Review for Exam 2: French Chapters 6 – 9

Physics 1422 (Dr. Yost)

Exam 2 will cover chapters 6 through 9 in French's text. You may use any calculator for this exam, but not notes. You will be given any constants and conversion factors needed. You should remember basic algebraic and geometric relationships and trigonometric identities. Any derivatives needed will be given, but you should know how to use them.

The essence of physics is learning to apply basic physical and mathematical concepts to analyzing new situations. Every situation is different, so memorizing specific solution techniques is pointless. What matters is to get as much practice as possible with a wide variety of problems to develop your analytical skills. You will want to remember the basic equations presented in these notes, but the exam will primarily test reasoning, not memorization. Physical equations are usually very easy to remember once you truly understand them. Drawing pictures often helps – remember this when you work the exam.

Chapter 6: Force, Inertia, and Motion

Sections skipped: The Invariance of Newton's Law, Invariance with Specific Force Laws, Newton's Law and Time Reversal.

Concepts: inertia, inertial frames, inertial mass, Newton's law, impulse and work,

In an **inertial reference frame**, an object free of a net external force moves in a straight line at constant velocity.

Newton's Law: $F = ma$.

Work is the effect of applying a force over a distance: $W = Fx$. Work is measured in Joules, where $1J = 1Nm$. Work applied to a stationary object causes it to acquire **kinetic energy** $\frac{1}{2}mv^2$ equal to the amount of work:

$$Fx = max = \frac{1}{2}mv^2. \tag{1}$$

Impulse is the effect of applying a force for a time interval: Ft. An impulse applied to a stationary object causes it to acquire **momentum** mv equal to the size of the impulse:

$$Ft = mat = mv. \tag{2}$$

These equations for work, kinetic energy, impulse and momentum are just different ways of expressing the effect of Newton's laws. They are expressed above for constant forces and acceleration in one dimension, but will be generalized in later chapters.

Chapter 7: Using Newton's Law

Sections skipped: Circular Paths of Charged Particles in Uniform Magnetic Fields, Charged Particle in a Magnetic Field, Mass Spectrographs, Fracture of Rapidly Rotating Objects, Detailed Analysis of Resisted Motion, Motion Governed by Viscosity, Growth and Decay of Resisted Motion.

Concepts: solving problems using Newton's law, centripetal force, curvilinear motion, resisted motion, simple harmonic motion

Problems are solved using a combination of Newton's laws and the kinematic equations governing velocity and acceleration studied in chapters 2 and 3. Remember especially the equations for constant acceleration,

$$v = v_0 + at, \qquad x = v_0 t + \frac{1}{2}at^2, \qquad v^2 - v_0^2 = 2ax, \tag{3}$$

and the equation for centripetal acceleration

$$a = v^2/R \tag{4}$$

in motion with radius of curvature R.

If there is no acceleration in a direction, the forces are balanced in that direction. This is true if the object is at rest or moving at constant velocity.

Always draw a diagram when solving force equations. Use **isolation diagrams** to clarify specifically which forces act on a component of interest. In problems with several components, you will need several isolation diagrams to solve all equations.

In addition to Newton's law, be sure to use any geometrical constraints in the problem.

The net force on any massless object must be zero. If a string is massless, the tension force is the same at either end (and any point in the middle). The tension on either side of a massless pulley must be equal.

A scale measures the normal force pressing on it.

If an object is moving in a circle of radius R, the net force toward the center of the circle must be $F_c = mv^2/R$. This is called the **centripetal force** on the object. It is not an independent force somehow arising from the motion, but a consequence of all forces acting on the object.

There may also be acceleration along the path, tangential to the circle. In this case, the tangential acceleration is due to the tangential component of force on the object, $F_t = ma_t$. This acceleration (and force) is always perpendicular to the centripetal acceleration (and force). The equation for centripetal force applies whether or not the object is acclererating along its path, and whether or not the radius of curvature is changing.

When an object is forced to move against fluid resistance, it will acquire a limiting speed, or terminal velocity, determined by balancing the force F pushing the object (which would be mg for a falling object) against the resistive force $R(v)$ due to viscosity and turbulence: $F = Av + Bv^2$. Often only one of the A and B terms is significant. For air resistance on normal-sized objects, the B term dominates

The force F exerted by a spring when it is extended a distance x is given by **Hooke's Law**: $F = -kx$, where k is the **spring constant** measured, typically, in N/m.

Simple harmonic motion: If an object on a spring is moved from its equilibrium position x_0 and then released, possibly with some initial velocity, it will oscillate about the point x_0 with frequency f, where

$$\omega = 2\pi f \quad \text{where} \quad \omega^2 = k/m. \tag{5}$$

The parameter ω is the **angular frequency** in radians per second. The frequency is the inverse of the period: $f = 1/T$. The equation of motion is

$$x - x_0 = A\sin(\omega t + \phi) \tag{6}$$

where A is the **amplitude**, ω and ϕ is an initial phase determined by the initial displacement and velocity. The velocity and acceleration are the first and second derivatives of the position:

$$v = A\omega\cos(\omega t + \phi), \qquad a = -A\omega^2\sin(\omega t + \phi). \tag{7}$$

There is no need to remember these if you can remember how to differentiate sines and cosines.

In simple harmonic motion, the maximum force occurs at the turning points, while the maximum speed occurs when the object crosses its equilibrium position.

Chapter 8: Universal Gravitation

Sections skipped: Finding the Distance to the Moon, The Gravitational Attraction of a Large Sphere, Finding the Distance to the Sun, The Discovery of Neptune, Einstein's Theory of Gravitation.

Concepts: universal gravitation, Kepler's third law, mass and weight, weightlessness

Newton's Law of Universal Gravitation:

$$F = G\frac{m_1 m_2}{R^2}. \tag{8}$$

Circular orbits: the force of gravity is the centripetal force keeping the planet in orbit:

$$G\frac{m_1 m2}{R^2} = \frac{m_2 v^2}{R} \tag{9}$$

if the mass m_2 is orbiting mass m_1.

Kepler's Third Law: The square of a planets orbital period is proportional the the third power of its orbital radius. Kepler's law is a consequence of the previous equations, using $T = 2\pi R/v$, so it doesn't need to be remembered independently. If R is the radius of a planet's orbit in astronomical units and T is its period in Earth years, $T^2 = R^3$. An astronomical unit is the distance of the earth from the sun: $1AU = 150 \times 10^6\ km$.

On any spherical planet of mass M and radius R, the acceleration of gravity on the surface is $g = GM/R^2$. For Earth, this gives $g = 9.8\ m/s^2$. An object's weight is mg, which depends both on its mass and on the strength of gravity. Kilograms are a unit of mass, but pounds are a unit of force, so the conversion between them depends on where you are. The MKS unit of weight is the Newton. An object inside a vessel which is falling with the acceleration of gravity will feel weightless.

French Chapter 9: Collisions and Conservation Laws

Concepts: momentum, conservation of momentum, collisions, impulse, force exerted by a stream of particles, elastic collisions, inelastic collisions, center of mass frame

Sections skipped: Rocket Propulsion, Inelastic and Explosive Processes, The Pressure of a Gas

Equations:

Momentum is defined to be $\mathbf{p} = m\mathbf{v}$.

Newton's second law of motion may be written $F = d\mathbf{p}/dt$.

Impulse is defined to be $\mathbf{J} = \int \mathbf{F}dt = \overline{\mathbf{F}}t$ where $\overline{\mathbf{F}}$ is the average force acting during the time t.

Impulse equals the change in momentum: $\mathbf{J} = \Delta\mathbf{P}$.

If particles of mass m travel at velocity v and transfer all their momentum to an object, the force on the object is $F = \mu v$, where μ is the mass per unit time striking the object. Total momentum is constant in any collision: $\mathbf{P}_i = \mathbf{P}_f$.

If the relative speed between two objects is the same before and after the collision, then the collision is said to be **elastic**. If the objects stick together, the collision is **inelastic**. In a one-dimensional elastic collision between two objects, the initial and final velocities are related by

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}. \qquad \text{(elastic 1-dim. collision)} \tag{10}$$

The kinetic energy is conserved in elastic collisions.

The location of the center of mass of a set of masses m_i at positions $\mathbf{r}_i$ is given by

$$\mathbf{r}_{\mathrm{CM}} = \frac{\sum m_i \mathbf{r}_i}{M} \tag{11}$$

where $M = \sum m_i$ is the total mass. For a continuous distribution of mass, an integral must be done,

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M} \int \mathbf{r} \rho(\mathbf{r}) d\mathbf{r}, \tag{12}$$

where $\rho(\mathbf{r})$ is the mass density at point r, and the total mass is $M = \int \rho(\mathbf{r}) d\mathbf{r}$.

A collection of objects acted on by a net external force $\mathbf{F}_{\text{ext}}$ will move in such a way that

$$\mathbf{F}_{\text{ext}} = \frac{d\mathbf{P}}{dt} = M\mathbf{a}_{\mathrm{CM}}, \tag{13}$$

where M is the total mass of the objects, $\mathbf{P}$ is the total momentum of the objects, and $\mathbf{a}_{\mathrm{CM}}$ is the acceleration of the center of mass of the objects.

The **center of mass frame** is the reference frame of an observer traveling along with the center of mass of a set of objects. The velocity of the CM is given by $\mathbf{V}_{\mathrm{CM}} = \mathbf{P}/M$ where $\mathbf{P}$ is the total momentum and M is the total mass of the system.

When a collision is viewed in the center of mass frame, the total momentum is always zero. For this reason, the center of mass frame is also called the **zero-momentum frame**. When a elastic collision of two objects is viewed in the center of mass frame, the incoming and outgoing speeds of each object are the same, but the direction of travel changes.