1	2	3	4	Total	Grade
8	12	8	12	40	100

Solutions

Physics 1422 **General Physics I-A**

Exam 2 - Version A

March 19, 2007

All numerical answers require an explanation for credit, which must show symbolically any formulas you used. Numerical results must include correct units for full credit. Non-numerical answers (multiple-choice or pictures) require an explanation only if one is explicitly requested. Answers and explanations must be legible for credit. Verbal explanations after the exam will not add points to your score.

Only work on the front of the page will be graded. You may use the backs of the pages as private scratch space to plan your strategy. This may help you write more readable calculations, which will maximize partial credit. In the event of a wrong answer, more partial credit can usually be assigned to an algebraic solution than a purely numerical one. Try not to do all the work on your calculator.

If you are not sure how to work a problem in terms of formulas, or do not have time to do so, a good verbal analysis may receive partial credit. If you need a result from an earlier part of a problem to work a later part, but do not have it, an algebraic solution will recieve most of the credit.

Useful Relations

Geometry:

circumference of circle: $2\pi R$

Newton's Law: $\vec{F} = m\vec{a}$

Friction: $F_f = \mu_k F_N$ (kinetic), $F_f \leq \mu_s F_N$ (static).

Acceleration of gravity on earth: $g = 9.8\ \text{m/s}^2$

Constant acceleration:

$$\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ v_f^2 &= v_i^2 + 2a(x - x_0) \end{aligned}$$

Centripetal acceleration: $a = v^2/R$

Newton's Law of Gravity: $F = Gm_1m_2/R^2$ with $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$.

Momentum: $\vec{p} = m\vec{v}$.

Kinetic energy: $K = \frac{1}{2}mv^2$.

Gravitational potential energy: $U = mgh$ (near earth), $U = -Gm_1m_2/R$ (in general)

Astronomical Distance: 1 astronomical unit (AU) $= 150 \times 10^6$ km

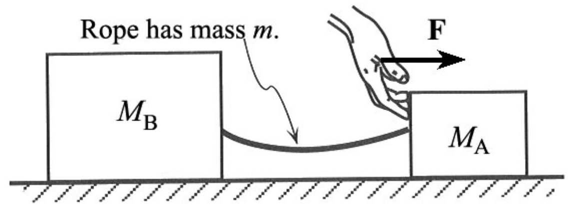
Earth Year: 1 Earth year $= 3.156 \times 10^7$ s

Kepler's Law is the relation between the radius R and period T for an orbiting object. For a circular orbit, it can be derived easily from Newton's Law, so it is not given here. For an eliptical orbit, the same relation between R and period T is obtained as for a circle, if R is taken to be the semi-major axis (half the longest distance across the ellipse).

Escape velocity is the speed a projectile must be given to escape a planet's gravity, meaning that it gets to infinite distance with zero velocity.

1. [8pt]

Two blocks on a frictionless table are connected by a heavy rope as shown. After they have been pushed for some time with constant force F, the rope settles into the constant shape shown.



(a) [2pt] Give an algebraic expression for the acceleration a of the entire system, as a function of F, M_A, M_B, and m.

By Newton's Law, the acceleration is the net force divided by the total mass,

$$a = \frac{F}{M_A + M_B + m}. \tag{1}$$

(b) [2pt] What is the horizontal component of the force pulling on the left end of the rope, in terms of F, M_A, M_B, and m?

Applying Newton's law to the block on the left shows that the net force on it is $F_A = M_A a$. By Newton's third law, this is also the horizontal component of the force on the left end of the rope. Using the equation for a in part (a), we find

$$F_A = \frac{M_A F}{M_A + M_B + m}. \tag{2}$$

(c) [2pt] What is the horizontal component of the force pulling on the right end of the rope, in terms of the same variables?

The same reasoning applied to the block on the right gives the net force on it, $F - F_B = M_B a$, where F_B denotes the horizontal force of the rope on block B. Therefore,

$$F_B = F - M_B a = F - \frac{M_B F}{M_A + M_B + m} = \frac{(M_A + m)F}{M_A + M_B + m}. \tag{3}$$

(Either of the last two expressions is an acceptable answer.)

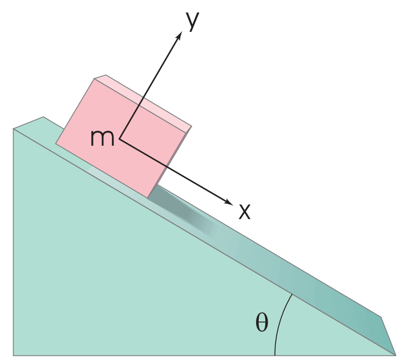
(d) [2pt] If the heavy rope were replaced by a very light string, and the same force F were applied, which of the following would increase? Indicate all correct answers.

(A) The acceleration of the system.
(B) The force on the left end of the rope.
C) The force on the right end of the rope.
D) None of the above.

The answers follow from replacing m in the previous equations by zero, and noting the effect. A lighter mass gives greater acceleration. Since F_A is proportional to a, increasing a also increases F_A. The first expression for F_B in the answer to (c) shows that when a increases, F_B decreases.

2. [12pt]

A 5.0 kg block is given an initial speed of 7.5 m/s up a rough plane inclined at $\theta = 20°$ as shown in the figure.



(a) [2pt] Write Newton's law for the block symbolically in terms of the frictional force F_f, the normal force F_N, the mass m of the block, and the angle θ. Give both the x and y components of Newton's law, using the coordinate system shown.

The net force components in the x and y directions are

$$F_x = F_f + mg\sin\theta = ma, \qquad F_y = F_N - mg\cos\theta = 0. \tag{4}$$

(b) [4pt] If the coefficient of friction is $\mu_k = 0.38$, how far up the plane does the box slide before coming to a stop? (Measure the distance along the plane.)

There are two possible solutions: using Newton's law or work and energy. First, consider Newton's Law. Using $F = ma$ in the x direction gives $a = F_f/m + g\sin\theta$, with $F_f = \mu_k F_N = \mu_k mg\cos\theta$. Note that gravity and friction both act downhill, in the $+x$ direction, since the block is sliding uphill. Then

$$a = g(\mu\cos\theta + \sin\theta) = (0.38\cos(20°) + \sin(20°))\,g = 0.699g = 6.85 \text{ m/s}^2. \tag{5}$$

The constant acceleration equation $v^2 = -2ax$ shows that x changes by $-v^2/2a = -4.11$ m. This means the block travels 4.11 m uphill before stopping.

To use work and energy, we can note that when the block travels a vertical distance h, the potential energy increases by $\Delta U = mgh = -mgx\sin\theta$, since x is measured downhill along the plane. The block is stopping during this motion, so $\Delta K = -\frac{1}{2}mv^2$. The change in total energy is the work done by friction, $W_f = \mu_k F_N x = \mu_k mgx\cos\theta$. (Remember that x is negative.) Then $W_f = \Delta K + \Delta U$ gives

$$\mu_k mgx\cos\theta = -\frac{1}{2}mv^2 - mgx\sin\theta. \tag{6}$$

Solving for x gives

$$x = \frac{-v^2}{2g(\mu_k\cos\theta + \sin\theta)}, \tag{7}$$

which is equivalent to the previous solution.

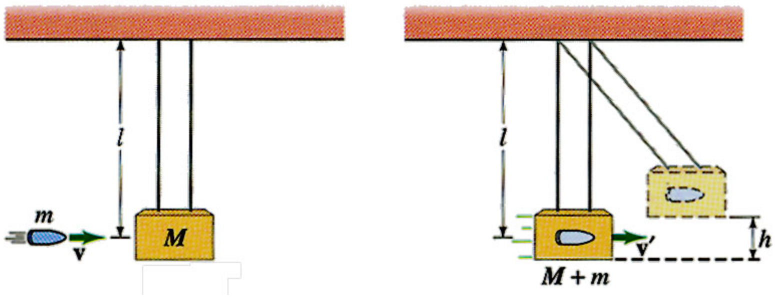
(c) [3pt] What is the total work done on the box during this motion, by all forces?

The work-energy theorem says that the work done by all forces, W, equals the change in kinetic energy $\Delta K = -\frac{1}{2}mv^2$. Therefore,

$$W = -\frac{1}{2}(5.0kg)(7.5 \text{ m/s})^2 = 141 \text{ J}. \tag{8}$$

(d) [3pt] Suppose the coefficient of static friction is $\mu_s = 0.41$. Will the box slide back down the inclined plane, or is it stuck once it stops? Explain you answer using what you know about friction and an appropriate calculation.

The box is stuck. To see this, note that to start moving, the downhill force $mg\sin\theta$ due to gravity would have to overcome static friction, which has a maximum value of $\mu_s F_N = \mu_s mg\cos\theta$. Therefore, to start moving, the angle must be great enough that $\sin\theta > \mu_s\cos\theta$, or $\tan\theta > \mu_s$. For $\theta = 20°$, $\tan\theta = 0.364$. This is smaller than $\mu_s = 0.41$, so the block doesn't move.



before **after**

3. [8pt] A 22 g rifle bullet traveling 180 m/s strikes a 3.60 kg pendulum block hanging on a 2.7 m long string.

(a) [2pt] What is the momentum of the bullet before it strikes the pendulum block?

$p = mv = (0.022 \text{ kg})(180 \text{ m/s}) = 3.96 \text{ kg m/s}.$

(b) [3pt] What is the speed v' of the pendulum block immediately after the bullet is embedded in it?

Momentum conservation implies that $p = (M + m)v'$, so

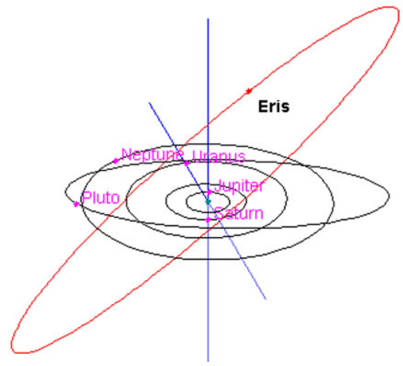
$$v' = \frac{3.96 \text{ kg m/s}}{3.622 \text{ kg}} = 1.09 \text{ m/s}. \tag{9}$$

(c) [3pt] What is the maximum height h reached by the pendulum?

Energy conservation implies that $\frac{1}{2}mv'^2 = mgh$, so

$$h = \frac{(1.09 \text{ m/s})^2}{2g} = 6.06 \text{ cm}. \tag{10}$$

4. [12pt] The ninth largest body known to be orbiting the sun is a dwarf planet named Eris, slightly larger than Pluto. It has at least one tiny moon, called Dysnomia, with an orbital radius of approximately 32,000 km and an orbital period of about 14.5 days.



(a) [4pt] Based on this information, estimate the mass of Eris.

The gravitational force on the moon is $F_g = GMm/R^2$, if M is the mass of Eris and m is the mass of Dysnomia. Newton's law implies $F_g = ma = mv^2/R$, where $R = 32 \times 10^6$ m and $v = 2\pi R/T$, with $T = 14.5$ days $= 1.25 \times 10^6$ s, giving $v = 161$ m/s. Setting $GMm/R^2 = mv^2/R$ and solving for M gives

$$M = \frac{v^2 R}{G} = \frac{(161 \text{ m/s})^2 (32 \times 10^6 \text{ m})}{6.67 \times 10^{11} \text{ Nm}^2/\text{kg}^2} = 1.24 \times 10^{22} \text{ kg}. \quad (11)$$

(b) [4pt] The semi-major axis of Eris' orbit is approximately 67.9 astronomical units (AU). How many earth-years does it take Eris to orbit the sun? *Hint:* Although the orbit is not nearly circular, a circular orbit of radius 67.9 AU would have the same period, according to Kepler's Law.

Kepler's law for a circular orbit may be written, as $F_g = mv^2/R$, which implies $GMm/R^2 = mv^2/R$, as in part (a). Setting $v = 2\pi R/T$ gives Kepler's law in terms of the radius and period,

$$GM = Rv^2 = 4\pi^2 \frac{R^3}{T^2}. \quad (12)$$

This is Kepler's Law, and it applies to any orbiting the sun, if M is the solar mass and R is the semi-major axis, whether or not the orbit is circular. Since it applies in particular to the earth, if we measure distance in astronomical units and time in earth years, we find $R^3 = T^2$ for all objects orbiting the sun. For Eris, this gives $T = \sqrt{(67.9)^3} = 560$ earth-years.

You could get the same result directly, using $M = 1.99 \times 10^{30}$ kg for the solar mass, and expressing the semi-major axis in meters, using 1 AU $= 150 \times 10^9$ m to get $R = 1.02 \times 10^{13}$ m. Solving Kepler's law to find T^2 then gives

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{4\pi^2 (1.02 \times 10^{13} \text{ m})^3}{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})} = 3.156 \times 10^{20} \text{ s}^2. \quad (13)$$

Then $T = 1.777 \times 10^{10}$ s, and using the fact that 1 earth-year is 3.156×10^7 s gives T =563 earth-years, which is the same answer to within the 2-digit precision of the calculation.

(c) [4pt] The current best estimate of the radius of Eris, from the best Hubble telescope photo, is about 1200 km. What is the escape velocity from Eris?

The escape velocity is calculated using energy conservation. When the object being launched is infinitely far away, it should have zero velocity. This gives a total energy of zero, using $U = -GMm/R$ for the gravitational potential energy. When the object is launched from the planet's surface, $U + K = 0$, giving $\frac{1}{2}mv^2 = GMm/R$, with R now the planet's radius and M its mass. The mass of Eris was found in part (a), so we have

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(1.24 \times 10^{22} \text{ kg})}{1.2 \times 10^6 \text{ m}}} = \sqrt{1.378 \times 10^6 \text{ (m/s)}^2} = 1.17 \text{ km/s}. \quad (14)$$