Physics 222 Exam 1

June 20, 2003

If you have trouble with an early part of a problem, you can still get most of the credit on the rest of the problem by solving it symbolically, even if you don't have the numbers needed for a complete answer. Show your work clearly to maximize partial credit. All non-numerical questions require explanations.

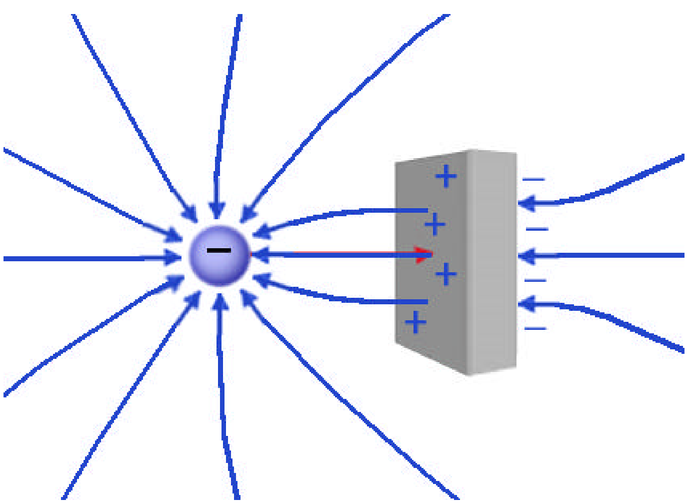
Constants:

k = 9.0 x 10^9 Nm^2/C^2 $\qquad$ ε_0 = 8.85 pF/m

e = 1.60 x 10^{-19} C $\qquad$ m_e = 9.31 x 10^{-31} kg

Problem 1   10 Points

(a) A negatively charged metal ball is placed near a neutral metal block as shown in the illustration. Draw the electric field lines, including arrows. Draw enough to show the essential features of the field lines, and write a brief explanation of the main points. Your explanation should cover any artistic deficiencies. (4 points)



The negatively charged ball induces a positive charge on the near surface of the conductor, and a negative charge on the far surface. Some of the field lines start and stop on these induced charges. Those that do must be perpendicular to the conductor at the surface. The number of field lines ending and starting on the metal block must be the same, since it is neutral. Field lines point from positive to negative charges.

(b) Does the metal ball change the charge distribution on the metal block? What can you say about the charge distribution on the metal block? Be as specific as possible. (3 points)

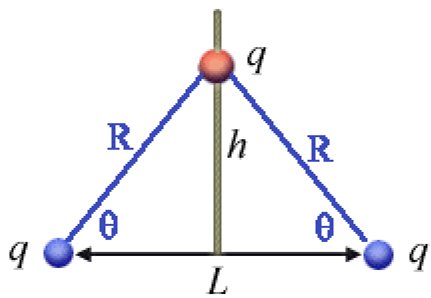
Yes. The metal ball attracts positive charge in the metal block, inducing a positive surface charge on the side closer to the ball, and an equal negative surface charge on the far side. All of the charge must be on the surface of the block, because it is a conductor.

(c) Does the conducting block attract or repel the charged ball, or does it have no effect on it. Explain your answer. (3 points)

The conducting block attracts the charged ball, because of the negative induced charge on its closer surface.

Problem 2 — 10 Points

Two equal charges q = 4.0 μC are placed a distance L = 60 cm apart horizontally, and a charged bead with the same charge q = 4.0 μC is allowed to slide up and down on an insulated rod midway between the first two charges as shown.



(a) If the force of gravity is balanced when the bead is a height h = 45 cm above the other charges, what is the mass of the bead? (5 points)

The force of gravity on the bead is $F_g = mg$, downward. The magnitude of the electrical repulsion due to each of the charges at the bottom acting on the bead is kq^2/R^2, where $R = ((L/2)^2 + h^2)^{1/2}$ = 54.1 cm is the distance between the bead and either of the bottom charges. The total upward electrical force on the bead is then

$$F_e = \frac{2kq^2}{R^2} \sin\theta = \frac{2kq^2 h}{R^3} ,$$

Since $\sin\theta = h/R$. Setting $F_g = F_e$ gives

$$m = \frac{2kq^2 h}{g R^3} = \frac{2 \times (9 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (4.0 \times 10^{-6} \text{ C})^2 \times 0.45 \text{ m}}{9.8 \text{ m/s}^2 \times (0.541 \text{ m})^3} = \mathbf{83.5 \text{ g}} .$$

(b) How much work does it take to push the bead down to a point midway between the other two charges? (5 points)

The work is equal to the change in potential energy, which is the sum of a gravitational term and an electrical term. The change in gravitational potential energy is $\Delta PE_g = -mgh$. The change in electrical potential energy is $\Delta PE_e = q(V_0 - V_h)$, where V_h is the electrical potential with the bead at height h, and V_0 is the electrical potential with the bead between the two other charges. Then

$$V_h = \frac{2kq}{R} \quad , \quad V_0 = \frac{2kq}{L/2} \quad .$$

The work required to push the bead down to a point midway between the other charges is then

$$W = \Delta PE_e + \Delta PE_g = \frac{4kq^2}{L} - \frac{2kq^2}{R} - mgh$$

$$= (9 \text{ x } 10^9 \text{ Nm}^2/\text{C}^2)(4.0 \text{ x } 10^{-6} \text{ C})^2 (6.67 \text{ m}^{-1} - 3.70 \text{ m}^{-1}) - (0.0835 \text{ kg})(9.8 \text{ m/s}^2)(0.45 \text{ m})$$

$$= 0.428 \text{ J} - 0.368 \text{ J} = \mathbf{0.060\ J}.$$

Problem 3 10 Points

A computer monitor creates a high voltage between two charged plates, which it uses to accelerate electrons toward the screen. The monitor operates on 120 VAC, which is boosted to high DC voltage electronically. It draws 300 W of power from the outlet.

(a) If the electrons start at rest from plate A and strike plate B with a speed of 2.00 x 10^7 m/s, what is the potential difference $V_B - V_A$? (2 points)

The change in kinetic energy plus the change in potential energy must be zero, so

$$\Delta KE = mv^2/2 = -\Delta PE = -(-e)(V_B - V_A).$$

Then

$$\Delta KE = mv^2/2 = (9.31 \text{ x } 10^{-31} \text{ kg})(2.00 \text{ x } 10^7 \text{ m/s})^2/2 = 1.862 \text{ x } 10^{-16} \text{ J}$$

and

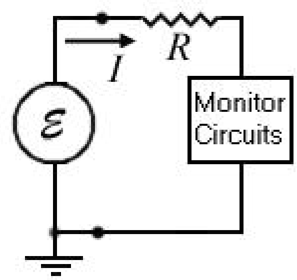
$$V_B - V_A = \Delta KE/e = 1.862 \text{ x } 10^{-16} \text{ J} / 1.60 \text{ x } 10^{-19} \text{ C} = \mathbf{1.16\ kV}.$$

(b) If the monitor uses 300 W of electricity and is 80% efficient in using that electricity to accelerate the electrons, how many electrons per second strike the monitor screen? (2 points)

The power used to accelerate the electrons is 0.80 x 300 W = 240 W. The work done on the electrons in one second is then 240 J. If n is the number of electrons striking the screen each second, then 240 J = $ne\,(V_B - V_A) = n\,\Delta KE$. Using the result from part (a) gives

$$n = 240\text{ J} / 1.862 \times 10^{-16}\text{ J} = \mathbf{1.29 \times 10^{18}} \text{ electrons per second.}$$

(c) Suppose the 20% lost power is due to a resistance R in the 120 VAC input circuit. See the picture at the right. The circle represents the 120 VAC power outlet, which is providing 300 W of power, and the remaining "monitor circuits" in the box are assumed to be 100% efficient. What is the value of R? (2 points)



The power lost in the resistor is 0.20 x 300 W = 60 W. Then 60 W = $R\,I^2$, where I is the r.m.s. current drawn from the outlet. Since 300 W = 120 V x I, the current drawn is I = 300 W/120 V = 2.5 A. Then R = 60 W / (2.5 A)^2 = **9.6 Ω**.

(d) If the supply voltage could be changed from 120 VAC to 240 VAC, but the resistance R remained the same, how much current would be needed to deliver the same power as before to the monitor circuits in the box of the figure above? Hint: at higher voltage, the monitor should draw less current. (2 points)

The average power delivered from the socket is $\mathcal{E}\,I$, where $\mathcal{E}$ and I are the r.m.s. voltage and current of the AC outlet. The power dissipated in the resistance is $R\,I^2$. The power P_M delivered to the monitor circuits must still be 240 W, as in part (b). Then we have the relation $\mathcal{E}\,I - R\,I^2 = P_M = 240$ W. This is a quadratic equation which must be solved for the unknown current I. Solving the quadratic equation $R\,I^2 - \mathcal{E}\,I + P_M = 0$ for I gives

$$I = \frac{\mathcal{E} - \sqrt{\mathcal{E}^2 - 4\,R\,P_M}}{2R} = \frac{240\text{ V} - 220\text{V}}{2 \times 9.6\Omega} = \mathbf{1.04\ A}\,.$$

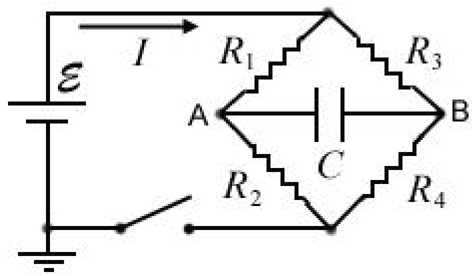
Note that a plus sign could have been used before the square root in solving the quadratic equation, but that would have given a current much greater than is drawn at 120 V, which doesn't make sense physically, since the power consumption should decrease at higher voltage, not increase.

(e) What is the efficiency of the monitor when operated at 240 VAC, *i.e.*, the ratio of the power used by the "monitor circuits" in the figure to the power supplied from the outlet? (2 points)

The power used by the monitor circuits is always 240 W. When the monitor is operated at 240 V, it draws 1.04 A of current, so the power drawn from the socket is 240 V x 1.04 A = 250 W. The efficiency is 240 W / 250W = 0.96, or **96%**.

Problem 4 — 10 Points

In the circuit shown, the capacitor is initially uncharged. Then the switch is turned on, and the capacitor is allowed to charge. The battery has and emf of $\mathcal{E}$ = 12 volts, the resistors are R_1 = 200 Ω, R_2 = 1kΩ, $R_3 = R_4$ = 470 Ω, and the capacitor has a value of C = 500 μF.



(a) When the switch is first closed and there is not yet any charge on the capacitor, what is the potential difference V_{AB} across the capacitor? (2 points)

Since there is no charge on the capacitor, the relation $Q = CV$ implies that there is no voltage across it. Therefore, V_{AB} = **0**.

(b) At this moment, how much current I is flowing from the battery? (2 points)

If there is no voltage across the capacitor, it is effectively behaving as an ordinary wire: it provides no resistance to current flowing into it. If the capacitor is replaced by a wire, then we just need to find the current into a network of resistors. In particular, R_1 is in parallel with R_3, and R_2 is in parallel with R_4. These parallel combinations have resistances $R_{13} = (R_1 + R_3)^{-1}$ = 140Ω, and $R_{24} = (R_2 + R_4)^{-1}$ = 320Ω. The total resistance across the battery is then $R_T = R_{13} + R_{24}$ = 460Ω. The current flowing from the battery is then

$$I = \mathcal{E} / R_T = 12\text{ V} / 460\Omega = \mathbf{26.1\ mA}.$$

(c) At the same instant, how much current I_C is flowing into the capacitor? (2 points)

We can determine the voltage $V_A = V_B$ by treating the circuit as a battery in series with two resistors, R_{13} and R_{24}. This gives

$$V_A = V_B = \mathcal{E} \; R_{24} / R_T = 12 \text{ V x } 320\Omega \;/\; 460\Omega = 8.3 \text{ V}.$$

Current conservation at point A shows that the current flowing into the capacitor is

$$I_C = I_1 - I_2 = (\mathcal{E} - V_A) / R_1 - V_A / R_2 = 3.7\text{V} / 200\Omega - 8.3\text{V} / 1000\Omega = \mathbf{10.2\ mA}.$$

(d) How much current I flows from the battery after the capacitor is fully charged? (2 points)

When the capacitor is fully charged, no current flows across it, so we can treat the circuit as if nothing at all were there. Then we effectively have two resistors $R_{12} = R_1 + R_2 = 1200\Omega$ and $R_{34} = R_3 + R_4 = 940\Omega$ in parallel. The total resistance is now given by the parallel combination $R'_T = (1/R_{12} + 1/R_{34})^{-1} = 527\Omega$. The current flowing from the battery is then

$$I = \mathcal{E} \,/\, R'_T = 12\text{V} / 527\Omega = \mathbf{22.8\ mA}.$$

(e) What is V_{AB} across the fully charged capacitor? (2 points)

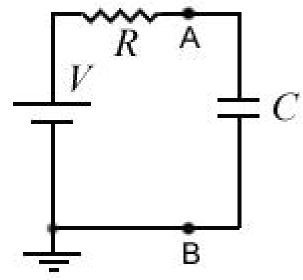
First, consider the loop through the battery, R_1 and R_2. This is just a series combination of resistors, so the voltage $V_A = \mathcal{E} \; R_2 / R_{12} = 12\text{V x } 1000\Omega \;/\; 1200\Omega = 10\text{V}$, relative to ground at the negative terminal of the battery.

Similarly, a loop through the battery, R_3 and R_4 is another series combination of resistors, giving $V_B = \mathcal{E} \; R_4 / R_{34} = 12\text{V x } 470\Omega \;/\; 940\Omega = 6.0\text{V}$.

The voltage across the charged capacitor is then $V_{AB} = V_A - V_B = \mathbf{4.0V}$.

Bonus Questions 5 Points Extra Credit

(1) Suppose an *RC* circuit is made as shown by attaching an uncharged capacitor *C* between the points A and B shown in the figure. At the time the capacitor is attached, there is no voltage across it. When the capacitor is fully charged, it reaches a maximum voltage $V_{AB} = V$. Calculate the ratio of the maximum charge Q_{max} on the capacitor after it is charged to the maximum current I_{max} flowing into the capacitor when it is just starting to charge, and show that this ratio is equal to the time constant, *i.e.*



$$Q_{max} / I_{max} = RC. \qquad \text{(2 points)}$$

The maximum charge on the capacitor is $Q_{max} = CV_{AB}$. The maximum current when the capacitor is first turned beginning to charge is $I_{max} = V_{AB} / R$. Therefore, $Q_{max} / I_{max} = RC$.

(2) This method can be used to calculate the time constant in problem 4 on the exam, which appears to be a much more complicated circuit. Although it may not be clear what to use for *R* to calculate the time constant, you can still calculate Q_{max} / I_{max} for the capacitor using results found in problem 4. Given this, how long does the capacitor in problem 4 take to reach 99% of its full charge? (3 points)

The maximum charge on the capacitor is $Q_{max} = CV_{AB}$, where $V_{AB} = 4.0$V from part (e) of problem 4. Since $C = 500\ \mu$F, this gives $Q_{max} = 2.0$ mC. The maximum current into the capacitor is given by part (c) of problem 4 to be $I_{max} = I_C = 10.2$ mA. Then the time constant is $\tau = Q_{max} / I_{max} = 0.196$ s.

The formula for the charge on the capacitor has the same form as the formula for the voltage on the capacitor, since Q = CV. The appropriate formula is for the charging capacitor, not the discharging one. Therefore, $Q = Q_{max}\left(1 - e^{-t/t}\right)$. At 99% of the final charge, Q / $Q_{max} = 0.99$, so $e^{-t/t} = 0.01$. Taking the natural logarithm of both sides gives $t / \tau = -\ln(0.01)$. Then the time to reach 99% of the maximum charge is

$$t = -\tau \ln(0.01) = (-0.196 \text{ s})(-4.61) = \mathbf{0.90\ s}.$$

Note: The simple circuit shown above at the beginning of the bonus problem is actually equivalent to the circuit in problem 4, provided $V = V_{AB} = 4.0$ V and the resistance is $R = V_{AB} / I_C = 392\ \Omega$. This is an example of a general theorem that any network of voltage sources and resistors can be replaced by a single emf in series with a resistance, as long as we only care about its properties between two terminals. Engineers call this a Thevanin equivalent circuit. In our case, the terminals are the two points A and B connected to the capacitor.