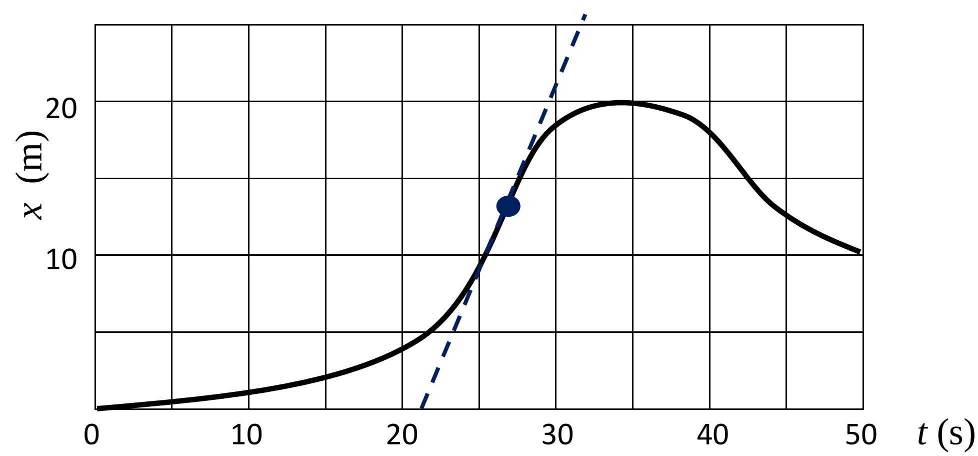
PHYSICS 221 EXAM 1: SOLUTIONS

September 15, 2008

Problem 1: [20pt] A rabbit runs through a straight pipe. The position x measured from the end of the pipe is shown in the following figure as a function of time.



Questions (a) – (d) **do not** require an explanation.

(a) [3pt] What is the rabbit's displacement for the 50 second trip?

$x_f - x_i =$ 10 m

(b) [3pt] How far does the rabbit travel during this time (total distance of trip)?

The total distance is 20 m out + 10 m back = 30 m.

(c) [3pt] What is the rabbit's average velocity?

Average velocity = displacement / time : 10 m/50 s = 0.20 m/s.

(d) [3pt] What is the rabbit's average speed?

Average speed = distance traveled / time : 30 m/50 s = 0.60 m/s.

Questions (e) – (f) require a **reason** as part of the answer.

(e) [4pt] Estimate the rabbit's maximum velocity using a line **you draw** on the above graph.

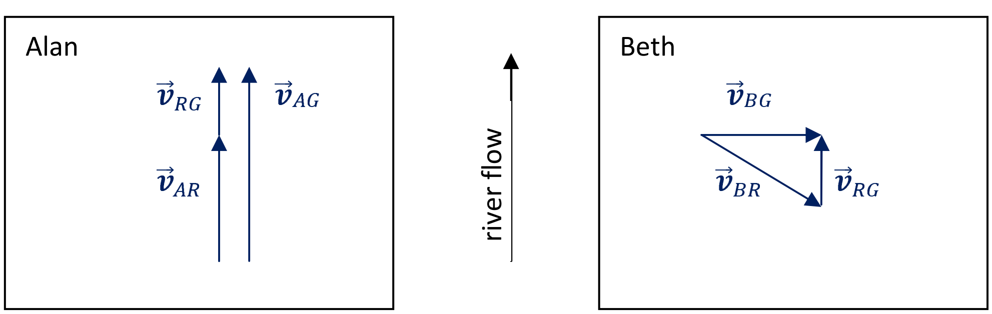
The maximum velocity is where the slope is steepest. Drawing a line tangent to the curve shows that this velocity is approximately 25 m / (32 – 21 s) = 23 m/s. Reasonable answers could be in the 20 m/s to 25 m/s range.
Pitfall: A lot of people picked a reasonable time, but then said $v = x/t$ = 13 m / 27 s, rather than correctly finding $v = \frac{\Delta v}{\Delta t}$ for a tangent line drawn along the curve.

(f) [4pt] Estimate the time when the rabbit had its greatest acceleration. **Explain** your choice.

The greatest acceleration is where the slope *changes* the fastest. This is happening in the region between about 20 and 24 seconds where the curvature is greatest. A good estimate of the time when the acceleration is greatest would then be about 22 s.
Pitfall: A lot of people said the acceleration was greatest where the slope was greatest. This is where the *velocity* is greatest, not the acceleration. The acceleration is actually zero where the slope is greatest, because $a = dv/dt = 0$ when $v = dx/dt$ is a local maximum!

Problem 2: [20pt] Alan and Beth start together at the same point on the bank of a 50-m wide stream that flows with speed 1.0 m/s. Both swimmers can swim at an equal speed of 2.5 m/s in still water. Alan swims downstream 50 m and then swims back, while Beth swims to a point directly across the river and returns to the starting point.

(a) [4pt] **Draw and label** vector diagram showing the following vectors **for each swimmer** in the labeled boxes as they travel away from the starting point: The velocity of the swimmer relative to the river, the velocity of the river relative to the ground, and the velocity of the swimmer relative to the ground. The direction of the river flow is shown. Assume both swimmers start on the left bank.



Note that

$$\vec{v}_{AG} = \vec{v}_{AR} + \vec{v}_{RG},$$

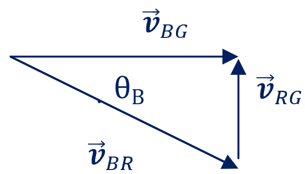
$$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG}.$$

(b) [4pt] How long does Alan take to return to the starting point (round trip)?

Downriver: t_1 = 50 m / (3.5 m/s) = 14.3 s.
Upriver: t_2 = 50 m / (1.5 m/s) = 33.3 s.
Total: $t_A = t_1 + t_2 =$ 47.6 s.

(c) [4pt] What are the magnitude and direction of Beth's velocity vector $\vec{v}_{BR}$ **relative to the river** on the way to the other side? Take the direction across the river to be zero degrees, and down the river to be 90°.

The magnitude of $\vec{v}_{BR}$ is Beth's speed relative to the water, 2.5 m/s. From the geometry of the second figure in part (b), the y component of $\vec{v}_{BR}$ is equal to the river speed, 1.0 m/s. (She has to oppose the current to make it straight across the river.) Therefore, her angle relative to the direction across the river is determined by



$$\sin\theta_B = \frac{1.0}{2.5} = 0.40,$$

giving θ_B = 23.6° upriver.

Pitfalls: A very common error was to draw $\vec{v}_{BR}$ pointing directly across the river. But Beth's path is along her velocity relative to the ground, not the river. The vector $\vec{v}_{BG}$ points along Beth's path, while $\vec{v}_{BR}$ points in the direction she swims. A related common error was to calculate v_{BR}, and not recognize that it was given information.

(d) [4pt] What are the magnitude and direction of Beth's velocity vector $\vec{v}_{BG}$ **relative to the ground** on the way to the other side?

The direction is given: 0°, directly across the river.
The magnitude is obtained from the same figure, using the Pythagorean Theorem,

$$v_{BG} = \sqrt{v_{BR}^2 - v_{RG}^2} = \boxed{2.29 \text{ m/s.}}$$

(e) [4pt] How long does Beth take to return to the starting point (round trip)?

Her speed relative to the ground will be the same both ways, so

t_B = 100 m / (2.29 m/s) = 43.7 s.

(Beth returns first.)

Problem 3: [20pt] A ball is tossed from window at a height of 136 m in a tall building. The ball is thrown at a speed of 9.0 m/s at an angle of 25° upward from the horizontal.

(a) [4pt] What are the ball's initial velocity components v_i^x and v_i^y? Take y to be positive upward.

$v_i^x = v_i \cos\theta = \boxed{8.16 \text{ m/s.}}$ $\quad v_i^y = v_i \sin\theta = \boxed{3.80 \text{ m/s.}}$

Pitfall: A lot of people tried to solve complicated equations here, but it was asking for something very simple. Do not solve more than the question is asking.

(b) [6pt] What are the ball's velocity components v_f^x and v_f^y just before it lands?

$$v_f^x = v_i^x = \boxed{8.16 \text{ m/s.}}$$

$$\left(v_f^y\right)^2 = \left(v_i^y\right)^2 + 2gh = 2680 \text{ m}^2/\text{s}^2$$

$$v_f^y = \boxed{-51.8 \text{ m/s.}}$$

You could also solve this using the time from part (d). This was a common approach, but is more work.

Pitfall: The acceleration is only in the y direction: Don't include it in the x motion!

(c) [4pt] What are the ball's speed and angle with respect to horizontal just before it lands?

$$v_f = \sqrt{\left(v_f^x\right)^2 + \left(v_f^y\right)^2} = \boxed{52.4 \text{ m/s.}}$$

$$\theta_f = \operatorname{atan}\left(\frac{v_f^y}{v_f^x}\right) = \operatorname{atan}(-6.35) = \boxed{-81.0^\text{o}.}$$

It is also true that $v_f^2 = v_i^2 + 2gh$, with $v_i = 9.0$ m/s. If the components and direction had not been requested, you could get the final speed directly this way.

(d) [3pt] How long does it take the ball to reach the ground?

This is determined by the vertical motion:

$$t = \frac{v_f^y - v_i^y}{-g} = \frac{(51.8 + 3.8)\text{m/s}}{9.8\ \text{m/s}^2} = \boxed{5.67\ \text{s.}}$$

This is the easiest way to get the speed, but you can also get it by solving a quadratic equation for the final height, $y_f = 0 = h + v_i^y t - \frac{1}{2}gt^2$, which numerically gives

$$4.9t^2 - 3.8t - 136 = 0$$

with t in seconds. The quadratic formula then gives

$$t = \frac{3.8 \pm \sqrt{(3.8)^2 + 4 \times 4.9 \times 136}}{2 \times 4.9}\text{s} = \frac{3.8 + 51.8}{9.8}\text{s} = 5.67\ \text{s.}$$

Notice that the square root in the quadratic formula just gave $|v_f^y|$, so that a solution using v_f^y directly is easier if this is already known. (This is true in general.)

Pitfall: You can't use the answer to part (c) here: $t \neq (v_f - v_i)/g$, because the acceleration is only in the y direction, and this is not a correct relationship for the speeds when the direction is changing.

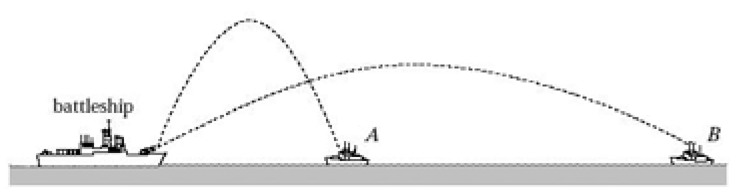
(e) [3pt] How far from the base of the building does the ball land?

$$x = v_x t = (8.16\ \text{m/s})(5.67\ \text{s}) = \boxed{46.3\text{m.}}$$

The following problem is extra credit, and can increase your score by a maximum of 3 points.

Bonus Question: [3pt] A battleship fires rounds simultaneously at ships A and B. The trajectory of each projectile is shown in the figure. Which ship is hit first? Justify your answer.

(a) Ship A.
(b) Ship B.
(c) They are hit simultaneously.
(d) More information is needed.



The time of flight is determined entirely by the vertical motion. The projectile with the higher trajectory will stay in the air longer. Therefore, ship B is hit first.