1	2	3	4	Total	Grade
9	12	10	9	40	100

Solutions

Physics 1422

Exam 3 – Version A

General Physics I-A

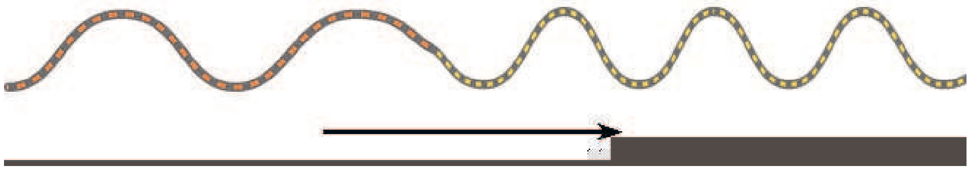
April 20, 2007

All numerical answers require some explanation for credit. Write any formulas used algebraically so that your strategy is clear. For full credit, be sure to show your units. Answers and explanations must be clear and legible for credit. Verbal explanations after the exam will not add points to your score. Only work on the front of the page will be graded unless you request otherwise. You can use the backs for private scratch work or to organize your thoughts.
To maximize partial credit in the event of a wrong answer, do as much of the work algebraically as your reasonably can, and avoid just doing work on your calculator, since I have no way to know what you did unless it is on paper.
If you run out of time and have no time to work a problem, a quick verbal explanation of the steps you would have done if you had time will receive partial credit, if correct. If you need a result from an earlier part of a problem to do a later part, but don't have it, working the problem algebraically will give you most of the credit.

Useful Relations

Sine wave:	$D(x,t) = D_M \sin(kx - \omega t)$ with $k = 2\pi/\lambda$, $\omega = 2\pi f$.
Wave speed:	$v = f\lambda$.
Wave speed string:	$v = \sqrt{F_T/\mu}$.
Harmonics: string or open tube:	$L = n\lambda_n/2$, $n = 1, 2, 3, \ldots$
closed tube:	$L = n\lambda_n/4$, $n = 1, 3, 5, \ldots$
Decibels:	$\text{dB} = 10\ \log_{10}(P/P_0)$ with threshold of hearing $P_0 = 10^{-12}$ W/m^2.
Rotational motion:	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
	$\omega_f = \omega_0 + \alpha t, \qquad \omega_f^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$
Torque:	$\vec{\tau} = \vec{R} \times \vec{F}, \qquad \tau = RF\sin\theta$
Angular momentum:	$\vec{L} = m\vec{R} \times \vec{v}, \qquad L = I\omega$
Relations:	$\vec{\tau} = d\vec{L}/dt, \qquad \tau = I\alpha$
Density:	$\rho = m/V$
Specific gravity:	$\text{SG} = \rho/\rho_w$ where ρ_w = density of water
Density of water:	1000 kg/m^3
Density of air:	1.29 kg/m^3
Pressure:	$P = F/A$
Units:	1 Pa = 1 N/m^2
Atmospheric pressure:	1 atm = 1.01×10^5 Pa
Hydrostatic pressure:	$P = \rho g h$
Buoyant force of water:	$F_B = \rho_w V$
Volume rate of flow:	$Q = Av$
Equation of continuity:	Q is constant
Bernoulli's principle:	$P + \frac{1}{2}\rho v^2 + \rho g h$ is constant

1. A wave travels from left to right, from the red rope to the yellow rope, as shown.



(a) [2pt] How do the frequencies in the two ropes compare?

(a) The frequency is greater in the rope on the left.
(b) The frequency is greater in the rope on the right.
(c) The frequency is the same in both ropes.

Why?

The point where the two ropes join oscillates up and down at the frequency of the wave on both sides of it. This means the frequencies of the waves in both ropes must be the same.

(b) [2pt] How do the wave speeds in the two ropes compare?

(a) The wave speed is greater in the rope on the left.
(b) The wave speed is greater in the rope on the right.
(c) The wave speed is the same in both ropes.

Why?

The wavelength is greater in the rope on the left. Recall that $v = \lambda f$. Since the frequencies are the same, the greater wavelength corresponds to the greater wave speed.

(c) [2pt] How do the linear mass densities of the two ropes compare?

(a) The density is greater in the rope on the left.
(b) The density is greater in the rope on the right.
(c) The density is the same in both ropes.

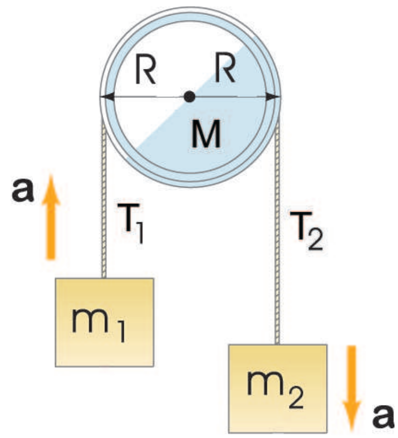
Why?

The wave speed is $v = \sqrt{F/\mu}$. The tension F is the same in both ropes, since they are in static equilibrium horizontally. Since the wave speed is greater on the left, the mass density must be less on the left.

(d) [3pt] If one mosquito at a distance of 1 m is just at the threshold of hearing, how loud would 1000 mosquitos be at the same distance, in decibels?

The threshold of hearing corresponds to 0 dB. If there are 1000 mosquitos at the same distance, the intensity is 1000 times as great, so the loudness is $10 \times \log_{10}(1000) = 30$ dB.

2. Two masses, $m_1 = 3.6$ kg and $m_2 = 5.2$ kg hang from the ends of a string passing over a pulley of mass $M = 12.0$ kg, which is a solid cylinder of radius $R = 31.0$ cm and moment of inertia $I = \frac{1}{2}MR^2$. Neglect friction.



(a) [4pt] Considering only the linear motion of the hanging masses, find relations between the two tensions T_1 and T_2 in the left and right sides of the rope, the masses m_1, m_2, and the acceleration a. Write them symbolically. (You should have two equations.)

The net upward force on m_1 is $T_1 - m_1 g$, and the net downward force on m_2 is $m_2 g - T_2$. Therefore, Newton's law for each mass gives

$$\begin{aligned} T_1 - m_1 g &= m_1 a \ , \\ m_2 g - T_2 &= m_2 a \ . \end{aligned}$$

Equivalently, these may be written as

$$\begin{aligned} T_1 &= m_1(a + g) \ , \\ T_2 &= m_2(g - a) \ . \end{aligned} \tag{1}$$

(b) [4pt] Relate the tensions T_1 and T_2 to the angular accleleration α of the pulley (symbolically) by considering the rotational motion of the pulley.

For the pulley, the net torque $\tau = I\alpha$, with $I = \frac{1}{2}MR^2$. The net torque in the clockwise direction is $\tau = T_2 R - T_1 R$, since $\vec{\tau} = \vec{R} \times \vec{F}$, ant T_2 pulls clockwise while T_1 pulls counterclockwise. Therefore,

$$(T_2 - T_1)R = \frac{1}{2}MR^2\alpha, \tag{2}$$

and cancelling a factor of R on both sides gives

$$T_2 - T_1 = \frac{1}{2}MR\alpha. \tag{3}$$

(c) [4pt] Find the acceleration a of the masses.

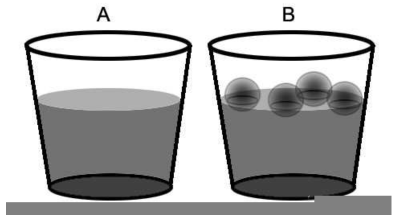
Substituting the expressions in eq. (1) on the left side of eq. (3) and using $R\alpha = a$ gives

$$m_2(g - a) - m_1(g + a) = \frac{1}{2}Ma. \tag{4}$$

Solving for a gives

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{1}{2}M} = \frac{1.6 \times 9.8 \text{ m/s}^2}{14.8} = 1.06 \text{ m/s}^2. \tag{5}$$

3. Two identical cups are filled to the same level, except that one has some plastic balls floating in it, as shown.



(a) [3pt] If 1/4 of each ball's volume is under water in cup B, what is the specific gravity of each ball? Explain.

Since 1/4 of each ball is under water, Archimedes' principle says that the weight of the water occupying 1/4 of the volume of a ball equals the weight of the entire ball. This means that water is 4 times as heavy as the ball, per unit volume, so that the specific gravity of the balls is $1/4 = 0.25$.

(b) [3pt] Which cup is heavier?

(a) The cup with balls is heavier.

(b) The cup without balls is heavier.

(c) Both cups are equally heavy.

Explain.

The weight of the balls in cup B is the same as the weight of the water that would occupy the portion of the balls that is under water. Replacing the balls by this much water would give the same situation as cup A. Therefore, the cups have the same weight.

(c) [4pt] A 5.0 cm diameter pipe gradually narrows to 4.0 cm while maintaining the same height. When water flows through this pipe, the gauge pressure in the two sections is 32.0 kPa and 23.0 kPa. What is the volume rate of flow in the pipe?

Bernoulli's equation says that if $P_1 = 32.0$ kPa on one end and $P_2 = 23.0$ kPa on the other, then

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \tag{6}$$

where $\rho = 1000$ kg/m^3 is the density of water. The volume rate of flow $Q = v_1 A_1 = v_2 A_2$ is the same in each section, where $A_1 = \pi(0.025\text{ m})^2 = 1.96 \times 10^{-3}\text{ m}^2$ and $A_2 = \pi(0.020\text{ m})^2 = 1.26 \times 10^{-3}\text{ m}^2$. Substituting $v = Q/A$ in Bernoulli's equation for both velocities, and putting the pressures on the left gives

$$P_1 - P_2 = \frac{1}{2}\rho Q^2 \left(\frac{1}{(A_2)^2} - \frac{1}{(A_1)^2}\right). \tag{7}$$

Therefore,

$$\begin{aligned} Q^2 &= \frac{2(9.0 \times 10^3\text{ N/m}^2)}{(1000\text{ kg/m}^3)[(1.26 \times 10^{-3}\text{ m}^2)^{-2} - (1.96 \times 10^{-3}\text{ m}^2)^{-2}]} \\ &= \frac{18.0\text{m/s}^2}{(6.30 \times 10^5 - 2.60 \times 10^5)\text{m}^{-4}} = 4.86 \times 10^{-5}\text{m}^6/\text{s}^2. \end{aligned} \tag{8}$$

Taking the square root gives $Q = 6.97 \times 10^{-3}$ m^3/s (or 6.97 L/s).

4. A 75.0 cm guitar string of mass 2.20 g is placed near a tube open at one end but closed at the other, also 75.0 cm long. The tension in the string is adjusted so that its fundamental frequency produces a resonance with the third harmonic of the tube.

(a) [3pt] What is the frequency of the guitar string when in resonance with the tube?

The frequency of the third harmonic of a tube of length $L = 0.75$ m closed on one end is

$$f_3 = \frac{3v_s}{4L} = \frac{3(343 \text{ m/s})}{3.0m} = 343 \text{ Hz}. \tag{9}$$

Therefore, the frequency of the guitar string in resonance with the tube is also 343 Hz.

(b) [3pt] What is the speed of wave propagation along the guitar string?

The speed of wave propagation on the string is $v = f\lambda$, with $f = 343$ Hz and $\lambda = 2L = 1.5$ m is the wavelength of the fundamental vibrational mode, for which half a wave length fits on the string. Therefore, $v = 515$ m/s.

(c) [3pt] How much tension must the string be under?

The wave velocity is given by $v = \sqrt{F_T/\mu}$, where F_T is the tension and

$$\mu = \frac{m}{L} = \frac{2.20 \times 10^{-3} \text{ kg}}{0.75 \text{ m}} = 2.93 \times 10^{-3} \text{ kg/m}. \tag{10}$$

Thus,

$$F_T = \mu v^2 = (2.93 \times 10^{-3} \text{ kg/m})(515 \text{ m/s})^2 = 778 \text{ N}. \tag{11}$$