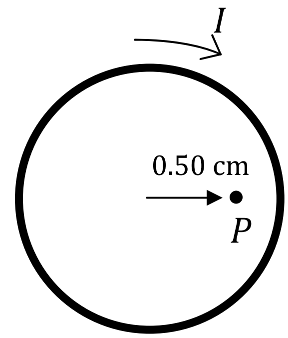
PHYSICS 222 - SECTION 1

EXAM 4: Solutions

April 27, 2009

1. [18pt] The figure shows the cross-section of a 30 cm long solenoid with a cross-sectional area of 2.4 cm^2. The solenoid has 1600 turns of wire. If the time t is given in Amperes, the current in the wire is $I = (40.0\,t^2 + 1.80)$ Amperes.



(a) [5pt] What is the magnetic flux through the cross section shown at $t = 0$? Is it into or out of the page, if the current circles clockwise about the solenoid?

$$\Phi_m = AB = A\mu_0 \frac{NI}{l} = (2.4 \times 10^{-4}\ \mathrm{m}^2)\left(4\pi \times 10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}}\right)\frac{1600\ (1.80\ \mathrm{A})}{0.30\ \mathrm{m}} = 2.90\ \mu\mathrm{Wb}.$$

(b) [4pt] What is the inductance of the solenoid?

$$L = \frac{N\Phi_m}{I} = \frac{1600\ (2.90\ \mu\mathrm{Wb})}{1.80\ \mathrm{A}} = 2.58\ \mathrm{mH}.$$

(c) [4pt] What emf is generated in the solenoid at time t = 0?

$$\mathcal{E} = L\frac{dI}{dt} = 0$$

(d) [5pt] What is the electric field at point P, a distance of 0.50 cm from the center? Give the magnitude and direction (up, down, left, right, into or out of the page).

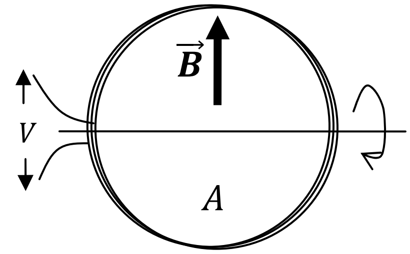
The emf about a circle of radius $r =$ 0.50 cm inside the solenoid is (with t in seconds)

$$\oint \vec{E} \cdot d\vec{s} = 2\pi r\,E = \frac{d\Phi_m}{dt} = \frac{L}{N}\frac{dI}{dt} = \frac{(2.58\ \mathrm{mH})(80\,t\ \mathrm{A/s})}{1600} = (129\,t)\ \mu\mathrm{V},$$

$$E = \frac{(129\,t)\ \mu\mathrm{V}}{2\pi(0.0050\ \mathrm{m})} = (\mathbf{4.11}\,\boldsymbol{t})\,\frac{\mathbf{mV}}{\mathbf{m}}.$$

The direction is such that a current following the emf would oppose the increase in flux into the page. Thus, the emf is counter-clockwise, to generate flux out of the page. At point P, the electric field is directed **upward** in the figure. At $t = 0, E$ vanishes. That answer would also be accepted.

2. A coil of area 0.100 m^2 with 125 turns of wire is rotating at 60.0 cycles per second with the axis of rotation perpendicular to a 0.025 T magnetic field, as shown. At this instant in the picture, the top of the coil is coming toward you and the bottom is moving away from you. The two wires coming out on the left are the ends of the coil.



(a) [6pt] What is the maximum voltage measured between the two lead wires as the coil rotates?

The emf generated in the loop is

$$V(t) = \left|\frac{Nd\Phi}{dt}\right| = N\frac{d}{dt}(AB\sin\omega t) = NAB\omega\cos\omega t,$$

so the maximum emf is

$$V_{\max} = NAB(2\pi f) = (125)(0.100\text{ m}^2)(0.025\text{ T})(2\pi \times 60\text{ s}^{-1}) = \mathbf{118\ V}.$$

(b) [2pt] What is the orientation of the coil with respect to the magnetic field when the maximum induced voltage occurs?

The maximum emf occurs when the loop coil is **parallel** to the magnetic field.

(c) [4pt] What is the magnitude and direction (clockwise, counter-clockwise, or zero) of the current flow if the coil is attached to a 100 Ω resistance, at the instant shown in the figure? Neglect the coil's resistance and inductance.

$$I = \frac{V}{R} = \mathbf{1.18\ A}.$$

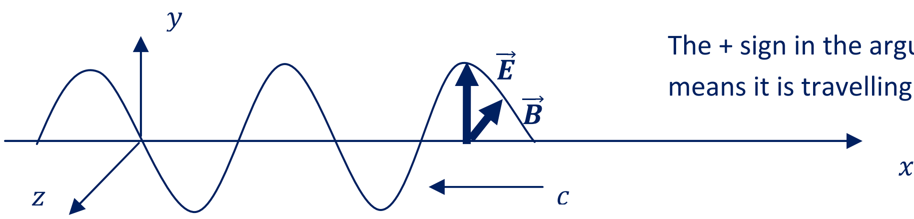
At the moment shown, the flux is zero but increasing away from you as the loop rotates toward you. According to Lenz's Law, a **counterclockwise current** will be generated to counteract this, by generating flux toward you.

(d) [4pt] What is the average power generated in the coil over a complete revolution?

The instantaneous power is $P = VI = V_{\max}I_{\max}\cos^2\omega t$, and the square of the cosine oscillates symmetrically about a value of ½ , so $P_{avg} = \frac{1}{2}V_{\max}I_{\max} = \mathbf{69.6\ W}$.

3. The electric field of a plane radio wave propagating in the x-direction points along the y-axis and has the functional form $E_y = (0.120\text{V/m})\ \sin(kx + \omega t)$. The frequency of the transmission is 900 MHz (Hz = cycles per second).

(a) [2pt] Which direction (left or right) is the wave moving along the x-axis?



The + sign in the argument means it is travelling **left**.

(b) [4pt] What is the wavelength of the wave, which is typical of a cellular telephone transmission?

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{m/s}}{9.00 \times 10^8 \text{s}^{-1}} = \mathbf{0.333\ m}.$$

(c) [4pt] What is the amplitude of the magnetic field vector?

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{0.120\ \text{V}}{3.00 \times 10^8 \text{m/s}} = \mathbf{4.00 \times 10^{-10}\ T}\ \ (= 4.00\ \mu\text{Gauss})$$

(d) [2pt] When the electric field has its maximum upward value, which way does the magnetic field point?

The Poynting Vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ points in the direction of the wave motion. The right-hand rule imples that when $\vec{E}$ is pointing up, $\vec{B}$ must point into the page in the figure above, in the **– *z* direction**. Then $\vec{S}$ will point to the left.

(e) [5pt] What is the average intensity (power per unit area) of the wave?

$$\frac{\text{P}_{\text{avg}}}{A} = S_{\text{avg}} = \frac{1}{2} S_{\text{max}} = \frac{1}{2\mu_0} E_{\text{max}} B_{\text{max}} = \frac{(0.120)(4.00 \times 10^{-10})}{8\pi \times 10^{-7}} \frac{\text{W}}{\text{m}^2}$$

$$= \mathbf{19.1 \frac{\mu W}{m^2}}\ \left(= 0.191 \frac{\text{W}}{\text{cm}^2}\right).$$