

# PHYSICS 222 - SECTION 1

## EXAM 3 - Solutions

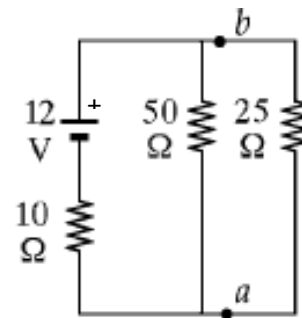
April 1, 2009

1. [20pt] Consider the circuit shown at right.

(a) [5pt] If the  $50\Omega$  and  $25\Omega$  resistor are considered together as a single resistor between points  $a$  and  $b$ , what is the resistance of this single resistor?

$$\frac{1}{R_{eff}} = \frac{1}{50\Omega} + \frac{1}{25\Omega} = 0.06\Omega^{-1},$$

$$R_{eff} = 16.7\Omega.$$



(b) [5pt] What is the electric potential  $V_{ba} = V_b - V_a$  of point  $b$  relative to point  $a$ ?

The total resistance of all three resistors is the sum of  $10\Omega$  in series with  $16.7\Omega$  from part (a), giving a total  $R_T = 26.7\Omega$ . The current supplied by the battery is  $I_T = \frac{12V}{R_T} = 0.450$  A. Thus,  $V_{ba} = I_T R_{eff} = 7.52$  V.

(c) [6pt] What is the current flowing through each of the three resistors? Put your answers in the appropriate box below.

The current through the  $10\Omega$  resistor is  $I_T = 0.450$  A from the solution to part (b). The currents through the other resistors are, respectively,  $\frac{V_{ba}}{50\Omega} = 0.150$  A and  $I_T - 0.150$  A =  $0.300$  A.

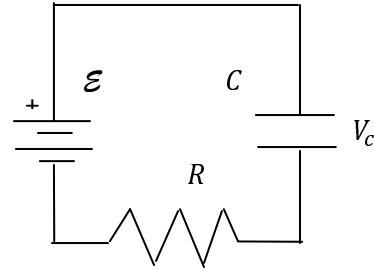
Resistor	$10\Omega$	$50\Omega$	$25\Omega$
Current	450 mA	150 mA	300 mA

(d) [4pt] What is the power  $\mathcal{P}$  supplied by the battery?

$$\mathcal{P} = (12\text{V})(0.450\text{ A}) = 5.4\text{ W}.$$


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2. [15pt] A capacitor  $C$  is charged by a battery with emf  $\mathcal{E} = 6.0\text{ V}$  through a resistance  $R = 500\text{ k}\Omega$  as shown. The current flowing from the battery after 3.0 seconds is half of initial current.



(a) [5pt] What is the initial current flowing in the circuit at the time the battery is first connected, and the capacitor is uncharged?

When the capacitor is uncharged,  $V_c = 0$ , so the initial current is

$$I_0 = 6.0\text{V}/R = 12\ \mu\text{A}.$$

(b) [5pt] What is the voltage  $V_c$  across the capacitor 3.0 seconds after it begins charging?

The current at this time is given as  $I = \frac{1}{2}I_0 = 3.0\ \mu\text{A}$ , by part (a). Thus,

$$V_c = 6.0\text{ V} - IR = 6.0\text{ V} - (3 \times 0.5)\text{ V} = 3.0\text{ V}.$$

(c) [5pt] What is the value of the capacitor  $C$  being charged?

The time dependence of the current is  $I(t) = I_0 e^{-t/RC}$ . At  $t = 3.0$  seconds,

$$\frac{I}{I_0} = \frac{1}{2} = e^{-(3.0\text{ s})/RC}$$

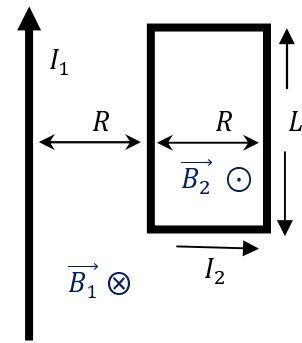
Taking the natural logarithm of both sides gives

$$\ln \frac{1}{2} = -\ln 2 = -\frac{3.0\text{ s}}{RC},$$

so that

$$C = \frac{3.0\text{ s}}{R \ln 2} = 8.66\ \mu\text{F}.$$

3. [15pt] In the figure at right, the current in the long, straight wire is  $I_1 = 20$  A and the wire lies in the plane of the rectangular loop, which carries the current  $I_2 = 12$  A. The dimensions are  $R = 0.10$  m and  $L = 0.30$  m.



- (a) [2pt] What is the direction of the magnetic field due to the long straight wire at any point to its right? [In all parts of this problem, specify direction as “left”, “right”, “up”, “down”, “into the page”, or “out of the page”.]

The direction of the magnetic field due to  $I_1$  can be found by pointing your right thumb in the direction of  $I_1$  and curling your fingers around the wire. The fingers point in the direction of the magnetic field  $\vec{B}_1$ , which is **into the page** to the right of the wire.

- (b) [2pt] What is the direction of the magnetic field due to the rectangular loop at any point inside the loop?

The same right-hand rule shows that the contribution from each side of the rectangle is directed **out of the page** at a point inside the rectangle. This is shown as  $\vec{B}_2$  in the figure above. Another right-hand rule for finding the direction of the magnetic field due to the current loop is that when you curl your fingers around the loop following the current, your thumb points in the direction of the magnetic field. This gives the same result.

- (c) [4pt] Give the direction of the magnetic force on each segment of the rectangular loop due to the current in the long straight wire. Enter “zero” if there is no force.

The magnetic force on a short length  $d\vec{l}$  of the rectangle is  $\vec{F} = I_2 d\vec{l} \times \vec{B}_1$ , where  $d\vec{l}$  points along the current, with  $\vec{B}_1$  into the page. Curling your right fingers from any of the segments into the page will cause your thumb to point inward, toward the center of the rectangle, giving the directions shown in the table on the next page.

Segment	Magnetic Force Direction
Left	Right
Right	Left
Top	Down
Bottom	Up

**(d) [5pt]** Calculate the magnitude of the total magnetic force on the rectangular loop due to the current in the long straight wire.

The forces on any segments of the top and bottom wire equidistant from current  $I_1$  will cancel, so there is no net force in the vertical direction. (This can also be inferred from the symmetry.) The force on the left segment is to the right, with positive value

$$F_L = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi R},$$

and the force on the right segment is to the left, with negative value

$$F_R = I_2 L B_1 = -\frac{\mu_0 I_1 I_2 L}{4\pi R},$$

which is half the strength of  $F_L$  since this segment is twice as far from  $I_1$ . The net force is then

$$F = F_L + F_R = \frac{\mu_0 I_1 I_2 L}{4\pi R} = \frac{(10^{-7} \text{N/A}^2)(20\text{A})(12\text{A})(0.30\text{m})}{0.10 \text{ m}} = 72.0 \mu\text{N}.$$

**(e) [2pt]** What is the direction of the force in part (d)?

As noted in the previous part, the net force is positive **to the right**.