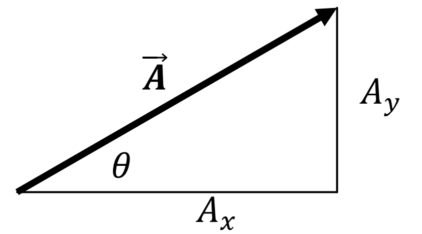
Equations for Physics 221

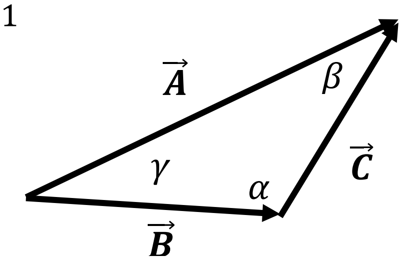
Mathematics

$Ax^2 + Bx + C = 0 \quad \Rightarrow \quad x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$



$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} \qquad A_x = A\cos\theta \qquad A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2} \qquad \tan\theta = \frac{A_y}{A_x} \qquad \sin^2\theta + \cos^2\theta = 1$



$\vec{\boldsymbol{A}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} \qquad A_x = B_x + C_x \qquad A_y = B_y + C_y$

$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = AB\cos\gamma = A_xB_x + A_yB_y$

$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = AB\sin\gamma\,\hat{\boldsymbol{k}} = (A_xB_y - A_yB_x)\hat{\boldsymbol{k}}$

$C^2 = \left(\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}\right)^2 = A^2 + B^2 - 2AB\cos\gamma \qquad \dfrac{\sin\alpha}{A} = \dfrac{\sin\beta}{B} = \dfrac{\sin\gamma}{C}$

Circle: circumference $= 2\pi r$, area $= \pi r^2$

Sphere: surface area $= 4\pi r^2$, volume $= \frac{4}{3}\pi r^3$

$\frac{dx^a}{dx} = ax^{a-1}, \qquad \frac{d}{dx}\sin ax = a\cos ax, \qquad \frac{d}{dx}\cos ax = -a\sin ax$

$\int x^a dx = \frac{x^{a+1}}{a+1}, \qquad \int \sin ax\,dx = -\frac{1}{a}\cos ax, \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$

Kinematics

$\vec{\boldsymbol{v}}_{\text{avg}} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} \qquad \vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{dt} \qquad \vec{\boldsymbol{a}}_{\text{avg}} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} \qquad \vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt}$

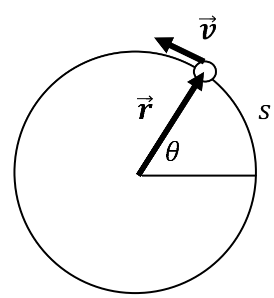
Constant acceleration: $\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t, \quad \vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2, \quad v_f^2 = v_i^2 + 2\vec{\boldsymbol{a}} \cdot (\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i)$

Relative velocities: $\vec{\boldsymbol{v}}_{CA} = \vec{\boldsymbol{v}}_{CB} + \vec{\boldsymbol{v}}_{BA}$

Circular motion:

$s = r\theta \qquad \omega = \frac{d\theta}{dt} \qquad v = r\omega \qquad a_c = \frac{v^2}{r} = r\omega^2$

Uniform: $v = \frac{2\pi r}{t}$ Nonuniform: $a_t = \frac{dv}{dt}$



Forces

$\vec{\boldsymbol{F}}_{\text{net}} = m\vec{\boldsymbol{a}}$ $\quad$ $\vec{\boldsymbol{F}}_{AB} = -\vec{\boldsymbol{F}}_{BA}$ $\quad$ Elasticity (Hooke's Law): $F = -kx$

Gravity: $F_g = \frac{Gm_1m_2}{R^2}$, $G = 6.67 \times 10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2}$. On Earth: $F_g = mg$, $g = \frac{GM_E}{R_E^2} = 9.80\ \text{m/s}^2$

Friction: $F < \mu_s F_N$ or $F = \mu_k F_N$. $\quad$ Viscosity: $F = -bv$. $\quad$ Air resistance: $F = \frac{1}{2}D\rho A v^2$.

Work and Energy

$\text{W} = \int \vec{\mathbf{F}} \cdot \text{d}\vec{\mathbf{r}}$ or $W = \vec{\boldsymbol{F}} \cdot \Delta\vec{\boldsymbol{r}}$ or $W = \int F(x)dx$ or $W = F\Delta x$

$K = \frac{1}{2}mv^2$ $\quad$ $\Delta K = W$ $\quad$ $\Delta U = -W$ $\quad$ $E = K + U$ $\quad$ $F = -\frac{dU}{dx}$

$\Delta E = W_{nc}$ $\quad$ $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$ $\quad$ Elasticity (Hooke's Law): $U = \frac{1}{2}kx^2$

Gravity: $U = -\frac{Gm_1m_2}{R}$ with $U = 0$ at $R = \infty$. On Earth: $U = mgh$ with $U = 0$ at $h = 0$.

Momentum, Impulse, and Center of Mass Motion

$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$ $\quad$ $\vec{\boldsymbol{F}} = \frac{d\vec{\boldsymbol{p}}}{dt}$ $\quad$ $\vec{\boldsymbol{I}} = \vec{\boldsymbol{F}}_{\text{avg}}\Delta t = \Delta\vec{\boldsymbol{p}}$

$\vec{\boldsymbol{x}}_{\text{cm}} = \frac{1}{M_{\text{total}}}\int \vec{\boldsymbol{x}}dm$ $\quad$ $\sum \vec{\boldsymbol{p}} = M_{\text{total}}\vec{\boldsymbol{v}}_{\text{cm}}$ $\quad$ $\sum \vec{\boldsymbol{F}} = M_{\text{total}}\vec{\boldsymbol{a}}_{\text{cm}}$

Collisions: $\sum \vec{\boldsymbol{p}}_i = \sum \vec{\boldsymbol{p}}_f$ $\quad$ Elastic collisions: $\sum K_i = \sum K_f$, $v_2^i - v_1^i = v_1^f - v_2^f$

Rigid Body Motion

Substitututions: $x \to \theta$, $v \to \omega$, $a \to \alpha$, $m \to I$, $\vec{\boldsymbol{F}} \to \vec{\boldsymbol{\tau}}$, $\vec{\boldsymbol{p}} \to \vec{\boldsymbol{L}}$

$\omega = \frac{d\theta}{dt}$ $\quad$ $\alpha = \frac{d\omega}{dt}$ $\quad$ Rolling: $v = r\omega$, $a = r\alpha$

$I = \int r^2 dm$ $\quad$ $K_{\text{rot}} = \frac{1}{2}I\omega^2$ $\quad$ $K = K_{\text{cm}} + K_{\text{rot}}$ $\quad$ $L = I\omega$

$\tau = rF\sin\theta$ $\quad$ $\tau = I\alpha$ $\quad$ $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$ $\quad$ $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$ $\quad$ $\vec{\boldsymbol{\tau}} = \frac{d\vec{\boldsymbol{L}}}{dt}$

Orbits

Circular orbit: $F = m\frac{v^2}{R} = \frac{GMm}{R^2}$ $K = \frac{1}{2}m_2 v^2 = \frac{GMm}{2R}$ $E = K + U = -\frac{GMm}{2R}$

Any orbit: $E =$ constant, $\vec{\boldsymbol{L}} = m\vec{\boldsymbol{r}} \times \vec{\boldsymbol{v}} =$ constant.

Earth: $M_E = 5.98 \times 10^{24}\text{kg}$, $R_E = 6.37 \times 10^6\text{m}$, $g = \frac{GM_E}{R_E^2} = 9.8\frac{\text{m}}{\text{s}^2}$

Fluids

Pressure: $P = \frac{F}{A}$ $P = P_0 + \rho g h$

Buoyancy: $B = \rho_{\text{fluid}}\, gV$

Atmospheric pressure: $1\text{ Atm} = 1.013 \times 10^5\text{ N/m}^2$

Density of water: $\rho_w = 1\text{ g/cm}^3 = 1000\text{ kg/m}^3$

Volume flux: $Av =$ constant

Bernoulli equation: $P + \frac{1}{2}\rho v^2 + \rho g h =$ constant

Oscillation

$$F = -kx, \quad a = -\frac{k}{m}x, \quad x = A\cos(\omega t + \phi_0), \quad v_{\text{max}} = \omega A, \quad a_{\text{max}} = \omega^2 A$$

$\omega = \sqrt{\frac{k}{m}}$, $T = \frac{2\pi}{\omega}$, $K = \frac{1}{2}mv^2$, $U = \frac{1}{2}kx^2$, $E = K + U =$ constant

Units

$1\text{ N} = 1\text{ kg m/s}^2$ $1\text{ J} = 1\text{ Nm}$ $1\text{ W} = 1\text{ J/s}$ $1\text{ h.p.} = 746\text{ W}$

1 A.U. = earth-sun distance = $1.496 \times 10^{11}\text{m}$ 1 year = $1.356 \times 10^7\text{s}$

1 degree = $\frac{\pi}{180}$ rad 1 Pa = 1 N/m^2