Review for Exam 2: Chapters 6 – 9

Physics 1425, Section 1 (Dr. Yost)

Exam 2 will cover chapters 6 through 9. You may use any calculator on the exam, but not notes. You will be given any constants you need, as well as any required derivatives. You should remember basic algebraic and geometric relationships and trigonometric identities. The only physics formulas from chapters 6 through 9 which you need to remember are those on this page. If you need any others, they can be derived quickly from these, or will be given. You may also need to know any of the equations you needed to know for Exam 1.

Chapter 6: Gravitation and Newton's Synthesis

Concepts: Newton's law of universal gravitation, gravity near the Earth's surface, satellite orbits, Kepler's Laws of planetary motion, gravitational fields, the principle of equivalence.

Equations:

Newton's Law of Universal Gravitation: The force between masses m_1 and m_2 is

$$F = G\frac{m_1 m_2}{r^2}. \tag{1}$$

where $G = 6.67 \times 10^{-11}$ N m^2/ kg^2.

Kepler's laws (in the case of circular orbits) are obtained by applying Newton's laws to a light object with mass m_2 orbiting a heavy object with mass m_1, and setting the centripetal force $F = m_2 v^2/r$ equal to the gravitational force.

The gravitational field acting on an object of mass m is $\mathbf{g} = \mathbf{F}/m$. This is also equal to the acceleration of the mass m due to gravity.

Einstein's principle of equivalence states that no local experiment can determine if a perceived acceleration is due to a gravitational field, or due to an accelerated reference frame.

Chapter 7: Work and Energy

Concepts: Work, scalar products, kinetic energy, the work-energy theorem.

Equations:

The scalar product of two vectors **A** and **B** is

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos\theta, \tag{2}$$

where θ is the angle between the two vectors.

The work done by a constant force **F** over a distance **d** is

$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos\theta, \tag{3}$$

where θ is the angle between the force and the displacement.

If the force is not constant, the work is an integral over the path from starting point A to endpoint B:

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{l} = \int_A^B (F_x dx + F_y dy + F_z dz) \tag{4}$$

Kinetic energy is defined to be $K = \frac{1}{2}mv^2$.

The work-energy theorem states that the change in kinetic energy is equal to the net work done on the object by all forces:

$$\Delta K = W_{\text{net}}. \tag{5}$$

Units: Work and kinetic energy are expressed in units of **Joules**, where 1 J = 1 Nm.

Chapter 8: Conservation of Energy

Concepts: Conservative forces, potential energy, energy conservation, dissipative forces, gravitational potential energy, escape velocity, potential energy diagrams, stable and unstable equilibrium.

Equations: A force is conservative if the work it does on an object moving from point A to point B depends only on the two points, and not on the path between them. For a conservative force, potential energy $U(\mathbf{x})$ is defined to keep track of the work done to get from A to B:

$$U(\mathbf{x}_B) - U(\mathbf{x}_A) = -W_{AB} = -\int_A^B \mathbf{F} \cdot d\mathbf{l} \tag{6}$$

Potential energy is normally defined with respect to a reference point A where $U(\mathbf{x}_A)$ is **defined** to be zero. The choice of the zero point of potential energy is always arbitrary, since only changes in potential energy keep track of the work done.

Gravitational potential energy at height h above Earth, with h small compared to Earth's radius:

$$U(h) = mgh. \tag{7}$$

Elastic potential energy for a spring with spring constant k compressed a distance x:

$$U(x) = \frac{1}{2}kx^2. \tag{8}$$

Gravitational potential energy for an object of mass m a distance r from the center of an object of mass M:

$$U(r) = -\frac{GMm}{r}. \tag{9}$$

The force on an object with potential energy $U(x, y, z)$ is given by a derivative of the potential energy function:

$$\mathbf{F}(x, y, z) = -\mathbf{i}\frac{\partial U}{\partial x} - \mathbf{j}\frac{\partial U}{\partial y} - \mathbf{k}\frac{\partial U}{\partial z}. \tag{10}$$

Conservation of energy: The total energy $E = K + U$ is constant if there are no other forces acting on the object than the ones accounted for by the potential energy U. If there are other forces $\mathbf{F}_{\text{ext}}$ not included in U, which do work W_{ext} on the object, then the change in the total energy is given by

$$\Delta E = \Delta K + \Delta U = W_{\text{ext}}. \tag{11}$$

The other forces $\mathbf{F}_{\text{ext}}$ necessarily include any nonconservative forces, since these cannot be expressed using a potential energy.

Average power is the work done per unit time: $\overline{P} = W/t$. Instantaneous power is

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}. \tag{12}$$

Units: Power is measured in **Watts**, where 1 W = 1 J s. Power can also be measured in **horsepower** (HP), with 1 HP = 746 W.

Chapter 9: Linear Momentum and Collisions

Concepts: momentum, conservation of momentum, collisions, impulse, elastic collisions, inelastic collisions, center of mass.

Equations:

Momentum is defined to be $\mathbf{p} = m\mathbf{v}$.

Newton's second law of motion is $F = d\mathbf{p}/dt$.

Impulse is defined to be $\mathbf{J} = \int \mathbf{F}dt = \overline{\mathbf{F}}t$ where $\overline{\mathbf{F}}$ is the average force acting during the time t.

Impulse equals the change in momentum: $\mathbf{J} = \Delta\mathbf{P}$.

Total momentum is constant in any collision: $\mathbf{P}_i = \mathbf{P}_f$.

Kinetic energy is not necessarily constant in a collision. If it is, the collision is said to be **elastic**. If not, the collision is **inelastic**. In a one-dimensional elastic collision between two objects, the initial and final velocities are related by

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}. \qquad \text{(elastic 1-dim. collision)} \tag{13}$$

The location of the center of mass of a set of masses m_i at positions $\mathbf{r}_i$ is given by

$$\mathbf{r}_{\text{CM}} = \frac{\sum m_i \mathbf{r}_i}{M} \tag{14}$$

where $M = \sum m_i$ is the total mass. For a continuous distribution of mass, an integral must be done,

$$\mathbf{r}_{\text{CM}} = \frac{1}{M}\int \mathbf{r}\rho(\mathbf{r})d\mathbf{r}, \tag{15}$$

where $\rho(\mathbf{r})$ is the mass density at point r, and the total mass is $M = \int \rho(\mathbf{r})d\mathbf{r}$.

A collection of objects acted on by a net external force $\mathbf{F}_{\text{ext}}$ will move in such a way that

$$\mathbf{F}_{\text{ext}} = \frac{d\mathbf{P}}{dt} = M\mathbf{a}_{\text{CM}}, \tag{16}$$

where M is the total mass of the objects, $\mathbf{P}$ is the total momentum of the objects, and $\mathbf{a}_{\text{CM}}$ is the acceleration of the center of mass of the objects.