Review for Exam 1: French Chapters 2 – 4

Physics 1422 (Dr. Yost)

Exam 1 will cover chapters 2 through 4 in French's text. You may use any calculator for this exam, but not notes. You will be given any constants and conversion factors needed. You should remember basic algebraic and geometric relationships and trigonometric identities. Any derivatives needed will be given, but you should know how to use them. There is a review of basic calculus at the end of these notes.

The essence of physics is learning to apply basic physical and mathematical concepts to analyzing new situations. Every situation is different, so memorizing specific solution techniques is pointless. What matters is to get as much practice as possible with a wide variety of problems to develop your analytical skills. You will want to remember the basic equations presented in these notes, but the exam will primarily test reasoning, not memorization. Physical equations are usually very easy to remember once you truly understand them. Drawing pictures often helps – remember this when you work the exam.

Chapter 2: Space, Time, and Motion

Concepts: frames of reference, coordinate systems, polar and spherical coordinates, vectors, vector addition, scalar product of vectors, average and instantaneous velocity and speed, relative velocity and relative motion

Equations:

For a two-dimensional vector $\mathbf{V}$ with components V_x, V_y, and angle θ measured counterclockwise from the x axis,

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta, \tag{1}$$

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \tag{2}$$

V is called the *magnitude* of the vector $\mathbf{V}$. When inverting the tangent to find the angle θ, remember to be sure to check what quadrant the answer should be in. The normal inverse tangent assumes $V_x > 0$. Otherwise, you must add 180° to the angle θ.

For a three-dimensional vector $\mathbf{V}$, the components V_x, V_y, V_z can be expressed in terms of the magnitude V, polar angle θ (measured from the north pole toward equator), and azimuthal angle ϕ (measured "east to west" around the equator) as

$$V_x = V \sin\theta \cos\phi, \quad V_y = V \sin\theta \sin\phi, \quad V_z = V \cos\theta, \tag{3}$$

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}, \tag{4}$$

$$\tan\theta = \frac{V_z}{\sqrt{V_x^2 + V_y^2}}, \qquad \tan\phi = \frac{V_y}{V_x}. \tag{5}$$

On the surface of the earth, positions can be represented by vectors of length R_e, the radius of the earth, with polar angles described by latitude and longitude. Specifically, latitude is $90^\circ - \theta$, and longitude is the azimuthal angle ϕ, measured to the west from the prime meridian. East longitudes have negative ϕ. The great-circle distance between two points 1 and 2 on the earth's surface is $R_e\theta_{12}$, where θ_{12} is the angle (in radians) between vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ pointing from the center of the earth to the two points on the surface: $\cos\theta_{12} = \mathbf{R}_1 \cdot \mathbf{R}_2 / R_e^2$.

When vectors are added or subtracted, each component of the vector is added or subtracted independently: $\mathbf{A} \pm \mathbf{B}_i = A_i \pm B_i$.

The *scalar product* (dot product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$. The magnitude of a vector is related to its square using this product: $A = \sqrt{\mathbf{A} \cdot \mathbf{A}}$. The scalar product of two vectors may be written in terms of the magnitudes A and B and the angle θ between the vectors as $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$. If $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, note that $\mathbf{A} \cdot \mathbf{B} = 0$.

Average velocity: $\overline{\mathbf{v}}$ = displacement vector/time = $\Delta\mathbf{r}/\Delta t$.

Average speed = total distance traveled (along path)/time.

The notation for average speed and velocity are often similar, so you must distinguish them by context. However, the concepts are distinct: The average velocity of a trip that begins and ends at the same point is always zero, but the average speed is the length of the path taken divided by the time.

Instantaneous velocity: $\mathbf{v} = d\mathbf{r}/dt$.

Take the derivative of each component of the position vector independently to get the components of the velocity vector. In one dimension, the velocity is the slope of a graph of the position as a function of time, $v = dx/dt$.

The *relative velocity* $\mathbf{v}_{12}$ of object 1 relative to object 2 is the velocity of object 1 in the reference frame of object 2. Relative velocities add. If object 1 is moving with velocity $\mathbf{v}_{12}$ with respect to object 2, and object 2 is moving with velocity $\mathbf{v}_{23}$ with respect to object 3, then the velocity of object 1 with respect to object 3 is the vector sum $\mathbf{v}_{13} = \mathbf{v}_{12} + \mathbf{v}_{23}$.

Chapter 3: Accelerated Motions

Concepts: acceleration, straight-line motion, two-dimensional trajectories, projectile motion, falling objects, uniform circular motion, acceleration in polar coordinates

Equations:

Average acceleration = change in velocity/time = $\Delta\mathbf{v}/\Delta t$.

Instantaneous acceleration: $\mathbf{a} = d\mathbf{v}/dt$.

Take the derivative of each component of the velocity independently to calculate the components of the acceleration. In one dimension, the acceleration is the slope of a graph of the instantaneous velocity as a function of time: $a = dv/dt = d^2x/dt^2$.

For constant acceleration:

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{v}_0 + \mathbf{a}t \\ \mathbf{r}(t) &= \mathbf{r}_0 + \mathbf{v}_0 t + \mathbf{a}t_2/2 \\ v^2 &- {v_0}^2 = 2\mathbf{a}\cdot(\mathbf{r}-\mathbf{r}_0) \end{aligned} \tag{6}$$

The acceleration due to gravity near the Earth's surface is $g = 9.80$ m/s^2 = 32.0 ft/s^2 directed downward. This is independent of the object.

For general acceleration:

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{v}_0 + \int_{t_0}^{t} \mathbf{a}(t)dt, \\ \mathbf{x}(t) &= \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(t)dt. \end{aligned} \tag{7}$$

Each component is integrated independently.

In projectile motion, the x component is described by a constant velocity, and the y component is described by constant downward acceleration g, as for any falling body. The two components of motion are completely independent.

The position of an object in uniform circular motion can be described by a vector $\mathbf{r}(t) = \mathbf{i}r\cos(\omega t + \phi) + \mathbf{j}r\sin(\omega t + \phi)$ where ϕ specifies the initial position on the circle at $t = 0$. The radial unit vector $\mathbf{e}_r = \mathbf{r}(t)/r$ points in the direction of the moving object.

A perpendicular vector $\mathbf{e}_\theta = -\mathbf{i}\sin(\omega t + \phi) + \mathbf{j}\cos(\omega t + \phi)$ points in the direction of the motion, tangential to the circle. The velocity in uniform circular motion may be written $\mathbf{v}(t) = v\mathbf{e}_\theta$.

The period T of uniform circular motion is the time to make one revolution. The angular velocity is defined as $\omega = 2\pi/T$. The ordinary velocity may be expressed as $v = R\omega$.

In uniform circular motion, there is an inward-pointing acceleration vector $\mathbf{a}_c = -\mathbf{e}_r v^2/R$ called the centripetal acceleration. The magnitude is $a_c = v^2/R = R\omega^2$.

Chapter 4: Forces in Equilibrium

Concepts: forces, static equilibrium, forces as vectors, units of force, action and reaction, rotational equilibrium, torque, weight, pulleys and strings

Equations:

Forces add as vectors. An object is in static equilibrium if the sum of all forces on it add to zero, so there is no *net force* ($\sum \mathbf{F}_i = 0$), and there is no net influence which would cause it to rotate. (Rotations are considered below.)

Forces are measured in *Newtons* (N) or *pounds* (lb). Newtons are a derived unit: 1 N = 1 kg m/s^2. Pounds are related to Newtons according to 1 lb = 4.45 N.

Any time an object 1 exerts a force $\mathbf{F}_{12}$ on object 2, object 2 exerts an equal and opposite force on object 1: $\mathbf{F}_{21} = -\mathbf{F}_{12}$. This is true whether or not the forces on either object are in equilibrium.

Forces can cause an object to rotate if they are applied at different points. The tendency of a force to induce rotation is quantified by torque. If a force $\mathbf{F}$ acts at a point described by a vector $\mathbf{r}$ measured from pivot point P, the torque due to $\mathbf{F}$ about the point P has magnitude $M = rF\sin\phi$, where ϕ is the angle from $\mathbf{r}$ to $\mathbf{F}$. Note that a force directed toward or away from P creates no torque about P.

The *vector product* (cross product) of any three-dimensional vectors $\mathbf{A}$ and $\mathbf{B}$ is defined by $\mathbf{A} \times \mathbf{B} = AB\sin\phi$, if ϕ is the angle from $\mathbf{A}$ to $\mathbf{B}$. The direction is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$, and the sign is obtained from the "right-hand rule," which says that if the fingers of the right hand are placed along $\mathbf{A}$ and curled toward $\mathbf{B}$, the thumb points in the direction of $\mathbf{A} \times \mathbf{B}$. In components,

$$\mathbf{A} \times \mathbf{B} = \mathbf{i}(A_y B_z - B_y A_z) + \mathbf{j}(A_z B_y - B_z A_y) + \mathbf{k}(A_x B_y - B_x A_y). \tag{8}$$

The vector cross product is bilinear, distributive and *anti*-commutative ($\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$). It is *not* associative, and exists *only* in three dimensions.

The torque due to a force $\mathbf{F}$ acting at a point given by a vector $\mathbf{r}$ measured from a pivot point P may be written as a vector product $\mathbf{M} = \mathbf{r} \times \mathbf{F}$. The torque $\mathbf{M}$ is directed along the axis of the rotation induced by the torque.

An object is in rotational equilibrium about a point P if the sum of all torques about P is zero: $\sum \mathbf{M}_i = 0$. In static equilibrium, torques balance about any pivot point, but it is often best to choose one at one of the points where a force acts, so that there is no torque contribution from that force. Begin by drawing a diagram to be sure you are accounting for all of the forces acting on an object.

The tension on a string is the force pulling on it. The tension always acts along the direction of the string. Pulleys can be used to change the direction of the string, and thereby redirect the force.

Weight is the force on an object due to gravity. For an extended object, it may be considered to act at a point called the "center of gravity" which may be found by balancing the object.

Basic Calculus

Calculus was invented by Isaac Newton to express the laws of physics in a mathematical form.

A *derivative* represent the instantaneous rate of change of a quantity. If a function $f(t)$ is graphed as a function of t, the derivative df/dt is the slope of the graph at point t. At "turning points" of the graph (local maxima or minima), $df/dt = 0$.

The second derivative of a function d^2f/dt^2 gives the rate of change of its slope. Geometrically, the second derivative of a function is positive when its graph curves upward, and negative when its graph curves downward.

A few useful derivatives are (you don't need to memorize them for the exam)

$$\frac{dt^n}{dt} = nt^{n-1},$$

$$\frac{d\sin(\omega t)}{dt} = \omega\cos(\omega t), \qquad \frac{d\cos(\omega t)}{dt} = -\omega\sin(\omega t) \tag{9}$$

$$\frac{de^{ct}}{dt} = ce^{ct}, \qquad \frac{d\ln(t)}{dt} = \frac{1}{t}. \tag{10}$$

In general, the derivative of an elementary function (any combination of polynomials, trigonometric functions, exponentials, and logarithms) is an elementary function.

The *chain rule* can be used to calculate the derivative of a function that depends on another function: If we have a function $f(x)$ and x in turn is a function of t, then

$$\frac{df}{dt} = \frac{df}{dx}\frac{dx}{dt}. \tag{11}$$

The *integral* of a function $f(t)$ from point t_1 to point t_2 is the area under the graph of the function f between the points t_1 and t_2. When $f(t)$ is negative, it makes a negative contribution to the area.

The *Fundamental Theorem of Calculus* states that integrals are the inverse of derivatives:

$$\int_{t_0}^{t} \frac{df}{dt}dt = f(t) - f(t_0). \tag{12}$$

The integral of an elementary function usually is not an elementary function. Many "special functions" are defined as integrals of elementary functions. When the integral does give an elementary function, finding it is a matter of working backwards to find what function's derivative is the function being integrated. This is something of an art, and is often done in practice by using integral tables. Integration techniques are the meat of most first courses in calculus. You will not be expected to do any challenging integrals on the exams in this course, but some may come up in homework.