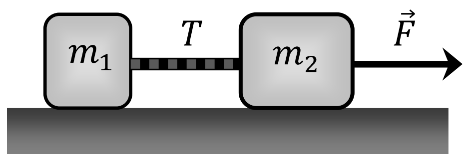
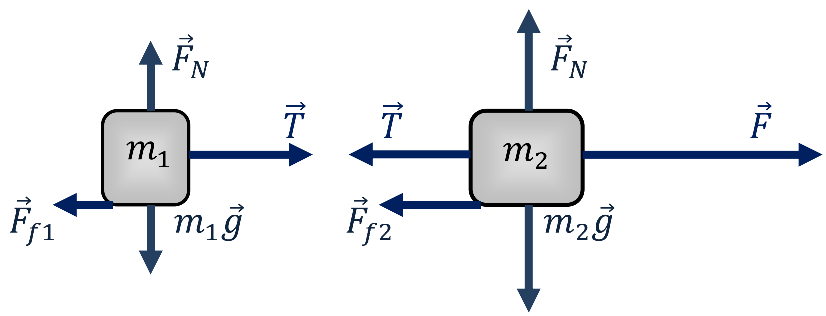
PHYSICS 221 EXAM 2: SOLUTIONS

October 15, 2008

Problem 1: [22pt] Two blocks of mass $m_1 = 7.2$ kg and $m_2 = 16.8$ kg are connected by a rope of negligible mass, and are being dragged by a horizontal force $\vec{F}$. The coefficient of kinetic friction between each block and the surface is $\mu = 0.286$.



(a) [4pt] Draw a free body diagram for each block, showing and labeling all forces acting on the block.



(b) [6pt] Write Newton's Law for the horizontal motion of each block. You should show two equations labeled to match your figure in part (a). Don't try to solve them yet.

Block 1: $\boxed{T - F_{f1} = m_1 a}$ Block 2: $\boxed{F - F_{f2} - T = m_2 a}$

(c) [6pt] Determine the acceleration of the system if $F = 108$ N.

Adding the previous equations gives $F - F_{f1} - F_{f2} = (m_1 + m_2)a$ with $F_{f1} = \mu m_1 g$ and $F_{f1} = \mu m_1 g$. Therefore, $(m_1 + m_2)a = F - \mu(m_1 + m_2)g$, giving

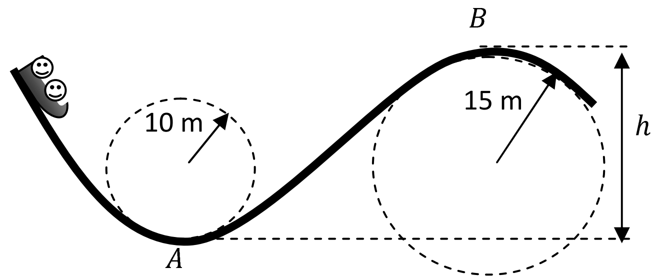
$$a = \frac{F}{m_1 + m_2} - \mu g = 4.50\,\frac{\mathrm{m}}{\mathrm{s}^2} - 2.80\,\frac{\mathrm{m}}{\mathrm{s}^2} = \boxed{1.70\ \mathrm{m/s^2}.}$$

(d) [6pt] Find the tension T in the rope in this case.

The result of part (b) for block 1 gives

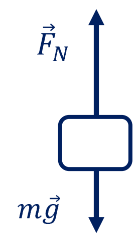
$$T = m_1 a + F_{f1} = m_1(a + \mu g) = 7.2\ \mathrm{kg} \times 4.50\,\frac{\mathrm{m}}{\mathrm{s}^2} = \boxed{32.4\ N.}$$

Problem 2: [18pt] A roller coaster car of mass 500 kg including passengers travels on a track as shown. The radius of curvature of the track at point A is 10.0 m, and the radius of curvature at point B is 15.0 m. Neglect friction.



(a) [6pt] If the car has a speed of 20.0 m/s at point A, what is the force of the track on the car at this point?

The centripetal acceleration is $v^2/R = 40$ m/s^2 upward. This is due to the net force, which is a sum of the normal force $\vec{F}_N$ of the track on the car and the weight $m\vec{g}$ of the car. Therefore,



$$F_N - mg = ma = 20{,}000 \text{ N}.$$

The weight of the car is $mg = 4900$ N, so $\boxed{F_N = 24{,}900 \text{ N.}}$

(b) [6pt] What is the maximum speed the car can have at point B for the passengers to remain in their seats without the help of their seat belts?

In the limiting case, the normal force vanishes, and the centripetal acceleration is due only to the force of gravity, so that

$$v^2/R = g = 9.8 \text{ m/s}^2.$$

Now $R = 15$ m, giving

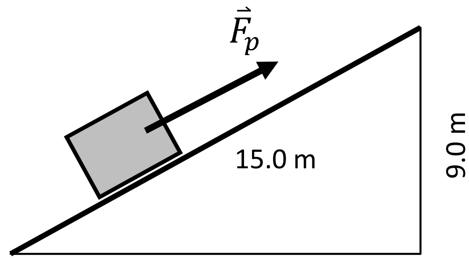
$$v = \sqrt{Rg} = \boxed{12.1 \text{ m/s.}}$$

(c) [6pt] If the car reaches point B at the speed found in part (b), how high is point B above point A? Find the distance h marked next to the figure. [If you don't have an answer to point (b), you can solve this assuming the speed at point B is 10 m/s.]

Energy conservation implies $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgh$. Therefore,

$$h = \frac{v_A^2 - v_B^2}{2g} = \boxed{12.9 \text{ m.}}$$ [alternative answer: 15.3 m]

Problem 3: [20pt] A crate of mass 10.0 kg is pulled up a rough incline of length 15.0 m, beginning at rest. The pulling force $\vec{F}_p$ has a magnitude of 99.0 N, directed parallel to the incline as shown. The coefficient of kinetic friction is $\mu = 0.400$. When answering the following questions, be careful to **include correct signs**.



(a) [4pt] How much work is done by $\vec{F}_p$ on the crate?

$$W_p = F_p \times 15\text{ m} = \boxed{1485\text{ J.}}$$

(b) [4pt] How much work is done by gravity on the crate?

$$W_g = -mg \times 9.0\text{ m} = \boxed{-882\text{J.}}$$

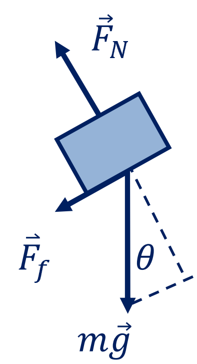
This is negative because gravity opposes the uphill motion.

(c) [6pt] How much work is done by friction on the crate?

$W_f = -F_f L$ with L =15 m. The frictional force is $F_f = \mu F_N = \mu mg\cos\theta$ where θ is the angle of incline of the ramp. Therefore,

$$W_f = -\mu mg\, L\cos\theta = \mu mg \times 12\text{ m}$$

$$= \boxed{-470\text{ J.}}$$



(d) [6pt] How fast is the crate moving at the top of the ramp?

The work-energy theorem implies that $K = W_p + W_g + W_f = 133$ J.

Then, since $K = \frac{1}{2}mv^2$,

$$v = \sqrt{\frac{2K}{m}} = \boxed{5.16 \text{ m/s}}$$

The following problem is extra credit, and can increase your score by a maximum of 3 points. You cannot get more than 60 points on this exam via extra credit, however.

Bonus Question: [3pt] A stone is launched into the air. In addition to the force of gravity, it is subject to air resistance. Taking this into account, the time the stone takes to reach the top of its path is

(a) greater than
(b) less than
(c) the same as

the time it takes to fall from the top of its trajectory to its original position.

Circle your answer [1pt] and explain it briefly [2pt].

On the way up, the weight and air resistance both act in the same direction, so the net force is *bigger* than the weight, giving $|a_{\text{up}}| > g$. On the way down, the weight and air resistance act in opposite directions, so the net force is *less* than the weight, giving $a_{\text{down}} < g$. Since the distance is $h = \frac{1}{2}at^2$ either way, the larger acceleration going up corresponds to a shorter time than coming down.