EXAM 2 – SOLUTIONS

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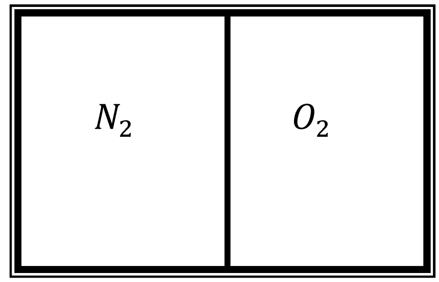
PHYS222 – Section 1, S. Yost

1. [10pt] A 2-liter container is divided into two equal parts. One side contains N_2, and the other O_2. Both are at 1 atm and 27^o C, and insulated from the surroundings. The partition is removed and the gas is allowed to mix freely.



(a) [3 pt] What is the final temperature of the mixture of gas after it reaches equilibrium? Explain your answer.

The final temperature is $\boxed{27^oC.}$ No work is done in the expansions, and no heat flows into or out of the insulated system, so the internal energy is constant, meaning the temperature is also constant. This can also be explained using kinetic theory: removing the partition does not change the instantaneous velocities of the gases, so the average kinetic energy is constant, implying that the temperature does not change.

(b) [7pt] What is the change in entropy of the system when the gases mix?

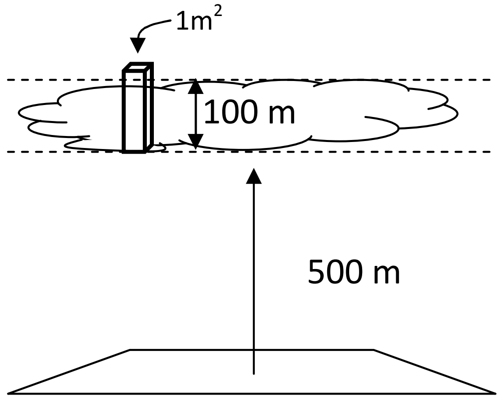
The process is not reversible, but the change in entropy is the same as for an isothermal process with the same initial and final conditions. Therefore, ΔS can be calculated for an isothermal expansion of the two gases. For each gas, $\Delta U = Q + W = 0$, so

$$\Delta S = \frac{Q}{T} = \frac{-W}{T} = \int \frac{PdV}{T} = \int \frac{nRdV}{V} = nR \ln\left(\frac{V_f}{V_i}\right).$$

Combining the two expansions and using $V_f = 2\,V_i$ in each case gives

$$\Delta S = (n_1 + n_2)R \ln 2 = \frac{PV_f}{T} = \frac{2\text{ atm} \cdot \text{L}}{300\text{ K}} \ln 2 = \boxed{0.469\,\frac{J}{K}}.$$

2. [20pt] A 100 m thick layer of clouds covers a region with the cloud base at an altitude of 500 m above the ground. The air below the clouds is clear, and may be approximated as charge free. However, static charge has built up in the clouds. An airplane measures an electric field of 110 N/C directed downward at the base of the clouds, and and 80 N/C downward at the top of the clouds.



(a) [5pt] What is the electric flux through a rectangular box of base 1 m² and height 100 m extending from the bottom of the cloud to the top? Assume the electric field is vertical throughout the cloud.

The outward flux through the top of the box is –80 Nm²/C, and the outward flux through the bottom of the box is 110 Nm²/C, giving a net flux of

$$\boxed{\Phi = 30\ \mathrm{Nm^2/C}}$$

through the box. (There is no flux through the sides, since the field is vertical.)

(b) [5pt] What is the average electric charge density (C/m³) inside the cloud? Include the correct sign.

Gauss's Law says that $\Phi = Q/\epsilon_0$ where $Q = \rho AL = (100\ \mathrm{m}^3)\rho$ in terms of the charge ρ per unit volume. Then

$$\rho = \frac{\epsilon_0 \Phi}{100\mathrm{m}^3} = \frac{\left(8.85\ \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}\right)\left(30\,\frac{\mathrm{Nm}^2}{\mathrm{C}}\right)}{100\ \mathrm{m}^3} = \boxed{2.66\,\frac{\mathrm{pC}}{\mathrm{m}^3}.}$$

(c) [5pt] What is the average surface charge density (C/m3) on the earth's surface below the cloud? Include the correct sign. You may assume the Earth is a good conductor.

Since there is no charge below the cloud, the electric field is constant between the bottom of the cloud and the Earth's surface: $E = 110\ N/C$ downward. This means the electric flux *out* of 1m^2 of the Earth's surface is $\Phi = -110\ Nm^2/C$. If the Earth is a good conductor, there is no flux just below the surface, implying that the charge in a square meter of the Earth's surface is

$$Q = \epsilon_0 \Phi = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}\right)\left(-110 \frac{\text{Nm}^2}{\text{C}}\right) = -974\ \text{pC}.$$

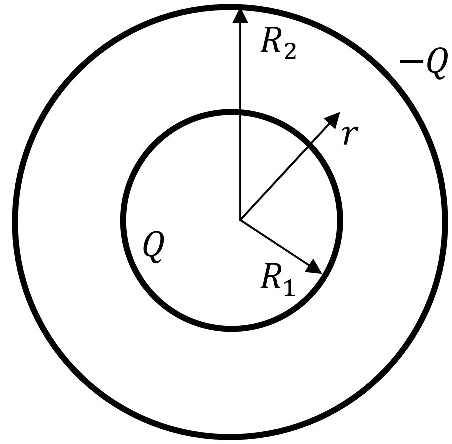
The surface charge density is then

$$\sigma = \frac{Q}{1\ m^2} = \boxed{-974 \frac{pC}{m^2}.}$$

(d) [5pt] If the ground is taken to have an electric potential of zero volts, what is the electric potential at the base of the clouds?

$$V = -Eh = \left(110 \frac{\text{V}}{\text{m}}\right)(500\ \text{m}) = \boxed{55\ \text{kV}.}$$

3. **[20 pt]** An air-filled spherical capacitor is constructed with an inner shell of radius $R_1 = 10.4\ \text{cm}$ and an outer shell of radius $R_2 = 13.6\ \text{cm}$.



(a) [6pt] If a charge of Q is placed on the inner conductor, and $-Q$ on the outer conductor, find the electric field as a function of r for all three regions:

$$r < R_1, \quad R_1 < r < R_2, \quad r > R_2.$$

Use $1/4\pi\epsilon_0$ for the electric force constant.

Gauss's Law implies that $E = 0$ inside the smaller sphere or outside the larger one. Between, them, Gauss's Law gives the same results as for a point charge at the center. Therefore:

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{if } R_1 < r < R_2, \qquad E = 0 \quad \text{elsewhere.}}$$

(b) [5pt] Find the electric potential difference between the inner and outer conductor symbolically.

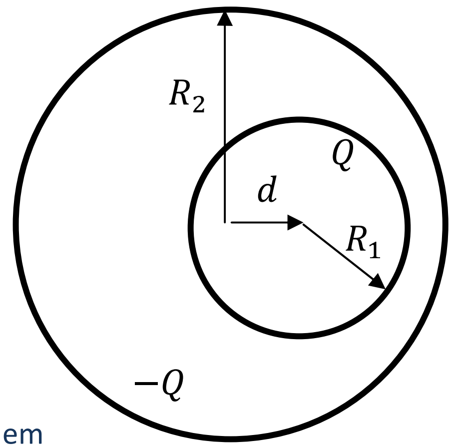
For $R_1 < r < R_2$, the definition of electric potential gives

$$V(r) = -\int_{R_2}^{R_1} E dr = \int_{R_1}^{R_2} \frac{Q dr}{4\pi\epsilon_0 r^2} = \boxed{\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).}$$

(c) [5pt] What is the capacitance of the spherical capacitor? Give a numerical result with units.

$$C = \frac{Q}{\Delta V} = 4\pi\epsilon_0\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1} = 12.57 \times \left(8.85\frac{\mathrm{pF}}{\mathrm{m}}\right) \times 0.442\ \mathrm{m} = \boxed{49.2\ \mathrm{pF}.}$$

(d) [4pt] If the inner conductor is moved a distance d to the right, as in the figure, how does each of the following change? The answer to each is "increases", "decreases", or "stays the same". No explanation is requested.



- The attractive force between the spheres?

  Increases. It was initially zero, but in unstable equilibrium, and the charges attract as they are brought together.

- The energy stored in the electric field?

  Decreases. Since opposite charges attract, bringing them together decreases the potential energy U, which may be considered to be the energy stored in the field.

- The potential difference between the conductors?

  Decreases. The stored energy $U = \frac{1}{2}Q\Delta V$ is proportional to the potential difference.

- The capacitance?

  Increases. The stored energy is inversely proportional to C for fixed charge: $U = Q^2/2C$. Since the stored energy decreases, the capacitance must increase.