quad \text{or} \quad I = \int dm (R(m))^2 \tag{7}$$

where each R is the perpendicular distance of a mass or mass element from the axis.

The moment of inertia depends on the choice of axis. If I is the moment of inertia about an arbitrary axis, and I_{CM} is the moment of inertia about a parallel axis through the center of mass, then the parallel axis theorem says that $I = I_{\text{CM}} + Mh^2$, where M is the mass of the object, and h is the distance between the axes. Note that it is important that one of the axes be through the center of mass.

Angular momentum is defined by

$$L = I\omega \tag{8}$$

If no external torques act on a system, angular momentum is conserved.

Rotational kinetic energy is defined by

$$K = \frac{1}{2} I\omega^2 \tag{9}$$

There is a rotational analog of the work-energy theorem: The work is given by $W = \int \tau d\theta$, and the chain rule implies that

$$W = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2. \tag{10}$$

The total kinetic energy of a rolling object is a sum of the translational kinetic energy $\frac{1}{2}mv^2$ and the rotational kinetic energy. If the object rolls without slipping, then $v = R\omega$, where R is the radius of the object.

Chapter 11: General Rotation

Concepts: Vector cross product, torque vector, angular momentum vector, rigid body rotation, conservation of angular momentum

Equations:

Angular velocity is considered to be a vector directed along the axis of rotation. The direction of angular velocity is determined by the right hand rule. If you curl your right hand fingers in the direction of rotation, your thumb points in the direction of the angular velocity. Angular acceleration works the same way.

The vector product $\vec{A} \times \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is defined so that $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$. The magnitude of $\vec{A} \times \vec{B}$ is $AB \sin\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$. The direction is determined by the right hand rule. If you curl your fingers from $\vec{A}$ toward $\vec{B}$, your thumb will point along the cross product. In components,

$$\vec{A} \times \vec{B} = \hat{\imath}(A_y B_z - B_y A_z) + \hat{\jmath}(A_z B_y - B_z A_y) + \hat{k}(A_x B_y - B_x A_y). \qquad (11)$$

The vector cross product is bilinear, distributive and *anti*-commutative ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$). It is *not* associative.

Torque and angular momentum are also vectors, in general. In terms of the cross product, the torque and angular momentum can be written as

$$\vec{\tau} = \vec{r} \times \vec{F}, \qquad \vec{L} = \vec{r} \times \vec{p} \qquad (12)$$

These apply to a point particle at position $\vec{r}$ relative to the point around which the torque or angular momentum is calculated. For multiple particles or extended bodies, a sum or integral over positions is taken. For a rigid body rotating about an axis of symmetry through the center of mass, $\vec{L} = I\vec{\omega}$, but this is not true in the most general case.

The torque and angular momentum are related by

$$\vec{\tau} = \frac{d\vec{L}}{dt}. \qquad (13)$$

when they are measured about an axis in an inertial frame, or an axis through the center of mass of an accelerating object. (For accelerated objects, the axis chosen must be through the center of mass to use this equation, or the other rotational motion relations. For objects at rest or moving at constant velocity, the choice of axis doesn't matter.) If there is no net torque on a system, the angular momentum is conserved.

Chapter 12: Static Equilibrium and Elasticity

Concepts: static equilibrium, balance of forces and torques, elasticity, elastic moduli, stress and strain, Young's modulus, shear modulus, bulk modulus.

Equations:

Statics problems are solved by combining balance of forces, $\sum \vec{F}_i = 0$, with balance of torques $\sum \vec{\tau}_i = 0$ about a carefully chosen axis. Although torques balance about any axis, it is best to choose one through one of the points where a force acts, or parallel to one of the forces, so that there is no torque contribution from that force. Always begin by drawing a free body diagram, to be sure you are accounting for all of the forces acting on an object. Write down enough equations to solve for all of the unknowns in the problem.

Materials behave *elastically* when the elongation is proportional to the force applied, meaning that Hooke's Law applies. The *stress* is defined to be the force per unit area, F/A, and the *strain* is defined to be the relative elongation, $\Delta L/L$. In the elastic limit, these are proportional:

$$\frac{F}{A} = E\frac{\Delta L}{L} \tag{14}$$

where the ratio E of stress/strain is called *Young's Modulus*. It is measured in $\mathrm{N/m^2}$.

The bulk modulus describes how much the volume of an object changes under pressure. Pressure is defined to be the force per unit area, $P = F/A$. In the elastic limit, the volume of an object decreases in proportion to the pressure applied,

$$\Delta P = -B\frac{\Delta V}{V}, \tag{15}$$

where B is called the *Bulk Modulus*. It is also measured in $\mathrm{N/m^2}$.

Units:

Strain is dimensionless. Stress is measured in $\mathrm{N/m^2}$.

Chapter 13: Fluids

Concepts: Density, specific gravity, pressure, Pascal's principle, barometers, buoyancy, Archimedes' principle, fluids in motion, flow rate, equation of continuity, Bernoulli's principle, airflow, lift

Equations:

Density is mass per unit volume: $\rho = m/V$. Specific gravity is the ratio $\rho/\rho_{\mathrm{H_2O}}$ of the density to the density of water, $\rho_{\mathrm{H_2O}} = 1\ \mathrm{g/cm^3} = 1000\ \mathrm{kg/m^3}$.

Pressure is force per unit area, $P = F/A$. It is measured in pascals, with 1 Pa = 1 $\mathrm{N/m^2}$. Atmospheric pressure is approximately $1.01 \times 10^5 Pa$. Gauge pressure is the difference between pressure and atmospheric pressure.

The pressure under a depth h of fluid of density ρ is $P = \rho g h$. This is just the force per unit area due to the weight of the fluid. If the fluid is compressible, the density may change with height, but at any height, $dP/dy = -\rho g$. The pressure in a faucet due to water in a storage tank is given by $P = \rho g h$ with ρ the density of water, and h the height of the top of the water in the tank above the faucet. The height h is called the *pressure head*.

Pascal's principle states that applying pressure to a confined fluid increases the pressure throughout by the same amount. This is used to construct hydraulic lifts. If force F_1 is applied to a piston of area A_1, the force on a second piston of area $A_2 > A_1$ will be $F_2 = F_1 A_2 / A_1 > F_1$.

A barometer can be made by placing an evacuated tube vertically in a reservoir of fluid. The fluid will rise in the tube until the weight of fluid in the tube equals the atmospheric pressure.

Archimedes' principle states that the buoyant force on an object is equal to the weight of water it displaces. An object will float in a fluid if its density is less than the density of the fluid. The fraction of the volume submerged is given by the ratio of the densities ρ_O/ρ_f of the object to the fluid.

Laminar flow occurs when a fluid flows in smooth paths with definite streamlines. Turbulent flow occurs when erratic eddies form, and the flow is chaotic, without regular streamlines. In this chapter, we are concerned only with Laminar flow.

The volume rate of flow of a fluid through a pipe of cross section A is $Q = Av$ where v is the velocity of the fluid. This measures the volume per unit time of fluid flowing through the pipe:

$$\frac{dV}{dt} = \frac{Adl}{dt} = Av. \tag{16}$$

The equation of continuity states that at any two different points in the pipe,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \tag{17}$$

Bernoulli's principle is a statement of the work-energy theorem for fluids, neglecting viscocity, which is the analog of friction for fluids. It states that the quantity

$$P + \frac{1}{2}\rho v^2 + \rho g h \tag{18}$$

is conserved. The pressure term represents the work per unit volume ΔV done on the moving fluid:

$$W = Fv\Delta t = F\Delta l = PA\Delta l = P\Delta V. \tag{19}$$

The term $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume, and $\rho g h$ is the potential energy per unit volume. Bernoulli's principle applies to both fluids and gasses. The main trick in applying it is knowing how to set the various variables in terms of the physics of the problem, so it is essential to review the examples and homework problems.

An example of Bernoulli's principle is the lift on an airfoil. An airfoil is designed so that the velocity v_1 of air flowing above the airfoil is greater than the velocity v_2 below. Bernoulli's principle then says that there will be a pressure difference, $P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$, which creates a net *lift* force $F_L = A(P_2 - P_1)$, where A is the area of the wing.

Viscosity is a retarding force due to the friction between adjacent layers of a fluid. It is defined as follows. Imagine two plates of area A, separated by a fluid layer of thickness l. If the top plate is moved at a speed v parallel to the bottom plate (so that the thickness l is constant), the force on the plate is $F = \eta Av/l$, where η is the *coefficient of viscosity.* "Thicker" fluids have a higher coefficient of viscosity.

Units:

Pressure is measured in Pascals, 1 Pa = 1 N/m^2.

Viscosity is measured in Poise (P), 1 P = 1 Ns/ m^2 = 1 Pas.