Lab Final Version 2

This is the answer key for Version 2 of the Spring 2002 Laboratory Final for sections 69555, 69571 and 69584 of Physics 221.

The exam is worth 100 points. Each question is worth 6 points, except the elastic/inelastic question of problem 6, which is worth 4 points. Partial credit is given only on the questions allowing multiple answers, with 2 points for each item correctly chosen or not chosen.

The correct answers are marked. As explained before the exam, the choices given do not always match the answer exactly. The answer closest to the correct result should be chosen. Explanations for the answers appear after the exam.

. . . .

1. A weight known to have a mass of exactly 50.000 g is placed on an electronic scale. The scale reads 51.13 g. The **precision** of the scale is most nearly

a. $\pm$0.02 g **b.** $\pm$1 mg **c.** $\pm$1.1 g <u>**d.** $\pm$0.01 g</u>

The **accuracy** of the scale is most nearly

a. 1% **b.** 0.02% <u>**c.** 2%</u> **d.** 0.2%

2. Momentum is mass times velocity. If the error in the mass is 2% and the error in the velocity is also 2%, the error in the momentum is

a. 4% <u>**b.** 3%</u> **c.** 2% **d.** 1%

3. When measuring centripetal force, a mass is spun around an axis by hand, while someone counts off 53 revolutions using a stopwatch that measures hundredths of a second (its precision is $\pm$0.01 s). Suppose the revolutions take 57.0 seconds, total, to complete. If the clock is accurate, the rotation is kept perfectly uniform and the counting is started and stopped at exactly the same point of the rotation, what is the best expression of the time for one period?

a. 1.08 s $\pm$0.2% **b.** 1.0754717 $\pm$ 0.0018867 s
<u>**c.** 1.075 s $\pm$0.01%</u> **d.** 1.08 s $\pm$2%

Suppose the person using the clock can make the starting and stopping points of the rotation match only to within half of a rotation (180 degrees). What is the error in the period measurement?

a. 5% **b.** 2% **c.** 0.5% <u>**d.** 1%</u>

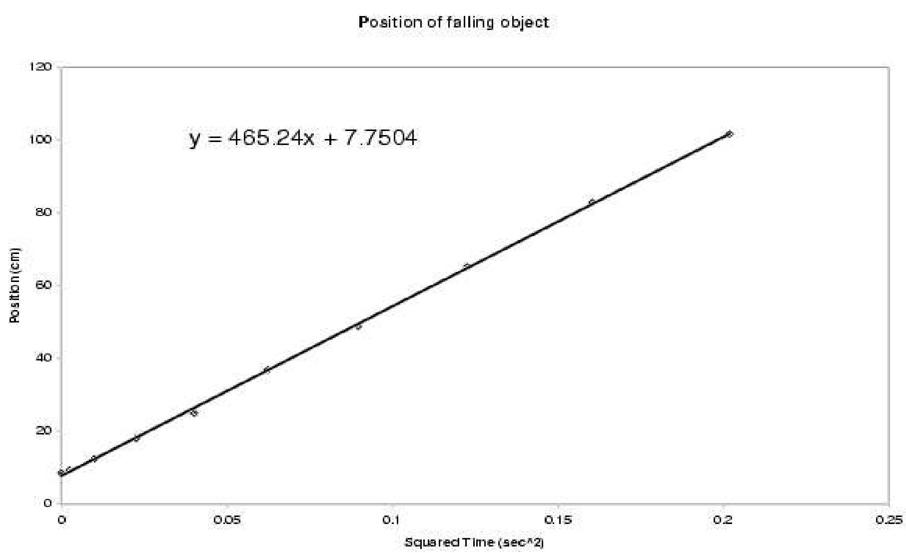
4. The acceleration of gravity could be measured by dropping a weight and photographing its location at a sequence of times. Assume the times are measured electronically to within a millisecond, and the distance traveled at each time can be reconstructed from film with an accuracy of 1%. The actual distances traveled, y, are then graphed as a function of the squared time,

t^2 as shown in the figure at the end of this problem. What is the measured acceleration due to gravity?
a. 9.30 m/s^2 **b.** 465.24 m/s^2 **c.** 930.48 m/s^2 **d.** 9.80 m/s^2

Which is closest to the percent error in the measurement of g?
a. 9.40% **b.** 94% **c.** 5% **d.** 50%

Approximately how much of the error can be reasonably attributed to the accuracy of the distance and timing measurements?
a. 0.1% **b.** 2% **c.** 10% **d.** 50%

Which of the following are possible explanations of the remaining error? Check **all answers** that apply.
a. Air resistance slowed the weight as it fell.
b. The timer was not started exactly when the weight was dropped.
c. The weight was not at $y = 0$ when it was dropped.



5. A cylindrical piston is used to investigate Boyle's Law. The length of the piston is 5 cm and a volume of 15 mL of air is enclosed with a pressure of 1 Atm. What is the cross-sectional area of the piston? (The volume is the length times the area.)
a. 3×10^{-6}m^2 **b.** 3×10^{-4}m^2 **c.** 3×10^{-2}m^2 **d.** 0.3m^2

A 500 g weight is applied to the piston in the cylinder to compress the air inside. What is the final volume of the cylinder, neglecting friction?
a. 0.9 mL **b.** 9.3 mL **c.** 4.64 mL **d.** 5.7 mL

6. Two 300 g gliders are pushed toward each other on an air track. When they go through photogates, glider 1 has velocity 1.3 m/s to the right, and the glider 2 has velocity 2.5 m/s to the left. What is the total initial momentum of the gliders? Positive velocity is toward the right.
a. -0.36 kg m/s **b.** 0.36 kg m/s **c.** -36 kg m/s **d.** 36 kg m/s

The gliders collide and go back through the photogates. The final velocity of glider 1 is 2.3 m/s to the left, and the final velocity of glider 2 is 1.2 m/s to the right. What is **change** in total momentum of the gliders?
a. +0.03 kg m/s **b.** -0.03 kg m/s **c.** +3 kg m/s **d.** -3 kg m/s

This collision is more nearly
a. elastic **b.** inelastic

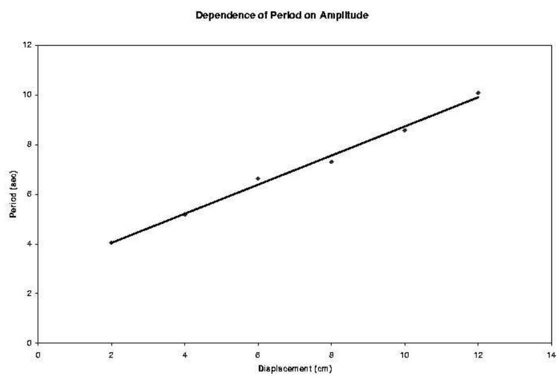
and the change in momentum can be explained best by

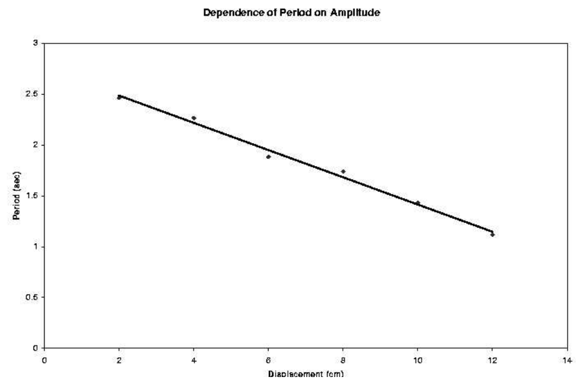
a. friction between the two gliders when they collide.
b. friction between the gliders and the air track.
c. no physical process: this must be a human error.

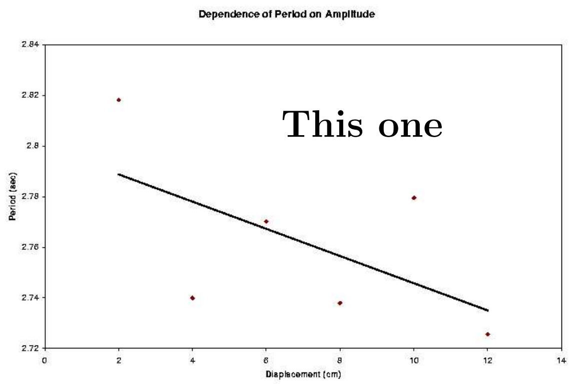
7. The latent heat of fusion of ice is measured by putting ice from a bucket at freezing temperature into an aluminum calorimeter containing warm water. Which of the following could cause you to **underestimate** the heat of fusion of the ice? Check **any or all** which apply.

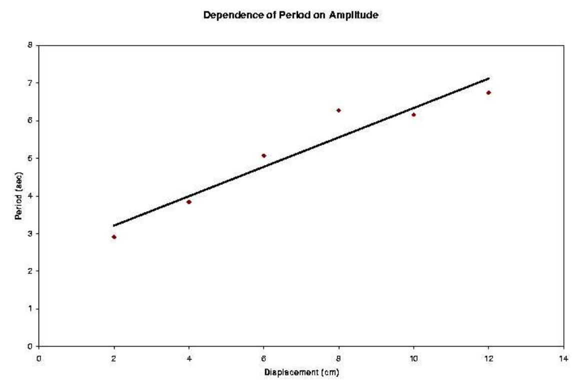
a. The temperature of the water in the calorimeter is too warm, well above room temperature.
b. The ice is wet, and not dried before adding to the calorimeter.
c. Some water splashes out of the calorimeter due to over-vigorous stirring while trying to melt the ice.

8. A mass is hung on a spring, and pulled and let go, initiating simple harmonic motion. The distance (amplitude) the spring was displaced from equilibrium is varied, and the period is measured as a function of the amplitude. Mark the graph below which most likely represents the outcome of this experiment.









Explanations for Answers

1. First Question: The precision is how many digits can be read from the apparatus. This scale could be read to hundredths of a gram, so the precision is ±0.01 g.

Second Question: The accuracy is how close the scale comes to the true measurement:

$$\frac{51.13 \text{ g} - 50.000 \text{ g}}{50.000 \text{ g}} = 0.0226 = 2.26\%. \tag{1}$$

The closest answer is 2%.

2. The momentum is given by $p = mv$ if m is the mass and v is the velocity. The error propagation equation for a product shows that the relative error in the momentum is

$$\frac{\epsilon_p}{p} = \sqrt{\left(\frac{\epsilon_m}{m}\right)^2 + \left(\frac{\epsilon_v}{v}\right)^2}, \tag{2}$$

where ϵ_p, ϵ_m and ϵ_v are the errros in p, m, and v, respectively. The relative errors in m and v are both

$$\frac{\epsilon_m}{m} = \frac{\epsilon_v}{v} = 2\% = 0.02. \tag{3}$$

This gives

$$\frac{\epsilon_p}{p} = \sqrt{(0.02)^2 + (0.02)^2} = 2\% \times \sqrt{2} = 2.83\%. \tag{4}$$

The closest answer is 3%.

3. First Question: The time for one period is the total time divided by the number of revolutions:

$$T = \frac{57.0 \text{ s}}{53} = 1.08 \text{ s} \tag{5}$$

The error in the period is the error in the total time, divided by 53 revolutions. Assuming the clock is read perfectly, its accuracy is limited only by its precision of ± 0.01 s. Then the error in the period is

$$\epsilon_T = \frac{\pm 0.01 \text{ s}}{53} = 1.9 \times 10^{-4} \text{ s}, \tag{6}$$

which gives a relative error of

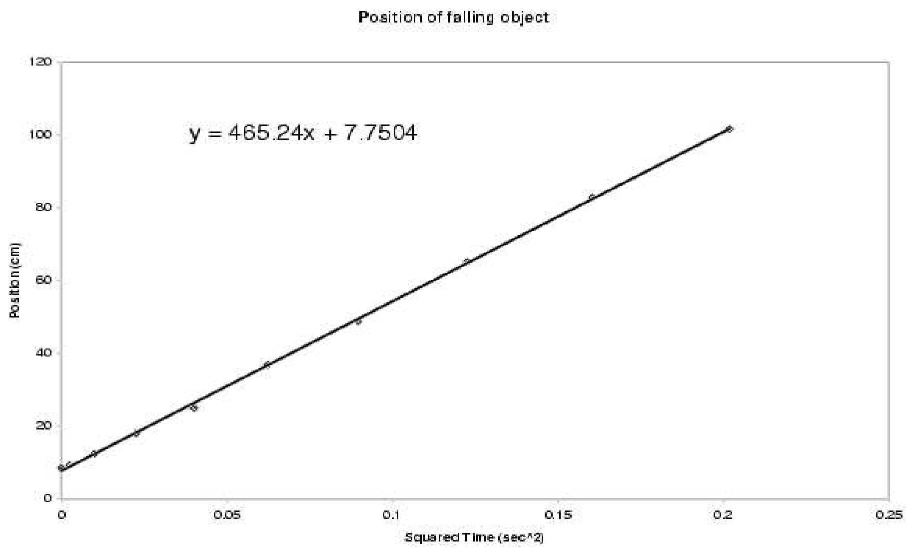
$$\frac{\epsilon_T}{T} = \pm 1.8 \times 10^{-4} = 0.018\%. \tag{7}$$

The closest answer is $1.075 \text{ s} \pm 0.01\%$. This is not a perfect match, but it is much closer than any other choice. In all the other choices, the error is at least 10 times too big. For example, the choice 1.0754717 ± 0.0018867 s has 10 times too much error. (It also shows far greater precision than is justified by the 3 digits measured.)

Second Question: If 53 rotations are measured, and the time is off by half a rotation, the relative error is $0.5/53 = 0.0094 = 0.94\%$. The closest answer is 1%.

4. First Question: The graph plots the position y of the falling object as a function of the squared time, t^2. The equation of motion of an object accelerating with acceleration g is $y = \frac{1}{2}gt^2$. Therefore, the slope of the graph is $\frac{1}{2}g$. The labels on the graph show that the times are measured in seconds, and the positions in cm. The trendline equation on the graph shows that the slope is 465.24 cm/s^2. This gives

$$\frac{g}{2} = 465.24 \text{ cm/s}^2 = 4.6524 \text{ m/s}^2. \tag{8}$$



Therefore, the measured value of g is 9.30m/s^2.

Second Question: The expected value of g is 9.80m/s^2. Comparing this to the measured value shows that the fractional error is

$$\left| \frac{9.30 \text{ m/s}^2 - 9.80 \text{ m/s}^2}{9.80 \text{ m/s}^2} \right| = 0.051 = 5.1\%. \tag{9}$$

The closest answer is 5%.

Third Question: The times are accurate to 0.001 s, and the average t^2 values are of order 0.1 s^2, which corresponds to times around 0.3 s, typically. So the relative error in times is about ± 0.001 s/0.3 s, or 0.3%. The relative error in the distances is about 1%. Without doing any calculations, a good ballpark estimate of the error in the slope, and consequently g, would then be around $1 - 2\%$. The closest answer is 2%. (The actual error would be somewhat less than this due to averaging over the 10 data points, but not a factor of 10 less.)

Fourth Question: Air resistance will tend to reduce g by an amount which depends on the surface area of the object. This **does** introduce some error into the g measurement.

Having the initial position different from $y = 0$ would change the intercept, but not the slope, of the graph. This does **not** affect the g measurement.

If the timer is not exactly at 0, but instead at time t_0 when the object is dropped, its true equation of motion would be changed to

$$y = \frac{1}{2}g(t - t_0)^2 = \frac{1}{2}gt^2 - gt_0 t + \frac{1}{2}gt_0^2. \tag{10}$$

This is no longer quite linear when plotted as a function of t^2. This will introduce an error in the slope, when attempting to fit a linear trendline. So not starting the timer at 0 **will** introduce an error into the g measurement.

The size of the error was not requested, but the relative error in g would turn out to be about $-2t_0/\overline{t}$, where $\overline{t}$ is an average value of t. A late start of 7.5 ms would be sufficient to explain a 5 % error in g.

5. First Question: The area A of the cylinder is the volume V divided by the length l:

$$A = \frac{V}{l} = \frac{15 \text{ mL}}{5 \text{ cm}} = \frac{15 \text{ cm}^3}{5 \text{ cm}} = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2 \tag{11}$$

since $1 \text{ cm}^2 = (0.01 \text{ m})^2 = 10^{-4} \text{ m}^2$.

Second Question: Boyles law says that if the temperature if fixed, the pressure times volume is a constant, so

$$P_1V_1 = P_2V_2. \tag{12}$$

The initial pressure is $P_1 = 1 \text{ Atm} = 1.015 \times 10^5 \text{ N/m}^2$. The 500 g weight adds an additional pressure of

$$\frac{mg}{A} = \frac{0.500 \text{ kg} \times 9.8 \text{ m/s}^2}{3 \times 10^{-4} \text{ m}^2} = 1.63 \times 10^4 \text{ N/m}^2, \tag{13}$$

giving a total pressure of $P_2 = P_1 + mg/A = 1.178 \times 10^5 \text{ N/m}^2$. Then

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right) = 15 \text{ mL} \times \frac{1.015}{1.178} = 12.9 \text{ mL}. \tag{14}$$

The closest answer is the largest: 9.3 mL.

6. First Question: Both gliders have mass $m = 0.300$ kg. Since positive velocity is to the right, the first glider has momentum

$$p_1^i = mv_1^i = 0.300 \text{ kg} \times 1.3 \text{ m/s} = 0.39 \text{ kg m/s} \tag{15}$$

and the second glider has momentum

$$p_2^i = mv_2^i = 0.300 \text{ kg} \times (-2.5 \text{ m/s}) = -0.75 \text{ kg m/s}. \tag{16}$$

The total initial momentum is then

$$p_{\text{total}}^i = p_1^i + p_2^i = -0.36 \text{ kg m/s}. \tag{17}$$

Second Question: After the collision, the first glider has momentum

$$p_1^f = mv_1^f = 0.300 \text{ kg} \times (-2.3 \text{ m/s}) = -0.69 \text{ kg m/s} \tag{18}$$

and the second glider has momentum

$$p_2^f = mv_2^f = 0.300 \text{ kg} \times 1.2 \text{ m/s} = 0.36 \text{ kg m/s}. \tag{19}$$

The total initial momentum is then

$$p_{\text{total}}^f = p_1^f + p_2^f = -0.33 \text{ kg m/s}. \tag{20}$$

The change in momentum is

$$p_{\text{total}}^f - p_{\text{total}}^i = -0.33 \text{ kg m/s} - (-0.36 \text{ kg m/s}) = +0.03 \text{ kg m/s}. \tag{21}$$

Third Question: In an elastic collision, energy is conserved. In an inelastic collision, it is not. If energy were conserved, we would have had $v_1^f = v_2^i$ and $v_2^f = v_1^i$, since the masses were equal. In the completely inelastic case, the gliders would stick together and leave with the same velocity, $v^f = p_{\text{total}}^f / 2m = -0.05$kgm/s. The velocities given are each within 10% of the elastic case, and it is plausible that this difference could be due to experimental error, so the collision is more nearly **elastic**.

Fourth Question: The correct answer is that friction between the gliders and air track most likely caused the change in momentum.

Friction between the gliders and the airtrack would slow down both gliders and cause the change in momentum observed. Note that the change in momentum was positive, even though friction caused both gliders to slow down. This is because of the change in directions.

Friction between the two gliders during the collision would change the energy, but not the momentum. This just determines how elastic or inelastic the collision is. It does not introduce error into momentum conservation.

7. All of the choices given could cause an error in the heat of fusion of ice. However, only one of them causes an underestimate. Equations are not really required to solve this problem, but are shown in the explanations for those who find them more convincing.

If the ice is wet and not dried before adding it to the calorimeter, then there is less ice in the calorimeter than expected. The change in energy would turn out to be the same, but when computing $L_f = \Delta Q / m_i$, the mass m_i of ice would be too big, since it included some water. This would lead to an underestimate of L_f.

If a fraction ϵ of the ice is water, with total mass m_i, then the actual mass of the ice is $m_i(1-\epsilon)$, and the remaining mass ϵm_i is actually water. Assuming the ice/water mixture is at temperature $T_i = 0^\circ$ C, the balance of energy equation becomes

$$(m_w c_w + m_c c_c)(T_f - T_o) + m_i c_w T_f + (1-\epsilon) m_i L_f = 0. \tag{22}$$

Here, m_w and m_c are the mass of the water and cup, and c_w, c_c are their specific heats. The original calorimeter temperature is T_o and its final temperature is T_f. The experimental value of the latent heat of fusion would be given by

$$\begin{aligned} m_i L_f^{\exp} &= (m_w c_w + m_c c_c)(T_o - T_f) - m_i c_w T_f \\ &= (1-\epsilon) m_i L_f, \end{aligned} \tag{23}$$

since the experimenter would assume all of the mass of the ice was really ice. The fractional error in the heat of fusion is then

$$\frac{L_f^{\exp} - L_f}{L_f} = -\epsilon < 0. \tag{24}$$

If the calorimeter were too warm, heat would be lost to the room. If this heat loss were not taken into account, the experimenter would think that the heat went toward melting the ice, and overestimate L_f. The heat Q_{lost} lost to the room to conduction is positive if the average temperature of the calorimeter is greater than the room during the experiment.

If m_i is the mass of the ice, the fractional error in the heat of fusion caused by heat loss to the room would be

$$\frac{L_f^{\exp} - L_f}{L_f} = \frac{Q_{\text{lost}}}{m_i} > 0. \tag{25}$$

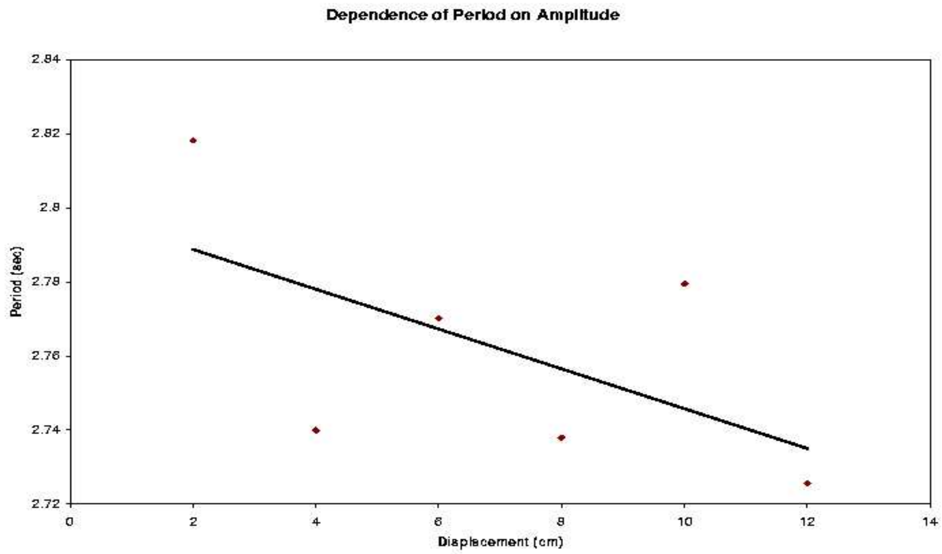
If water splashed out of the inner cup of the calorimeter, less water would remain, so the ice would cool it more. If the experimenter didn't know some water splashed out, this larger change in temperature would be attributed to more cooling by the ice, which would lead to an overestimate of L_f.

Suppose a fraction ϵ of the water, with mass ϵm_w, splashes out of the inner cup, where m_w is the total mass of the water. Assuming the ice, with mass m_i, is initially at 0° C, the initial calorimeter temperature is T_o, and the specific heat of water is c_w, then the relative error turns out to be

$$\frac{L_f^{\exp} - L_f}{L_f} = \frac{\epsilon}{1 + m_i/m_w - \epsilon}\left(1 + \frac{c_w T_o}{L_f}\right) > 0. \tag{26}$$

8. The period of a weight on a spring undergoing simple harmonic motion depends on the mass of the weight and spring and on the spring constant. It does not depend on the amplitude of the motion. An experiment to find how the period depends on the amplitude should show little correlation between the two, apart from experimental error. The graph shown has the least correlation between the amplitude (displacement) and the period.

Any data set can be fit to a trendline, but fit is not always very good. That is the case for the data shown. Given the variation among the data points, any slope can be attributed to random errors, so the experimenter could



conclude that the period was independent of the amplitude, within experimental limitations. Note that the time scale shows that the total variation of the period is less than 4% while the amplitudes varied from 2 to 12 cm.

All of the other choices shown had a much stronger correlation between the amplitude and period, as can be seen by the relatively good fit of the data to the trendline in those graphs. Therefore, those graphs were inconsistent with the expectation that the period of simple harmonic motion should not depend on the amplitude.