

Chapter 12: Suspension Bridge

Scott A. Yost
Baylor University

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Consider a section of a suspension bridge shown, with mass M , suspended from cables as shown. The right end of the cable is a height h above the left end, supported by a tower. If the length of the bridge is $l = 343$ m and the angle of the cable at the top of the bridge is $\theta = 30.0^\circ$, we would like to find the height of the tower.

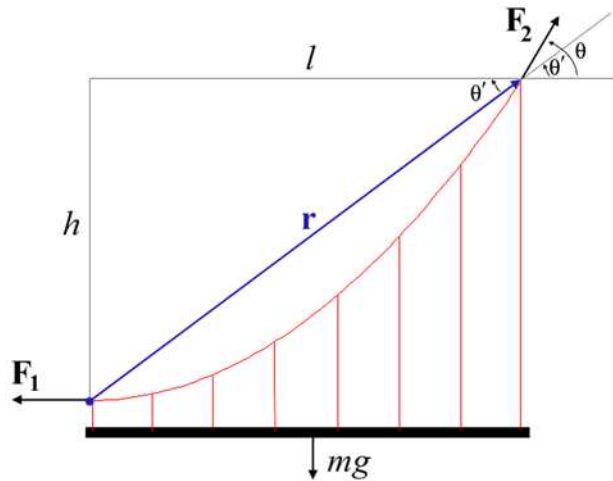


Figure 1: Suspension Bridge

The height can be determined using balance of torques about an axis, which for convenience, we take through

the point where F_1 acts. Then F_1 does not contribute to the torque. The weight of the bridge contributes a torque $Mgl/2$ in the clockwise direction, since the weight acts vertically downward a horizontal distance $l/2$ from the axis. The force F_2 contributes a torque $\mathbf{r} \times \mathbf{F}_2$, which acts in the counter-clockwise direction with magnitude $hF_2^x - lF_2^y = F_2(l \sin \theta - h \cos \theta)$. Therefore, balance of torques shows that

$$\frac{1}{2}Mgl = F_2(l \sin \theta - h \cos \theta). \quad (1)$$

We can determine F_2 in terms of the weight of the bridge by balancing forces in the vertical direction. This gives $F_2 \sin \theta = Mg$. Then the previous equation can be rewritten as

$$\frac{1}{2}Mgl = Mg \left(l - \frac{h}{\tan \theta} \right). \quad (2)$$

The masses cancel, and solving for h gives

$$h = \frac{l}{2} \tan \theta = 0.50(343 \text{ m})(0.577) = 99.0 \text{ m}. \quad (3)$$

The forces at the ends of the bridge can also be expressed in terms of the weight of the bridge. We already know $F_2 = Mg/\sin \theta = 2Mg$. Balance of forces in the horizontal

direction gives $F_1 = F_2 \cos \theta$, so $F_1 = Mg / \tan \theta = \sqrt{3}Mg$. In other words, the tensions on the ends of the cable are $F_1 = 1 / \tan \theta = \sqrt{3}$ and $F_2 = 1 / \sin \theta = 2$ in units of the bridge weight.

Since we are assuming the bridge has a uniform weight per unit length, the analysis would apply to any section of the bridge of length x , with $x = 0$ starting at the left of the bridge. If the weight per unit length is written $w = Mg/l$, the weight of the section from 0 to x would be wx , and the height of the cable at point x may be written as $y(x)$, with y varying from 0 to h between the two ends of the bridge. The angle $\theta(x)$ of the cable at point x varies from 0 at $x = 0$ to 30° at $x = l$. F_1 is fixed, but the tension $F_2(x)$ at point x varies. Balancing forces gives

$$F_1 = F_2(x) \cos \theta(x), \quad wx = F_2(x) \sin \theta(x). \quad (4)$$

Combining these and eliminating $F_2(x)$ gives $\tan \theta(x) = wx/F_1$. Using the same reasoning as above, the height of the cable at point x is

$$y = \frac{x}{2} \tan \theta(x) = \frac{wx^2}{2F_1} = \frac{x^2}{2l} \tan \theta(l) \quad (5)$$

where $\theta(l) = \theta = 30^\circ$ is the angle at the right end of the entire cable. This shows that the shape of the cable is parabolic, because the height y is proportional to x^2 . The textbook gets the same result using calculus, but is missing the factor of 2 in the denominator of the last equation.