

PHYSICS 221 EXAM 4: SOLUTIONS

December 8, 2008

Problem 1: [15pt] A crate of mass $M = 184$ kg suspended at the end of beam of mass $m = 72$ kg and length $L = 1.20$ m. A wire is attached to the wall as shown, with $\theta = 30^\circ$.

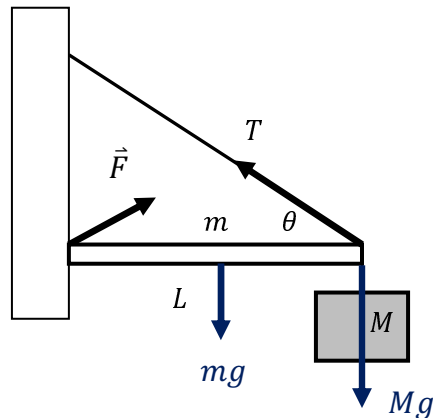
(a) [5pt] Find the tension T in the wire.

Balancing torques about the left end of the rod gives

$$\frac{1}{2} L m g + L M g = L T \sin \theta$$

with $\sin \theta = \frac{1}{2}$. Then

$$T = (m + 2M)g = \boxed{4312 \text{ N.}}$$



(b) [10pt] Find the components F_x, F_y of the force \vec{F} supporting the left end of the beam.

Balancing forces in the x direction gives

$$F_x = -T \cos \theta = \boxed{3734 \text{ N.}}$$

Balancing forces in the y direction gives

$$F_y = (M + m)g - T \sin \theta = \frac{1}{2} m g = \boxed{353 \text{ N.}}$$

Alternatively, balancing torques about the right end of the rod would give the same result, since

$$L F_y - \frac{1}{2} L m g = 0.$$

Problem 2: [15pt] A satellite weighing 1200 N on Earth is placed in a circular orbit around the Earth. The radius of the orbit is 4.0 Earth radii, so that the satellite is at a constant altitude of 3.0 earth radii above Earth's surface. The Earth's radius is $R_E = 6370$ km.

(a) [5pt] What is the magnitude of the gravitational force on the satellite in orbit?

By the inverse square law,

$$F_g = mg/16 = \boxed{75 \text{ N.}}$$

It is a lot more work, but you could also use Newton's law directly, finding $F_g = GM_E m/R^2$ with $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, $M_E = 5.98 \times 10^{24} \text{ kg}$, satellite mass $m = 1200 \text{ N} / 9.8 \text{ m/s}^2 = 122 \text{ kg}$, and $R = 4R_E = 25.5 \times 10^6 \text{ m}$. Numerically, this gives the same result.

(b) [10pt] How much work was needed to put the satellite into orbit? Neglect the Earth's rotation, and assume it started at rest on the Earth's surface.

The work is the change in the total energy of the satellite to go from the ground into orbit. On the ground, the energy is potential: $E_i = -GM_E m/R_E$. In orbit, the sum of the kinetic and potential energy is $E_f = -\frac{1}{2}GM_E m/R = -\frac{1}{8}GM_E m/R_E$. The work done to put the satellite into orbit is then

$$W = E_f - E_i = \frac{7GM_E m}{8R_E} = \frac{7}{8}mgR_E = \frac{7}{8} \times 1200 \text{ N} \times (6.37 \times 10^6 \text{ m}) = \boxed{6.69 \times 10^9 \text{ J.}}$$

The fact that $g = GM_E m/R_E^2$ was used to simplify the calculation. Using the numerical values $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, $M_E = 5.98 \times 10^{24} \text{ kg}$, and $m = 1200 \text{ N} / 9.8 \text{ m/s}^2 = 122 \text{ kg}$ would give an equally good result, $W = 6.68 \times 10^9 \text{ J}$.

Note that the equation for the energy in orbit is obtained by writing the kinetic energy as $K = \frac{1}{2}mv^2 = \frac{1}{2}RF_G = \frac{1}{2}GM_E m/R$, which follows from the centripetal force equation $F_G = mv^2/R$. Adding the gravitational potential energy $U = -GM_E m/R$ gives total energy $E = -\frac{1}{2}GM_E m/R$ in a circular orbit of radius R .

Problem 3: [15pt] Water is kept in a large open tank with a hose attached to the bottom. The hose has a diameter of 2.4 cm. The top of the water in the tank is 7.5 m above the hose.

- (a) [6pt] When the hose is plugged with a rubber stopper, what is the friction force needed to hold the stopper in?

The pressure due to the water on the stopper is

$$P = \rho gh = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) (7.5 \text{ m}) = 7.35 \times 10^4 \text{ N/m}^2.$$

The cross-sectional area of the hose is $A = \pi R^2 = \pi \times (0.012 \text{ m})^2 = 4.52 \times 10^{-4} \text{ m}^2$.

The force needed to hold the stopper in is then $F = PA = \boxed{33.2 \text{ N}}$.

- (b) [9pt] When the stopper is removed, what is the flow rate in kg/s of water from the hose?

Comparing a point at the top of the water in the tank to a point just outside the hose, and using Bernoulli's equation, gives $P + \rho gh = P + \frac{1}{2}\rho v^2$, where $P = 1 \text{ atm}$ in either case. (The speed at which water drops at the top of the tank can be neglected because it is said to be large.) Therefore, the flow speed is $v = \sqrt{2gh} = 12.1 \text{ m/s}$. The mass rate of flow from the pipe is the volume rate of flow times the density ρ of the water:

$$\rho v A = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(12.1 \frac{\text{m}}{\text{s}}\right) (4.52 \times 10^{-4} \text{ m}^2) = \boxed{5.47 \frac{\text{kg}}{\text{s}}}.$$

Problem 4: [15pt] A 0.50 kg object is attached to a light-weight spring with force constant 8.00 N/m. It vibrates in simple harmonic motion with an amplitude of 12.0 cm.

- (a) [5pt] What is the maximum acceleration of the mass?

If $A = 12.0 \text{ cm}$ is the amplitude, the maximum force is $F_{\text{max}} = kA$, and the maximum acceleration is

$$a_{\text{max}} = kA/m = (8.00 \text{ N/m}) (0.12 \text{ m}) / 0.50 \text{ kg} = \boxed{1.92 \text{ m/s}^2}.$$

(b) [5pt] What is the maximum speed of the mass?

Energy conservation implies that $\frac{1}{2} kA^2 = \frac{1}{2} mv_{\max}^2$, so

$$v_{\max} = A \sqrt{\frac{k}{m}} = 0.12 \text{ m} \times \sqrt{\frac{8.00 \text{ N/m}}{0.50 \text{ kg}}} = \boxed{0.48 \text{ m/s.}}$$

(c) [5pt] How long does the object take to complete one oscillation (back and forth)?

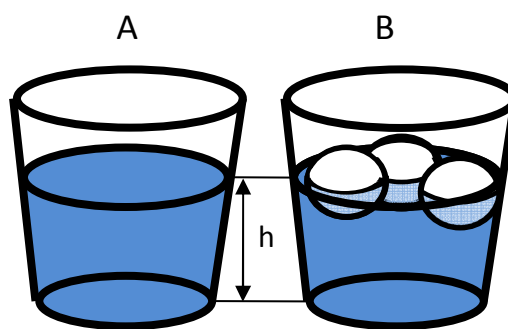
The period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.50 \text{ kg}}{8.00 \text{ N/m}}} = \frac{\pi}{2} \text{ s} = \boxed{1.57 \text{ s.}}$$

The following problem is extra credit, and can increase your score by a maximum of 3 points. You cannot get more than 60 points on this exam via extra credit, however.

Bonus Question: [3pt] Two identical glasses are filled to the same level. Glass A contains only water, and glass B contains water with some plastic balls floating in it. Which glass weighs more?

- (a) Glass A.
- (b) Glass B.
- (c) Their weights are equal.
- (d) More information is needed to decide.



Circle your answer [1pt] and explain it briefly [2pt] using physical concepts.

The fact that the balls in glass B are floating means they weigh as much as the water that would occupy their submerged portions. But this is the same amount of water that replaces the balls in glass A. Therefore, the two glasses have the same weight.