Review for Exam 3: French Chapters 10 – 11, Giancoli Chapters 15 – 16

Physics 1425, Section 1 (Dr. Yost)

Exam 3 will cover chapters 10 and 11 in French, and Chapters 15 and 16 in Giancoli. Expect 2 – 3 problems. You may use any calculator on the exam, but not notes. You will be given any constants you need, as well as any required derivatives. You should remember basic algebraic and geometric relationships and trigonometric identities. The only physics formulas from the covered chapters which you need to remember are those on this page. If you need any others, they can be derived quickly from these, or will be given. You may also need to know any of the equations you needed to know for Exams 1 and 2.

French Chapter 10: Energy Conservation in Dynamics; vibrational motions

Concepts: Work, energy, power, potential energy, energy in one-dimensional motion, energy of simple harmonic oscillators.

Sections skipped: The Linear Oscillator as a Two-Body Problem, Collision Processes Involving Energy Storage, The Diatomic Molecule

Equations:

The work done by a constant one-dimensional force F over a distance x is

$$W = Fx, \tag{1}$$

If the force F is not constant, the work is an integral of the force $F(x)$ between the starting point x_1 and endpoint x_2

$$W = \int_{x_1}^{x_2} F(x)dx \tag{2}$$

A force does no work if there is no displacement.

The work-energy theorem states that the change in kinetic energy $K = \frac{1}{2}mv^2$ is equal to the net work done on the object by all forces:

$$\Delta K = W_{\text{net}}. \tag{3}$$

Power is the rate of doing work. Average power is the work done per unit time: $\overline{P} = W/t$. In one dimension, Instantaneous power is

$$P = \frac{dW}{dt} = Fv. \tag{4}$$

A force is conservative if the work it does on an object moving from point x_1 to point x_2 depends only on the two points, and not on the path between them. For a one-dimensional conservative force, potential energy $U(x)$ is as the work done against the force to get from x_1 to x_2:

$$U(x_2) - U(x_1) = -W = -\int_{x_1}^{x_2} F(x)dx \tag{5}$$

Potential energy is normally defined with respect to a reference point x_0 where $U(x_0)$ is **defined** to be zero. The choice of the zero point of potential energy is always arbitrary, since only changes in potential energy keep track of the work done.

Conservation of energy: The total mechanical energy $E = K + U$ is constant if there are no other forces acting on the object than the ones accounted for by the potential energy U. If there are other forces $\mathbf{F}_{\text{ext}}$ not included in U, which do work W_{ext} on the object, then the change in the total energy is given by

$$\Delta E = \Delta K + \Delta U = W_{\text{ext}}. \tag{6}$$

The other forces $\mathbf{F}_{\text{ext}}$ necessarily include any nonconservative forces, such as friction, since these cannot be expressed using a potential energy.

Gravitational potential energy at height h above Earth, with h small compared to Earth's radius:

$$U(h) = mgh. \tag{7}$$

Elastic potential energy for a spring with spring constant k compressed a distance x:

$$U(x) = \frac{1}{2}kx^2. \tag{8}$$

If a mass m is in simple harmonic motion, so that $x = A\cos(\omega t)$, then its total energy is $\frac{1}{2}m\omega^2 = \frac{1}{2}kA^2$, where k is the spring constant, with $\omega = \sqrt{k/m}$. (Even if the harmonic motion is not due to an actual spring, an effective spring constant k can be defined by the relation $k = m\omega^2$).

In one dimension, the force on an object with potential energy $U(x)$ is given by a derivative of the potential energy function: $F(x) = -dU/dx$. The force is zero at a local minimum (stable equilibrium point) or local maximum (unstable equilibrium point) of the potential energy. Near a local minimum, the motion can be approximated as a harmonic oscillator. The effective spring constant is $k_{\text{eff}} = -F(x_0)/(x - x_0) = d^2U/dx^2$ at the equilibrium point x_0.

Properties of the motion of a particle acted on by a conservative force can be predicted from a graph of $U(x)$. If the particle has energy E, the motion is confined to a region where $E > U(x)$, and the kinetic energy at point x is $K(x) = E - U(x)$. When $E = U(x_0)$, the particle must stop: this is a *turning point* of the motion. If the particle moves between two turning points on either side of a local minimum of the potential, it will oscillate back and forth continuously. If the amplitude of the motion is small enough, the motion will be approximately simple-harmonic with $k = d^2U/dx^2$, evaluated at the equilibrium point.

Units: Work and energy are expressed in units of **Joules**, where 1 J = 1 Nm. Other common units are the calorie and electron volt. You do not need to remember the conversions, but 1 cal = 4.186 J and 1 eV = 1.602×10^{-19} J. English units are foot-pounds and British thermal units (Btu), where 1 ft·lb = 1.36 J and 1 Btu = 1054 J.

Power is expressed in **Watts**, where 1 W = 1J/s. Another common unit is the horse-power, with 1 hp = 746 W.

French Chapter 11: Conservative Forces and Motion in Space

Concepts: Conservative forces, using force and energy methods together, the pendulum, gravitational potential energy in space, escape velocity

Sections skipped: Gravitating Spherical Shell, A Gravitating Sphere, Fields, Equipotential Surfaces and the Gradient of Potential Energy, Motion in Conservative Fields, The Effect of Dissipative Forces, Gauss's Law

Equations:

The concepts of work and energy can be extended to multi-dimensional systems using vectors:

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta, \tag{9}$$

where θ is the angle between the force and the displacement. If the force is not constant, the work is an integral over the path from starting point A to endpoint B:

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{s} = \int_A^B (F_x dx + F_y dy + F_z dz) \tag{10}$$

Instantaneous power may be written

$$P = \frac{dW}{dt} = vF_{\text{parallel}} = \mathbf{F} \cdot \mathbf{v}. \tag{11}$$

where F_{parallel} is the component of the force parallel to the velocity.

In three dimensions, potential energy is a function of the three coordinates of space: $U(x, y, z)$. The potential can only be defined for conservative forces, for which the work done does not depend on the path. This condition is equivalent to the statement that the work done around any closed path is zero: $W = \int \mathbf{F} \cdot d\mathbf{s} = 0$ for a path ending at its starting point.

The period of small oscillations of a pendulum with length L is $T = 2\pi\sqrt{L/g}$. For large oscillations, the period is somewhat longer than this. (A pendulum is only approximately a harmonic oscillator.)

Gravitational potential energy for an object of mass m a distance r from the center of an object of mass M:

$$U(r) = -\frac{GMm}{r}. \tag{12}$$

The constant in the potential energy is chosen so that $U(r) \to 0$ at $r \to \infty$.

The **escape velocity** is the initial speed needed for an object to escape a gravitational field (to $r = \infty$) starting from a given radius $r = R$ with no additional force. The condition is $K + U(R) = 0$ initially, which means the object gets to $r = \infty$ with zero kinetic energy. The required velocity is $v = \sqrt{2GM/R}$ for an object of mass m. This can also be written as $v = \sqrt{2gR}$, where g is the gravitational acceleration at radius R.

If a satellite is in a circular orbit at radius R, its gravitational and potential energy are related: $U(R) = -2K$. Its total energy is $E = K + U = \frac{1}{2}U(R)$.

Chapter 15: Wave Motion

Sections skipped: Sec. 15-10, 15-11 (and most of 15-5)

Concepts: pulses, waves, amplitude, wavelength, frequency, period, longitudinal waves, transverse waves, wave velocity, energy transmitted by waves, the wave equation, principle of superposition, reflection and transmission, standing waves, resonance, harmonics

Equations:

Any wave or pulse can be represented by a *displacement function* $D(x, t)$ showing how far (D) a particle at position x will be displaced from equilibrium at time t. If the wave or pulse travels with speed v and holds its shape, then $D(x, t) = D(x - vt)$. When v is positive, the wave moves to the right. For a sine wave, the wave velocity is related to the frequency and wavelength by $v = f\lambda = \omega/k$, where the wave number is defined to be $k = 2\pi/\lambda$. A sine wave of amplitude D_M can be represented as $D(x, t) = D_M \sin(kx - \omega t)$. If F_T is the tension on a cord, and μ is the linear mass density (kg/m), then the velocity of transverse waves on the cord is $v = \sqrt{F_T/\mu}$.

The velocity of longitudinal waves on a rod is $v = \sqrt{E/\rho}$, where E is elastic modulus (Young's modulus) and ρ is the mass density (kg/m^3) of the rod. The elastic modulus is defined by the relation $F/A = E\Delta L/L$, where F/A is the force per unit area pulling on a rod, and $\Delta L/L$ is the fractional change in length.

For longitudinal waves in a liquid or gas of density ρ and bulk modulus B, the velocity is $v = \sqrt{B/\rho}$. The bulk modulus is defined by the relation $\Delta P = -B\Delta V/V$, where $\Delta V/V$ is the fractional change in volume when a pressure ΔP (force per unit area) is applied.

The sum of two waves with speed v is also a wave with speed v. The sum of waves is called the *superposition* of the waves. If the amplitude of the sum is greater, the waves are said to *interfere constructively*. If it is less, then they interfere *destructively*.

A wave is reflected from a free end without a phase change, but is reflected from a fixed end with a 180^o phase shift (which means it is inverted).

A standing wave vibrates in place. If the ends of a string are fixed so that the displacement is $y = 0$ at $x = 0$ and $x = L$, then the vibrations can occur at wavelengths $\lambda_n = 2L/n$, $n = 1, 2, 3, \ldots$. The frequency of these vibrations is $f_n = nv/2L$, where v is the speed of wave propagation on the string. The frequency f_n is called the n^{th} *harmonic*, and f_1 is called the *fundamental frequency*.

For a wave on a string, the speed at which a particle at position x moves up and down as the wave goes by at time t is given by

$$v_y(x, t) = -vD'(x - vt) \tag{13}$$

where $D'(x) = dD(x)/dx$ is the derivative of the displacement function giving the shape of the wave at $t = 0$. The transverse force per unit length on the string is given by

$$F_y(x, t) = -\mu v^2 D'(x - vt) = F_T D''(x - vt) \tag{14}$$

where $D''(x) = d^2D(x)/dx^2$ is the second derivative of the displacement function, μ is the linear mass density of the string, and F_T is the string tension.

The power carried by a wave is

$$P = \frac{1}{2} vA\rho\omega^2 D_M^2 \tag{15}$$

where $\omega = 2\pi f$ is the angular frequency and D_M is the amplitude, ρ is the density of the medium, v is the wave speed, and A is the cross-sectional area. The *intensity* is the average power per unit area: $I = \overline{P}/A$. The most important features of the power relation are that the intensity is proportional to the square of the frequency and the square of the amplitude.

The intensity of a spherical wave variea according to the *inverse square law*. At two distances r_1 and r_2, the intensities are related by the inverse square law, $I_2/I_1 = (r_1/r_2)^2$. Since the intensity is proportional to the square of the amplitude, the amplitude is inversely proportional to the distance from the source: $D_{M2}/D_{M1} = r_1/r_2$.

The frequency of a wave is measured in Hertz, 1 Hz = 1 s^{-1}. The intensity of a wave is measured in W/m^2.

Chapter 16: Sound

Sections skipped: 16-7, 16-8 (and the derivations in 16-2)

Concepts: loudness, pitch, audible range, pressure waves, decibels, string instruments, wind instruments, overtones, harmonics, open pipes, closed pipes, interference, beats

Equations:

The speed of sound in air is approximately $v \approx (331 + 0.60\ T)$ m/s, with T in Celcius degrees. At room temperature, 20°C, the speed of sound is 343 m/s. You don't need to remember this.

The **pitch** of a sound is determined by its frequency. Higher musical notes have higher frequency. Music is divided into octaves, with each octave corresponding to a doubling of frequency. The range of normal human hearing is approximately 20 Hz to 20 kHz.

The **sound level** in decibels is given by

$$\beta(\text{in dB}) = 10 \log_{10} \frac{I}{I_0} \tag{16}$$

where I is the intensity of the sound wave in W/m^2 and I_0 is a reference intensity, normally taken to be the threshold of human hearing, or 1.0×10^{-12} W/m^2. An increase of 10 dB corresponds to a factor of 10 in intensity. A doubling of power corresponds to about 3 dB. The threshold of pain is about 120 dB, which corrseponds to an intensity of 1 W/m^2.

The fundamental vibrational mode of a stringed instrument is $\lambda = 2L$, where L is the length of the string. The overtones or harmonics are at wavelengths $\lambda_n = 2L/n$, for $n = 1, 2, 3, \ldots$.

A tube open on both ends also produces wavelengths $\lambda_n = 2L/n$. A tube closed on one end produces wavelengths $\lambda_n = 4L/(2n - 1)$. An open tube produces all integer multiples of the fundamental frequency $f_1 = v/2L$, while a tube closed on one end produces only the odd integer multiples of $f_1 = v/4L$, so that $f_n = nf_1$ with $n = 1, 3, 5, \ldots$. Whether the tube is open or closed, the integer $n = f_n/f_1$ is called the *order* the harmonic, and the difference in frequency between two successive harmonics is $\Delta f = v/2L$.

If two sounds are produced simultaneously with frequencies f_1 and f_2, they will interfere and produce "beats" with frequency $|f_1 - f_2|$.

Units:

Decibels (dB) are used to measure sound intensity logarithmically.