EXAM 1 – SOLUTIONS

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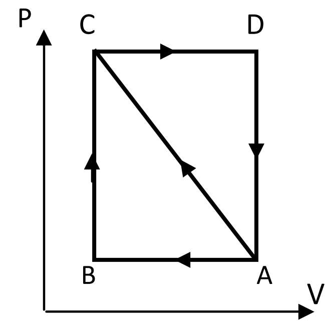
1. **Problem 1: [20pt]** A gas is kept in a cylinder fitted with a piston. In the figure, the change in internal energy of a gas that is taken from A to C along the diagonal path is +800 J. The work done on the gas when it is compressed isobarically from point A to point B is +500 J.



(a) [5pt] How much heat is added to or removed from the gas as it goes from A through B and on to C?

Use the First Law of Thermodynamics:

$E_C - E_A = 800\text{J} = Q_{ABC} + W_{ABC} = Q_{ABC} + 500\text{J}$ implies $Q_{ABC} = 300\text{J}$.

(b) [5pt] If the pressure at point C is five times that of point B, how much work does the gas do on the piston when it expands from C to D?

The work done on the gas is $W_{CD} = -P_C|\Delta V|$ and $W_{AB} = P_A|\Delta V|$, so

$$W_{CD} = -W_{AB}\frac{P_C}{P_A} = 500\text{J} \times 5 = -2500\text{J}.$$

The work done on the piston is then 2500 J.

(c) [5pt] How much heat flows into or out of the gas when it goes from C to D and back to A. Specify whether the flow is into or out of the gas.

$Q_{CDA} + W_{CDA} = E_A - E_C = -800\text{J}$. Since $W_{CDA} = W_{CD} = -2500\text{J}$, $Q_{CDA} = 1700\text{J}$.

This heat flows into the gas.

(d) [5pt] How much work is done on the gas in the diagonal process AC?

In the closed cycle ABCD, the net work done on the gas is $W_{AB} + W_{CD} = -2000$ J. The work is minus the area inside the rectangle. This implies that the work done in the process ACD must be half this: $W_{ACDA} = -1000\text{ J} = W_{AC} + W_{CD} = W_{AC} - 2500\text{J}$, giving $W_{AC} = 1500$ J. Using the triangle ABC gives the same result, since $W_{AB} + W_{CA} = 500\text{ J} - 1500\text{ J} = -1000\text{ J} = W_{ABCA}$. ($W_{CA} = -W_{AC}$.)

2. [15pt] During the compression stroke of a certain gasoline engine, the pressure increases from 1.00 atm to 20.0 atm. The process is adiabatic and the fuel-air mixture behaves as a diatomic ideal gas. The volume of the piston is 500 cm^3 at the beginning of the compression stroke.

(a) [5pt] What is the volume of the gas at the end of the compression stroke?

In an adiabatic process, PV^γ is constant, with $\gamma = \frac{c_P}{c_V} = \frac{7}{5}$ for a diatomic gas. Therefore, at the end of the adiabatic compression,

$V_f^\gamma = V_i^\gamma \frac{P_i}{Pf_i} = \frac{V_i^\gamma}{20} = 0.050\, V_i^\gamma$. Therefore,

$$V_f = (0.050)^{5/7} V_i = 0.118 \times 500 \text{ cm}^3 = 58.8 \text{ cm}^3.$$

(b) [5pt] If the gas starts out at 30.0°C, what is its temperature at the end of the compression stroke? [You may express your answer in °C or K.]

Since $T_i\ V_i^{\gamma-1} = T_f\ V_f^{\gamma-1}$, $\quad T_f = \left(\frac{V_i}{V_f}\right)^{0.4} T_i = (0.118)^{-0.4}\, T_i = 2.35\, T_i$.

This expression is for absolute temperatures, so $T_i = 303\ K$, and $T_f = 712 \text{ K} = 439^\text{o}\text{C}$.

(c) [5pt] Find the work done on the gas to compress the gas. [If you don't have an answer to part (b), use the final temperature 630°C.]

Since the process is adiabatic, $Q = 0$ by definition. The work and change in internal energy are the same by the first law of thermodynamics:

$$W = \Delta E = nC_V \Delta T = \frac{5}{2} nR(T_f - T_i) = \frac{5}{2} nRT_i\left(\frac{T_f}{T_i} - 1\right) = \frac{5}{2} P_i V_i \left(\frac{T_f}{T_i} - 1\right)$$

$$= 2.5\left(1.01 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)(500 \times 10^{-6} \text{m}^3) \times 1.34 = 169 \text{ J}.$$

Note: $1 \text{ cm}^3 = (0.01 \text{ m})^3 = 10^{-6} \text{m}^3$.

[The alternative answer for $T_f = 630^o C = 903$ K is 250 J.]

3. [15pt] A refrigerator has a coefficient of performance of 3.00. The ice tray compartment is at $-20.0°\text{C}$, and the room temperature is $16.0°\text{C}$. The refrigerator can convert 36.0 g of water at 16.0°C to 36.0 g of ice cubes at $-20.0°\text{C}$ each minute.

(a) [5pt] How much heat must be removed from the water each minute to freeze it into ice cubes at the given temperature?

The heat removed from the water each minute to freeze it into ice is

$$Q_C = mc_w(T_w - 0^o) + mL_f + mc_i(0^o - T_i)$$

$$= (36.0\text{ g})\left[16 \times 4.186\frac{\text{J}}{\text{g}} + 333\frac{\text{J}}{\text{g}} + 20 \times 2.090\frac{\text{J}}{\text{g}}\right] = 15.9\text{ kJ}.$$

(b) [5pt] If the coefficient of performance of the refrigerator is 3.00, how much power, in Watts, is needed to run it? [If you don't have an answer to part (a), assume 20.0 kJ of heat is removed per minute.]

The coefficient of performance is $\text{CoP} = 3.00 = Q_C/W$, so the work per minute is $W = \frac{Q_C}{3.00} = 5301\,J$. The power in Watts is the work per second, $\mathcal{P} = \frac{W}{60s} = 88.4\text{ W}$.

[The alternative answer for $Q_C = 20.0\text{ kJ}$ is $W = 6667\text{ J}$, $\mathcal{P} = 111\text{ W}$.]

(c) [5pt] If the refrigerator had the maximum theoretical efficiency for the given temperatures, how much power, in Watts, would be needed to run it?

The most efficient process is the Carnot cycle, for which

$$\text{CoP} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C} = \frac{253\text{ K}}{36\text{ K}} = 7.03.$$

The power needed to run the refrigerator is inversely proportional to the CoP, so the power needed is $\mathcal{P} = \left(\frac{3.00}{7.03}\right) \times 88.4\text{W} = 37.7\text{ W}$. Note that you must use absolute temperatures in this problem, not Celcius.