Physics 222 Exam 1

June 21, 2002

Constants:

$k = 9.0 \times 10^9$ Nm^2/C^2 $\varepsilon_0 = 8.85$ pF/m

$e = 1.60 \times 10^{-19}$ C $m_e = 9.31 \times 10^{-31}$ kg

Problem 1 8 Points

(a) Can a charged rod attract a neutral piece of paper? Explain. (2 points)

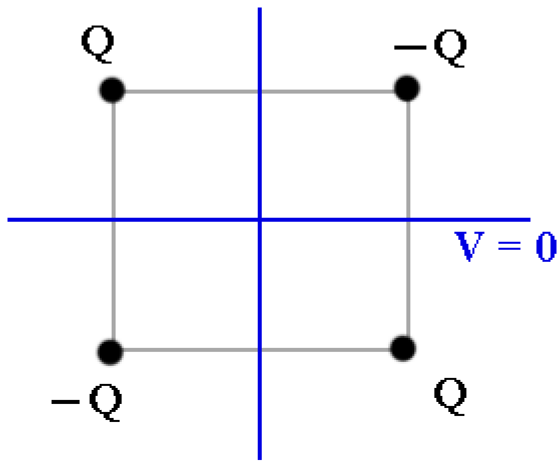
Yes. If the rod is positively charged, it will attract electrons in the paper. Although the paper is an insulator, the electrons can move a little closer to the rod, leaving a net negative charge on the side of the paper nearer the rod, and a net positive charge on the side of the paper further from the rod. Since the negative charge is closer to the rod, the total force will be attractive.

(b) If a capacitor is held at a fixed voltage while a dielectric is inserted between the plates, does the stored energy increase or decrease? Why? (2 points)

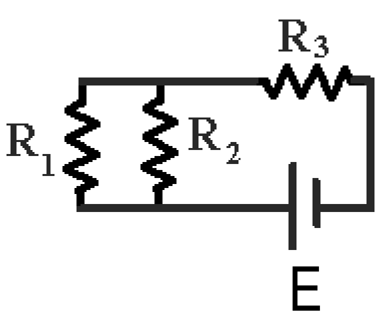
A dielectric increases the capacitance C, and for fixed voltage, the energy is given by $U = CV^2/2$. Therefore, the energy stored **increases**. This is because the dielectric allows more charge to be held in the capacitor for the same voltage, increasing the energy it can store. Note that if a dielectric were inserted while the charge on the capacitor were held fixed instead, the stored energy would decrease, because $U = Q^2/(2C)$ in that case. The reason is that the dielectric partially shields the charges from each other, so V decreases.

(c) Draw the $V = 0$ equipotentials for the charges shown, where $V = 0$ at infinity. (2 points)

The lines midway between the opposite charges are equipotentials at $V = 0$. This configuration of charges is an "electric quadrupole".



(d) What happens to the currents through R_2 and through R_3 if R_1 increases? (2 points)



Let R_{12} be the parallel combination of R_1 and R_2. Increasing R_1 increases R_{12}. Therefore, the total resistance of the circuit increases, so the current going through R_3 **decreases**. If E is the emf of the battery, $I_3 = \mathsf{E}/(R_{12} + R_3)$ flows through R_3.

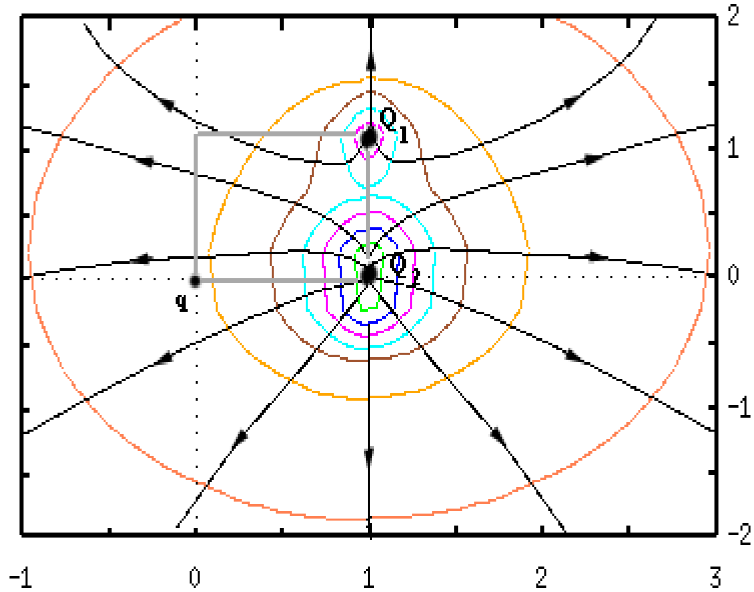
The current through R_2 is V_{12}/R_2, so that

$$I_2 = \left(\frac{\mathsf{E}}{R_2} \right) \frac{R_{12}}{R_{12} + R_3}$$

This increases when R_{12} increases, so the current through R_2 **increases**.

Problem 2 12 Points

Two charges $Q_1 = 4.0\ \mu C$ and $Q_2 = 12.0\ \mu C$ are arranged on the corners of a square of side a = 5.0 cm as shown. A test charge $q = 0.5\ \mu C$ is placed on a third corner as shown.



The black lines with arrows show the electric field lines. The colored lines are equipotentials, with lines of the same color at the same potential. The test charge q is not included when drawing the field lines and equipotentials due to Q_1 and Q_2.

(a) Draw some equipotentials on the figure for the field generated by the charges Q_1 and Q_2 for arbitrary equally spaced voltages. Near which charge are they closest together? (2 points)

The potential varies fastest where the electric field is strongest, so the equipotentials are closest together **near Q_2.**

(b) Draw some field lines on the figure, indicating their direction. What is the ratio of the number of field lines beginning or ending on each of the two charges? (2 points)

The number of field lines starting on a charge is proportional to the charge, so there are **3 times** as many field lines starting on charge Q_2 as on charge Q_1.

(c) Find the Coulomb force on q due to Q_1 and Q_2. Specify both the magnitude and direction, in terms of an angle measured counterclockwise from the segment connecting q to Q_2. (4 points)

All of the charges are positive, so they all repel. The force $\mathbf{F}_1$ of Q_1 on q acts 45^o below the left axis. The distance between these charges is the diagonal of the square, $2^{½}\, a$, where a = 0.050 m. The components of F1 are therefore

$$\mathbf{F}_1^{\,x} = \mathbf{F}_1^{\,y} = -\, k\, qQ_1 \cos(45^o)/2a^2 = -2.55 \text{ N}.$$

The force $\mathbf{F}_2$ of Q_2 on q acts to the left. Its components are

$$\mathbf{F}_2^{\,x} = -\, k\, qQ_2 \cos(45^o)/a^2 = -21.6 \text{ N}, \qquad \mathbf{F}_1^{\,y} = 0.$$

The components of the total force $\mathbf{F}_T$ are then

$$\mathbf{F}_T^{\,x} = \mathbf{F}_1^{\,x} + \mathbf{F}_2^{\,x} = -24.15 \text{ N}, \qquad \mathbf{F}_T^{\,y} = \mathbf{F}_1^{\,y} + \mathbf{F}_2^{\,y} = -2.55 \text{ N}.$$

The magnitude of the total force is $F_T = [(\mathbf{F}_T^{\,x})^2 + (\mathbf{F}_T^{\,y})^2]^{½}$ **= 24.3 N.** The tangent of the angle measured downward from the left axis is $q = \tan^{-1}(\mathbf{F}_T^{\,y} / \mathbf{F}_T^{\,x})$ **= 6°, or 186°** counterclockwise from the right-facing axis.

(d) Find the electric potential V at the location of the test charge q. (2 points)

The potential is the sum of two terms for charges Q_1 and Q_2:

$$V = k\, Q_1/(2^{½} a) + k\, Q_2/a = 26.6 \times 10^5 \text{ J/C} = \mathbf{2.66\ MV}.$$

(e) If the charge q is allowed to move, what is its velocity far away from Q_1 and Q_2? Assume its mass is 5.0 g. (2 points)

The initial potential energy of charge q is PE = qV = 1.33 J. The final potential energy far away from Q_1 and Q_2 is 0. Therefore, by conservation of energy, the final kinetic energy is KE = 1.33 J = ½ mv^2. Solving for the velocity v then shows the final velocity of the charge q, with mass m = 0.0050 kg, to be $v = (2\text{PE}/m)^{½}$ = **23.1 m/s**.

Problem 3 10 Points

(a) If a charge $Q = 7.0\ \mu C$ is deposited on a solid conducting sphere of radius $r_1 = 10$ cm, where will the charge be located on the sphere, and how will it be distributed? What is the electric field inside the conducting sphere? (2 points)

The charge will spread out as much as possible, symmetrically, so it will be distributed **uniformly on the surface of the sphere**. The electric field inside a conductor is **0**.

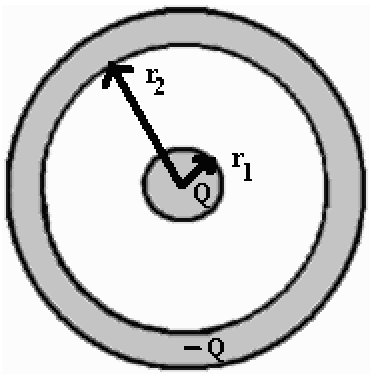
(b) Gauss's law shows that the electric field outside a uniformly charged sphere is the same as the field due to a point charge of the same magnitude at the center of the sphere. What is the electric potential at the surface of the sphere relative to $V = 0$ at infinity? (2 points)

The field due to a point charge of magnitude Q at the center of the sphere would be

$$V_1 = kQ/r_1 = (9\text{x}10^9\ \text{Nm}^2/\text{C}^2)\ (7\text{x}10^{-6}\ \text{C})\ /\ 0.10\ \text{m} = 6.3\ \text{x}\ 10^5\ \text{V}.$$

As the problem states, Gauss's law shows that the electric field must be due to this potential, which is defined so that its value is $V = 0$ when r becomes infinite. Therefore, the value of the electric potential at the surface of the sphere is **6.3×10^5 V**.

(c) A second conducting sphere with inner radius $r_2 = 12$ cm and charge $-Q$ is placed around the first sphere as shown. How will the charge be distributed on the outer conductor, and what will be the electric field outside the pair of conductors? (2 points)



The negative charge will be attracted toward the inner sphere, and become **uniformly distributed over the inside of the outer sphere**. There is no other charge on the outer sphere: the combination is neutral. In particular, there is *not* a positive charge on the outside of the sphere.

Since the spheres taken together are neutral, and the net electric field outside the spheres is **zero**.

(d) What is the potential difference between the inner and outer conducting spheres, assuming the inner radius r_2 of the outer sphere is 12 cm? (2 points)

The field in the region between the two spheres is entirely due to the charge on the inner sphere, by Gauss's Law. Therefore, the potential difference between the two spheres is the difference between the potential of a charge Q at the center, measured at r_1 and r_2:

$$V_{12} = V_1 - V_2 = kQ/r_1 - kQ/r_2 = V_1\,(1 - r_1/r_2)$$
$$= 6.3 \times 10^5 \text{ V } (1 - 10/12) = \mathbf{1.05 \times 10^5\ V}.$$

(e) What is the capacitance of the pair of conducting spheres? (2 points)

The capacitance is defined by $Q = C\,V_{12}$. Therefore,

$$C = 7.0 \times 10^{-6} \text{ C}/1.05 \times 10^5 \text{ V} = 6.67 \times 10^{-11} \text{ F} = \mathbf{66.7\ pF}.$$

Note that it is the charge on either plate that defines the capacitance: do not double-count. Also, the parallel plate expression $C = \varepsilon_0\,A/d$ does not apply directly in this case. However, if the area is defined to be the *geometric mean* of the areas of the outer surface of the inside sphere and the inner surface of the outside sphere, so that $A = 4\pi r_1 r_2$, and $d = r_2 - r_1$, then the correct result is obtained. This follows from the algebra in parts **(b)** and **(d)**, which shows that

$$C = Q/V_{12} = 4\pi\varepsilon_0 \left(\frac{r_1 r_2}{r_2 - r_1} \right).$$

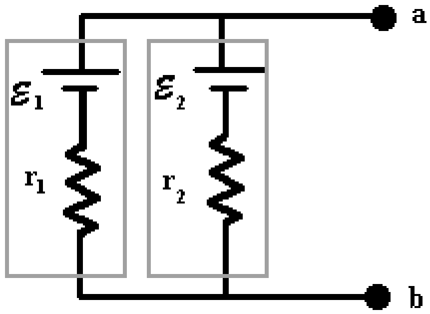
Problem 4 10 Points

(a) Two batteries have voltages V_1 = 3V and V_2 = 2.5V when no current flows. Battery 1 delivers a current of 3A if its terminals are shorted, and battery 2 delivers a current of 5 A if its terminals are shorted together. What is the emf and internal resistance of each battery? (2 points)

The emf is the terminal voltage when no current flows from the battery. So the emfs of the two batteries are $\mathcal{E}_1$ **= 3 V** and $\mathcal{E}_2$ **= 2.5 V**, respectively. When the terminals are shorted, all current flows through the internal resistance, so Ohm's law gives the value of the internal resistance:

$r_1 = \mathcal{E}_1 / I_1$ =3V/3A = **1** W, $r_2 = \mathcal{E}_2 / I_2$ =2.5V/5A = **0.5** W.

(b) If the two batteries of part (a) are connected in parallel, what is the terminal voltage V_{ab} of the pair? Why may this be considered to be the emf E_{12} of this pair of batteries? (3 points)



Let I flow in the clockwise direction. You can use

$$V_{ab} = E_1 - I\,r_1 \quad \textbf{or} \quad V_{ab} = E_2 + I\,r_2 .$$

Since the batteries are joined in parallel, these must be equal. Kirchoff's loop rule can be used to find the current I flowing between the batteries. Then

$$E_1 - E_2 - I\,r_1 - I\,r_2 = 0.$$

Solving for the current gives $I = (E_1 - E_2)/(r_1 + r_2)$.
Then $V_{ab} = E_1 - I\,r_1 = (r_2E_1 + r_2E_2)/(r_1 + r_2)$.

Numerically, V_{ab} = **2.67V**. We may consider V_{ab} to be the emf E_{12} of this pair of batteries because it is the voltage measured between the terminals when no current is flowing. This is the definition of emf. Note that the combined emf of the batteries in parallel depends not only on the emfs of the individual batteries, but also on their internal resistances.

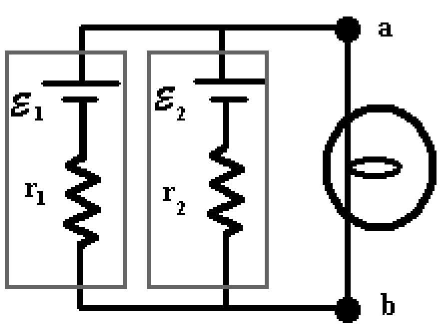
(c) If terminals **a** and **b** are shorted together with a wire, how much current flows through the wire? Use this to find the internal resistance r_{12} of the pair of batteries. The pair of batteries is equivalent to a single emf E_{12} in series with an internal resistance r_{12}.
(2 points)

Now, points **a** and **b** are connected together with a wire. Let I be the current flowing through the wire. It is the sum of the currents I_1 and I_2 flowing from batteries 1 and 2:

$$I = I_1 + I_2 = (E_1/r_1) + (E_2/r_2) = (3.0\text{V}/1.0\Omega) + (2.5\text{V}/0.5\Omega) = \mathbf{8.0\ A}.$$

The internal resistance can be found as in part **(a)**, as the emf for the pair of batteries divided by the current that flows when the terminals are shorted. Using the result of **(b)** for the emf E_{12} gives an internal resistance $r_{12} = E_{12}/I$ = 2.67V / 8.0A = **0.33**W. Note that algebraically, $r_{12} = r_1\,r_2/(r_2 + r_2)$ is the parallel combination of the two internal resistances of the batteries.

(d) The batteries are connected in parallel across a bulb rated to use 600 mA at 3V. Assuming its resistance is constant, what is the bulb's current consumption when connected to the terminals **a** and **b** as shown? (3 points)



The bulb resistance is determined by Ohm's Law to be $R = 3\text{V}/0.600\text{A} = 5.0\Omega$. Parts (c) and (d) show that the pair of batteries is equivalent to a single battery with emf E_{12} = 2.67V and internal resistance $r_{12} = 0.33\Omega$. The current flowing through the bulb is then

$$I = E_{12}/(r_{12} + R) = 2.67\text{V}/5.33\Omega = 0.50\text{A}.$$

Therefore, the bulb draws **500 mA**. You could also solve this using Kirchoff's rules for two loops.