Review for Exam 3: French Chapters 9 – 11, Giancoli Chapters 15 – 16

Physics 1425, Section 1 (Dr. Yost)

Exam 3 will cover chapters 9 through 11 in French, and Chapters 15 and 16 in Giancoli. Expect 2 – 3 problems. You may use any calculator on the exam, but not notes. You will be given any constants you need, as well as any required derivatives. You should remember basic algebraic and geometric relationships and trigonometric identities. The only physics formulas from the covered chapters which you need to remember are those on this page. If you need any others, they can be derived quickly from these, or will be given. You may also need to know any of the equations you needed to know for Exams 1 and 2.

French Chapter 9: Collisions and Conservation Laws

Concepts: momentum, conservation of momentum, collisions, impulse, force exerted by a stream of particles, elastic collisions, inelastic collisions, center of mass frame

Sections skipped: Rocket Propulsion, Inelastic and Explosive Processes, The Pressure of a Gas

Equations:

Momentum is defined to be $\mathbf{p} = m\mathbf{v}$.

Newton's second law of motion may be written $F = d\mathbf{p}/dt$.

Impulse is defined to be $\mathbf{J} = \int \mathbf{F} dt = \overline{\mathbf{F}} t$ where $\overline{\mathbf{F}}$ is the average force acting during the time t.

Impulse equals the change in momentum: $\mathbf{J} = \Delta\mathbf{P}$.

If particles of mass m travel at velocity v and transfer all their momentum to an object, the force on the object is $F = \mu v$, where μ is the mass per unit time striking the object. Total momentum is constant in any collision: $\mathbf{P}_i = \mathbf{P}_f$.

Kinetic energy is defined to be $K = \frac{1}{2}mv^2$. The total kinetic energy is not necessarily constant in a collision. If it is, the collision is said to be **elastic**. If not, the collision is **inelastic**. In a one-dimensional elastic collision between two objects, the initial and final velocities are related by

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}. \qquad \text{(elastic 1-dim. collision)} \qquad (1)$$

The location of the center of mass of a set of masses m_i at positions $\mathbf{r}_i$ is given by

$$\mathbf{r}_{\text{CM}} = \frac{\sum m_i \mathbf{r}_i}{M} \tag{2}$$

where $M = \sum m_i$ is the total mass. For a continuous distribution of mass, an integral must be done,

$$\mathbf{r}_{\text{CM}} = \frac{1}{M} \int \mathbf{r} \rho(\mathbf{r}) d\mathbf{r}, \tag{3}$$

where $\rho(\mathbf{r})$ is the mass density at point r, and the total mass is $M = \int \rho(\mathbf{r}) d\mathbf{r}$.

A collection of objects acted on by a net external force $\mathbf{F}_{\text{ext}}$ will move in such a way that

$$\mathbf{F}_{\text{ext}} = \frac{d\mathbf{P}}{dt} = M \mathbf{a}_{\text{CM}}, \tag{4}$$

where M is the total mass of the objects, $\mathbf{P}$ is the total momentum of the objects, and $\mathbf{a}_{\text{CM}}$ is the acceleration of the center of mass of the objects.

The **center of mass frame** is the reference frame of an observer traveling along with the center of mass of a set of objects. When a collision is viewed in the center of mass frame, the total momentum is always zero. For this reason, the center of mass frame is also called the **zero-momentum frame**. When a elastic collision of two objects is viewed in the center of mass frame, the incoming and outgoing speeds of each object are the same, but the direction of travel changes.

French Chapter 10: Energy Conservation in Dynamics; vibrational motions

Concepts: Work, energy, power, potential energy, energy in one-dimensional motion, energy of simple harmonic oscillators.

Sections skipped: Some Examples of the Energy Method, The Linear Oscillator as a Two-Body Problem, Collision Processes Involving Energy Storage, The Diatomic Molecule

Equations:

The work done by a constant force F over a distance d is

$$W = Fd\cos\theta, \tag{5}$$

where θ is the angle between the force and the displacement. A force does no work if there is no displacement. A force perpendicular to the motion also does no work.

If the force F is not constant, the work is an integral of the x component F_x of the force between the starting point x_1 and endpoint x_2

$$W = \int_{x_1}^{x_2} F_x(x)dx \tag{6}$$

The work-energy theorem states that the change in kinetic energy $K = \frac{1}{2}mv^2$ is equal to the net work done on the object by all forces:

$$\Delta K = W_{\text{net}}. \tag{7}$$

Power is the rate of doing work. Average power is the work done per unit time: $\overline{P} = W/t$. Instantaneous power is

$$P = \frac{dW}{dt} = vF_{\text{parallel}} \tag{8}$$

where F_{parallel} is the component of force parallel to v.

A force is conservative if the work it does on an object moving from point x_1 to point x_2 depends only on the two points, and not on the path between them. For a conservative force, potential energy $U(x)$ is defined to keep track of the work done to get from x_1 to x_2:

$$U(x_2) - U(x_1) = -W = -\int_{x_1}^{x_2} F_x dx \tag{9}$$

Potential energy is normally defined with respect to a reference point x_0 where $U(x_0)$ is **defined** to be zero. The choice of the zero point of potential energy is always arbitrary, since only changes in potential energy keep track of the work done.

Conservation of energy: The total mechanical energy $E = K + U$ is constant if there are no other forces acting on the object than the ones accounted for by the potential energy U. If there are other forces $\mathbf{F}_{\text{ext}}$ not included in U, which do work W_{ext} on the object, then the change in the total energy is given by

$$\Delta E = \Delta K + \Delta U = W_{\text{ext}}. \tag{10}$$

The other forces $\mathbf{F}_{\text{ext}}$ necessarily include any nonconservative forces, since these cannot be expressed using a potential energy.

Gravitational potential energy at height h above Earth, with h small compared to Earth's radius:

$$U(h) = mgh. \tag{11}$$

Elastic potential energy for a spring with spring constant k compressed a distance x:

$$U(x) = \frac{1}{2}kx^2. \tag{12}$$

If a mass m is in simple harmonic motion, so that $x = A\cos(\omega t)$, then its total mechanical enrgy is $\frac{1}{2}m\omega^2 = \frac{1}{2}kA^2$, where k is the spring constant, with $\omega = \sqrt{k/m}$. (Even if the harmonic motion is not due to an actual spring, an effective spring constant k can be defined by the relation $k = m\omega^2$).

In one dimension, the force on an object with potential energy $U(x)$ is given by a derivative of the potential energy function: $\mathbf{F}(z) = -dU/dx$. The force is zero at a local minimum (stable equilibrium point) or local maximum (unstable equilibrium point). Near a local minimum, the motion can always be approximated as a harmonic oscillator.

Units: Work and energy are expressed in units of **Joules**, where 1 J = 1 Nm. Other common units are the calorie and electron volt. You do not need to remember the conversions, but 1 cal = 4.186 J and 1 eV = 1.602×10^{-19} J. English units are foot-pounds and British thermal units (Btu), where 1 ft·lb = 1.36 J and 1 Btu = 1054 J.

Power is expressed in **Watts**, where 1 W = 1J/s. Another common unit is the horse-power, with 1 hp = 746 W.

French Chapter 11: Conservative Forces and Motion in Space

Concepts: Conservative forces, using force and energy methods together, the pendulum, gravitational potential energy in space, escape velocity

Sections skipped: An Experiment by Galileo, Mass on a Parabolic Track, A Gravitating Spherical Shell, A Gravitating Sphere, Fields, Equipotential Surfaces and the Gradient of Potential Energy, Motion in Conservative Fields, The Effect of Dissipative Forces, Gauss's Law

Equations:

The concepts of work and energy can be extended to multi-dimensional systems using vectors:

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta, \tag{13}$$

where θ is the angle between the force and the displacement. If the force is not constant, the work is an integral over the path from starting point A to endpoint B:

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{s} = \int_A^B (F_x dx + F_y dy + F_z dz) \tag{14}$$

Instantaneous power may be written

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}. \tag{15}$$

In three dimensions, potential energy is a function of the three coordinates of space: $U(x, y, z)$. The potential can only be defined for conservative forces, for which the work done does not depend on the path. This condition is equivalent to the statement that the work done around any closed path is zero: $W = \int \mathbf{F} \cdot d\mathbf{s} = 0$ for a path ending at its starting point.

The period of small oscillations of a pendulum with length L is $T = 2\pi\sqrt{L/g}$. For large oscillations, the period is somewhat longer than this. (A pendulum is only approximately a harmonic oscillator.)

Gravitational potential energy for an object of mass m a distance r from the center of an object of mass M:

$$U(r) = -\frac{GMm}{r}. \tag{16}$$

The **escape velocity** is the initial speed needed for an object to escape a gravitational field (to $r = \infty$) starting from a given radius $r = R$ with no additional force. The condition is $K + U(R) = 0$ initially, which means the object gets to $r = \infty$ with zero kinetic energy. The required velocity is $v = \sqrt{2GM/R}$ for an object of mass m. This can also be written as $v = \sqrt{2gR}$, where g is the gravitational acceleration at radius R.

Chapter 15: Wave Motion

Sections skipped: Parts of Sec. 15-2: Velocity of Transverse Waves, Velocity of Longitudinal Waves; Sec. 15-10, Sec. 15-11

Concepts: pulses, waves, amplitude, wavelength, frequency, period, longitudinal waves, transverse waves, wave velocity, energy transmitted by waves, the wave equation, principle of superposition, reflection and transmission, standing waves, resonance, harmonics

Equations:

The wave velocity is related to the frequency and wavelength by $v = f\lambda = \omega/k$, where the wave number is defined to be $k = 2\pi/\lambda$. A sine wave of amplitude D_M can be represented as $D(x,t) = D_M \sin(kx - \omega t)$. If F_T is the tension on a cord, and μ is the linear mass density (kg/m), then the velocity of transverse waves on the cord is $v = \sqrt{F_T/\mu}$.

The intensity of a spherical wave is the average power transmitted per unit area: $I = \overline{P}/A$. The power transmitted is proportional to f^2 and A^2, if f is the frequency and A is the amplitude. At two distances r_1 and r_2, the intensities are related by the inverse square law, $I_2/I_1 = (r_1/r_2)^2$.

If t and x are the time and position where a wave is measured, and y is the wave's displacement (in any units relevant for the wave), the wave equation says that

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial t^2} = 0, \tag{17}$$

where v is the wave velocity.

A wave is reflected from a free end without a phase change, but is reflected from a fixed end with a 180^o phase shift (which means it is inverted).

A standing wave vibrates in place. If the ends of a string are fixed so that the displacement is $y = 0$ at $x = 0$ and $x = L$, then the vibrations can occur at wavelengths $\lambda_n = 2L/n$, $n = 1, 2, 3, \ldots$. The frequency of these vibrations is $f_n = nv/2L$, where v is the speed of wave propagation on the string. The frequency f_n is called the n^{th} harmonic, and f_1 is called the fundamental frequency.

If one end of the string is open, then $\partial y/\partial x = 0$ at that end of the string, and the vibrations occur at $\lambda_n = 4L/(2n-1)$, $n = 1, 2, 3, \ldots$. The frequency of these vibrations is $f_n = (2n-1)v/4L$.

The frequency of a wave is measured in Hertz, 1 Hz = 1 s^{-1}. The intensity of a wave is measured in W/m^2.

Chapter 16: Sound

Sections skipped: 16-2, 16-7, 16-8, 16-9 **Concepts:** loudness, pitch, audible range, pressure waves, decibels, string instruments, wind instruments, overtones, harmonics, open pipes, closed pipes, interference, beats

Equations:

The speed of sound in air is approximately $v \approx (331 + 0.60T)$ m/s, with T in Celcius degrees. At room temperature, 20°C, the speed of sound is 343 m/s. You don't need to remember this.

The **pitch** of a sound is determined by its frequency. Higher musical notes have higher frequency. Music is divided into octaves, with each octave corresponding to a doubling of frequency. The range of normal human hearing is approximately 20 Hz to 20 kHz.

The **sound level** in decibels is given by

$$\beta(\text{in dB}) = 10 \log_{10} \frac{I}{I_0} \tag{18}$$

where I is the intensity of the sound wave in W/m^2 and I_0 is a reference intensity, normally taken to be the threshold of human hearing, or 1.0×10^{-12} W/m^2. A doubling of power corresponds to about 3 dB, which is perceived as a small change in loudness. The threshold of pain is about 120 dB, which corrseponds to an intensity of 1 W/m^2.

The fundamental vibrational mode of a stringed instrument is $\lambda = 2L$, where L is the length of the string. The overtones or harmonics are at wavelengths $\lambda_n = 2L/n$, for $n = 1, 2, 3, \cdots$.

A tube open on both ends also produces wavelengths $\lambda_n = 2L/n$. A tube closed on one end produces wavelengths $\lambda_n = 4L/(2n-1)$. An open tube produces all integer multiples of the fundamental frequency $f_1 = v/2L$, while a tube closed on one end produces only the odd integer multiples of $f_1 = v/4L$. If two sounds are produced simultaneously with frequencies f_1 and f_2, they will interfere and produce "beats" with frequency $|f_1 - f_2|$.

Units:

Decibels (dB) are used to measure sound intensity logarithmically. They are dimensionless. A 1 dB increase in power corresponds to increasing the intensity by a factor of $10^{0.1} \approx 1.259$.