

Physics 222 Exam 2

July 12, 2002

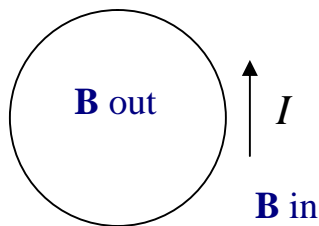
Problem 1

10 Points

- (a) If the north pole of a magnet points north, where is the north pole of the earth, magnetically speaking, *i.e.*, where is the earth's north magnetic pole? (2 points)

Opposite magnetic poles attract, so the earth's magnetic pole is near the **geographic south pole**.

- (b) If current flows in a wire as shown, what direction is the magnetic field inside the wire loop? What direction outside the wire loop? (2 points)



The right-hand rule shows that the field lines make counter-clockwise loops about the direction of current flow in the wire. The magnetic field is **out of the page** inside the current loop, and **into the page** outside the current loop.

- (c) The left circle below represents a wire coming through the paper, carrying a current of $I = 25\text{A}$ toward the reader. An electron is a distance $r = 1.0\text{ cm}$ right of the wire, headed toward the wire at a speed $v = 7.5 \times 10^7\text{ m/s}$. Find the magnetic field (magnitude and direction) at the position of the electron. (3 points)

The diagram shows a blue circle representing a wire with the text 'wire out of paper' next to it. An arrow labeled 'v' points from a red dot representing an electron towards the wire. An upward-pointing arrow labeled 'B' indicates the magnetic field direction.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Tm/A} \times 25 \text{ A}}{2\pi \times 0.010 \text{ m}}$$

$$= 5.0 \times 10^{-4} \text{ T} = \mathbf{500 \text{ mT}}$$

The direction is **upward**, as shown.

(d) Find the magnetic force (magnitude and direction) acting on the electron. (3 points)

$$F = evB = 1.60 \times 10^{-19} \text{ C} \times 7.5 \times 10^7 \text{ m/s} \times 5.0 \times 10^{-4} \text{ T} = \mathbf{6.0 \times 10^{-15} \text{ N}}$$

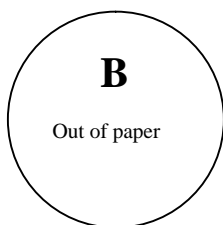
The right-hand rule shows that the force is directed **out of the paper**. Remember that since the electron is negatively charged, you have to reverse the direction compared to a positive charge.

Problem 2

10 Points

A uniform magnetic field of magnitude 0.75 T points out of the paper. A 20 cm long wire is made into a single loop and placed in the field.

(a) How many Webers of magnetic flux pass through the wire loop? (2 points)

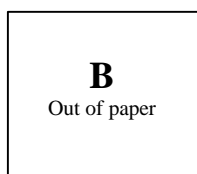


The length l of the wire is related to the radius r by $l = 2\pi r$. The enclosed area is then $A_{\text{circle}} = \pi r^2 = \pi (l/2\pi)^2 = l^2 / 4\pi = 31.8 \text{ cm}^2$.

The magnetic flux is then

$$\Phi_B = BA_{\text{circle}} = 0.75 \text{ T} \times 31.8 \times 10^{-4} \text{ m}^2 = \mathbf{2.39 \times 10^{-3} \text{ Wb}}$$

(b) If the same wire is pulled into the shape of a square, what is the change in magnetic flux through the wire loop? (3 points)



The length of the wire is the same, so the square is 5.0 cm on a side. The area of the square is then $A_{\text{square}} = 25.0 \text{ cm}^2$. The change in flux is then

$$\begin{aligned} \Delta\Phi_B &= B (A_{\text{square}} - A_{\text{circle}}) = \Phi_B [(A_{\text{square}} / A_{\text{circle}}) - 1] \\ &= 2.39 \times 10^{-3} \text{ Wb} \times [(25.0\text{cm}^2/31.8\text{cm}^2) - 1] = \mathbf{-5.1 \times 10^{-4} \text{ Wb}} \end{aligned}$$

(c) If the resistance of the wire is 0.020Ω , and it takes 0.50 seconds to pull the wire into the shape of a square, what is the average current that flows in the wire, in magnitude and direction? (3 points)

By Faraday's Law, the emf generated by the changing flux is

$$\mathcal{E} = -\Delta\Phi_B/\Delta t = 5.1 \times 10^{-4} \text{ Nm/A} / 0.5 \text{ s} = 1.02 \text{ mV}$$

The current is given by Ohm's Law: $I = \mathcal{E} / R = 1.02 \text{ mV} / 0.020 \Omega = \mathbf{51 \text{ mA}}$. Lenz's Law says that the current flows to oppose the change in flux. Since flux is decreasing, it must generate a magnetic field out of the paper, which means it flows **counterclockwise**.

- (d) After the wire has been stretched into the shape of a square, the magnet is slowly turned off. If the wire is very flexible, what will its final shape be after the current is turned off? Why? (2 points)

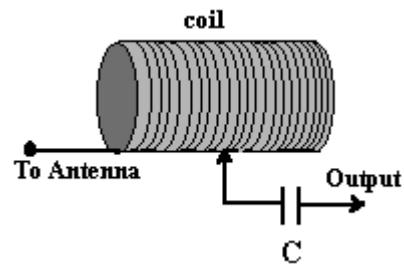
When the flux decreases, the wire will go back to being a **circle**, because the circle has the largest area for fixed circumference, and therefore preserves the most magnetic flux.

In more detail, the decreasing flux induces a counterclockwise current in the wire. The force of the magnetic field on this current is outward, by the right-hand rule, since **B** is out of the paper. This outward force will put tension on the wire, stretching it into the shape of a circle.

Problem 3

10 Points

An AM radio tuner must tune frequencies between 530 kHz and 1600 kHz. One way to make an LC oscillator to tune these frequencies could be to take a fixed 2400 pF capacitor and connect it to a coil as shown in the figure. A sliding contact can change the size of the inductance, by changing the number of wire loops in the coil.



- (a) Calculate the value of the inductance at both ends of the AM dial for the given capacitance. Which end of the AM dial corresponds to the maximum coil length? (3 points)

The resonance condition for the LC tuner is $\omega^2 = 1/LC$, where $\omega = 2\pi f$ for frequency f . Then the inductance is given by $L = [(2\pi f)^2 C]^{-1}$. For 530 kHz, this gives

$$L = [(2\pi \times 5.30 \times 10^5 \text{ Hz})^2 \times 2.40 \times 10^{-9} \text{ F}]^{-1} = \mathbf{37.5 \text{ mH}}$$

For 1600 kHz, the inductance is $L = 37.5 \mu\text{H} \times (530\text{kHz}/1600\text{kHz})^2 = \mathbf{4.11 \text{ mH}}$.

The inductance of a solenoid with length l , cross-sectional area A and N turns is $L = \mu_0 N^2 A / l = \mu_0 n l A$ if $n = N/l$ is the number of turns per unit length, which is fixed. (In words, if the coil density is fixed, a bigger solenoid has more inductance.)

Therefore, the longest coil corresponds to the largest inductance, and the smallest frequency. The maximum coil length therefore corresponds to the **low end** of the AM dial, 530 kHz.

- (b) If the radio is picking up a station which broadcasts 65 kW of power isotropically from an antenna 10 km away, what is the size of the electric field it receives? (4 points)

The area of a sphere of radius $r = 10$ km enclosing the transmitter is $A = 4\pi r^2 = 1.25 \times 10^9 \text{ m}^2$. The signal contains total power $P = 65$ kW, so the intensity at 10 km is $P/A = 5.17 \times 10^{-5} \text{ W/m}^2$. This is the average magnitude of the Poynting vector \mathbf{S} , which is related to the rms value of the electric field by $S = \epsilon_0 c E_{\text{rms}}^2 = E_{\text{rms}}^2 / \mu_0 c$, using the relation $c = \epsilon_0 \mu_0$. Then the rms value of the electric field received is

$$E_{\text{rms}} = (\mu_0 c S)^{1/2} = (4\pi \times 10^{-7} \text{ Tm/A} \times 3.0 \times 10^8 \text{ m/s} \times 5.17 \times 10^{-5} \text{ W/m}^2)^{1/2} = \mathbf{0.14 \text{ V/m.}}$$

- (c) What is the minimal antenna length needed to pick up this station if the radio requires a 50 mV input signal? (3 points)

At the minimum, we need an rms voltage $V_{\text{rms}} = 50$ mV across the ends of the antenna, which is related to the field by $V_{\text{rms}} = l E_{\text{rms}}$, if the antenna has length l . The minimal antenna length is then $l = V_{\text{rms}} / E_{\text{rms}} = 0.050 \text{ V} / 0.14 \text{ V} = \mathbf{36 \text{ cm.}}$

Problem 4

10 Points

- (a) Microwaves of frequency 1.0 GHz fall on a slit of width 1.0 cm. What is the angle between the central peak and the first intensity minimum? (3 points)

The wavelength of 1.0 GHz microwaves is

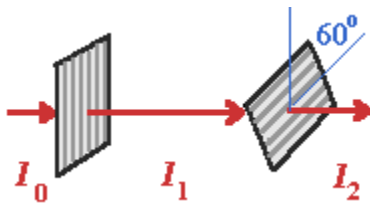
$$\lambda = c/f = 3.0 \times 10^8 \text{ m/s} / 1.0 \times 10^9 \text{ Hz} = 0.30 \text{ m} = 30 \text{ cm.}$$

The wavelength is greater than the slit width, so **there will not be an intensity minimum**. The angle of the first minimum occurs when $\sin \theta = \lambda/D$, where λ is the wavelength and D is the slit width. The sine of an angle cannot be greater than 1, so there are no solutions for θ . The diffraction is very great when the wavelength is larger than the slit, making only a wide peak with no minima.

(b) Can sound be diffracted? Can it be polarized? Explain. (2 points)

Sound **can be diffracted** because any wave is subject to diffraction. Sound **cannot be polarized**, because it is a longitudinal wave, and only transverse waves can be polarized.

(c) If unpolarized light of intensity I_0 enters the polarizer on the left and passes through another polarizer at angle 60° with respect to the first as shown, what is the ratio of outgoing to incoming intensities I_2/I_0 ? (2 points)

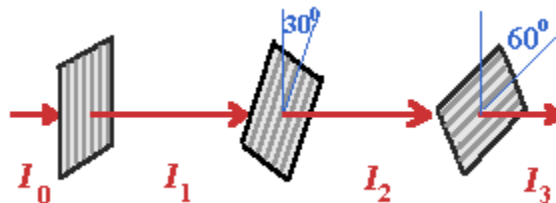


If unpolarized light arrives from the left, then half of the intensity comes through the first polarizer: $I_1 = I_0/2$. For the second polarizer,

$$I_2 = I_1 \cos^2 60^\circ = I_1/4.$$

Therefore, $I_2/I_0 = 1/8 = 12.5\%$.

(d) If a third polarizer is inserted between the previous two at an angle of 30° to vertical, what is the ratio of outgoing to incoming intensities I_3/I_0 ? How does this compare to the result of (c)? (3 points)



As in part (c), $I_1 = I_0/2$. For the second polarizer, $I_2 = I_1 \cos^2 30^\circ = 3I_1/4$. For the third polarizer, it is the change in polarization angle that is relevant. The light already arrives at a 30° angle with respect to vertical, so the angle of the third polarizer with respect to the light is 30° again. Therefore, $I_3 = 3I_2/4$. Multiplying the three factors together gives $I_3/I_0 = 9/32 = 28.1\%$.

Comparing this result with part (c) shows that **more intensity** is transmitted when the polarizer at 30° is inserted between the ones already present. That is because the additional polarizer in the middle rotates the direction of polarization to an angle where more intensity can be transmitted by the final polarizer. The net change in intensity is greater in this case, though it could go either way depending on the two polarizer angles.