1	2	3	4	5	Total	Grade
4	6	9	9	12	40	100

Solutions

Physics 1422 — **General Physics I-A**

Exam 1 - Version A — February 5, 2007

All numerical answers require an explanation for credit, which must show symbolically any formulas you used. Numerical results must include correct units for full credit. Non-numerical answers (multiple-choice or pictures) require an explanation only if one is explicitly requested. Answers and explanations must be legible for credit. Verbal explanations after the exam will not add points to your score.

Only work on the front of the page will be graded. You may use the backs of the pages as private scratch space to plan your strategy. This may help you write more readable calculations, which will maximize partial credit. In the event of a wrong answer, more partial credit can usually be assigned to an algebraic solution than a purely numerical one. Try not to do all the work on your calculator.

If you are not sure how to work a problem in terms of formulas, or do not have time to do so, a good verbal analysis may receive partial credit. If you need a result from an earlier part of a problem to work a later part, but do not have it, an algebraic solution will recieve most of the credit.

Useful Relations

Geometry:

circumference of circle:	$2\pi r$
area inside circle:	πr^2
area of sphere:	$4\pi r^2$
volume inside sphere:	$\frac{4}{3}\pi r^3$

Mass and weight: 1 kg weighs 9.8 N on Earth; $g = 9.8$ N/kg

Radius of Earth: 6380 km

Acceleration of gravity: $g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2$.

Vector components:

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta$$
$$A = \sqrt{A_x^2 + A_y^2} \qquad \tan\theta = A_y/A_x$$

for a vector $\mathbf{A}$, if the angle θ is measured counterclockwise from the $+x$ axis.

Velocity and Acceleration: $\mathbf{v} = d\mathbf{r}/dt, \qquad \mathbf{a} = d\mathbf{v}/dt$

$$\mathbf{v}_{\text{avg}} = (\mathbf{r}_f - \mathbf{r}_i)/t, \qquad \mathbf{a}_{\text{avg}} = (\mathbf{v}_f - \mathbf{v}_i)/t.$$

Constant acceleration:

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v_f^2 = v_i^2 + 2a(x - x_0)$$

Torque in 2 dimensions:

The torque due to a force $\mathbf{F}$ acting at a displacement $\mathbf{r}$ from a given point is

$$r_x F_y - r_y F_x = rF\sin\theta$$

where θ is the angle from $\mathbf{r}$ to $\mathbf{F}$. The distance $L = r\sin\theta$ is called the "lever arm" for the torque.

1. [4pt] Which is the best estimate of the number of air molecules in the Earth's atmosphere? Note that the density of air at sea level is about 2.5×10^{25} molecules per cubic meter. Use reasonable estimates for any other data you may need.

A) 10^{33}
(B) 10^{44}
C) 10^{55}
D) 10^{66}

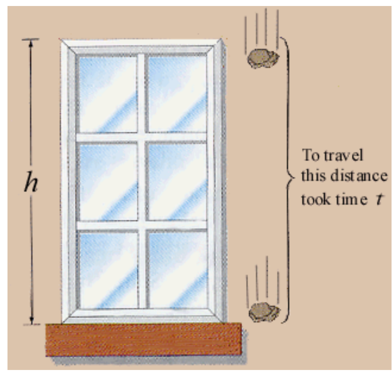
Explain your reason.

For a rough estimate, we can assume the air density is comparable to that at sea level up to some reasonable elevation, but negligable above that. Taking the thickness of the atmosphere to be $h = 10$ km, which is the height where jet fly would give a volume of the atmosphere $V = 4\pi R_e^2 h$, where $R_e = 6830$ km is the radius of the earth. Approximating $4\pi \approx 10$ and $R_e \approx 10{,}000$ km $= 10^7$ m, we find $V \approx 10 \times 10^14$ m$^2 \times 10^4$m$= 10^{19}$ m^3. Multiplying this by the given density gives a result of order $10^{(25+19)} = 10^{44}$.

---

2. [6pt]

A stone is dropped from the roof and takes time $t = 0.30$ s to cross a window of height $h = 2.1$ m.



(a) [3pt] How fast was the stone falling when it crossed the top of the window?

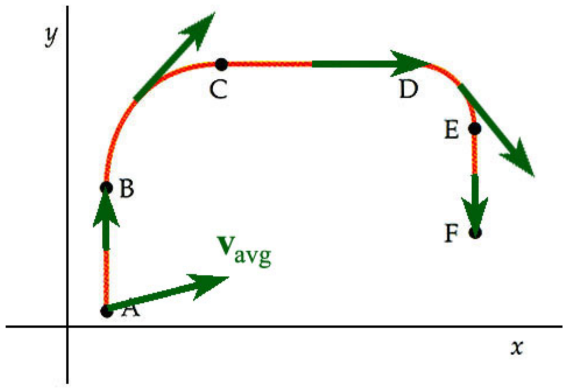
Using the constant acceleration equation $h = v_0 t + \frac{1}{2}gt^2$ and the given values for h and t gives the speed v_0 at the top of the window:

$$v_0 = \frac{h}{t} - \frac{1}{2}gt = 7.0 \text{ m/s} - 0.98 \text{ m/s} = 6.0 \text{ m/s}.$$

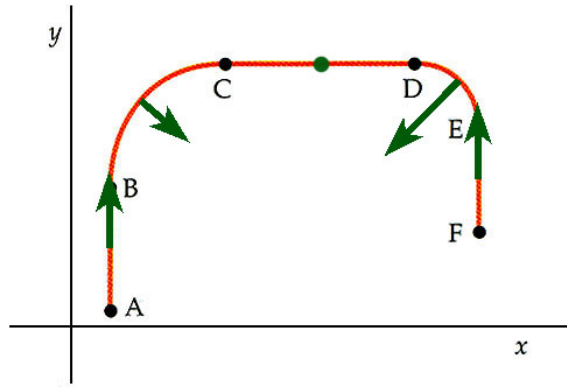
(b) [3pt] How far is the roof above the top of the window?

The constant acceleration equation $v_f^2 - v_0^2 = 2gy$ gives the distance y to the roof, if for v_0 we use the initial speed 0 and for v_f we use the speed 6.0 m/s at the top of the window. Then

$$y = \frac{(6.0 \text{ m/s})^2}{2 \times 9.8 \text{ m/s}^2} = 1.8 \text{ m}.$$



Parts (a,d)



Part (b)

3. [9pt] An automobile follows the path shown in the xy plane, starting from rest at point A, accelerating until it reaches point B, then travelling at constant speed through points C and D to point E. Then the automobile begins to slow down, and stops at point F.

(a) [2.5pt] On the first figure, draw the instantaneous velocity vector at the midpoint of each of the segments in the diagram (AB, BC, CD, DE, and EF). Place the tail of each vector at the midpoint, and be careful to show the direction correctly and to draw vectors with equal speeds with the same magnitude. Zero velocity may be marked by a heavy dot.

The velocity vector always points in the direction of motion, tangential to the path. Between B and E, it has the same magnitude, and in AB and EF, it will have a fraction of this magnitude.

(b) [2.5pt] On the second figure, draw the instantaneous acceleration vector at the midopoint of each segment. Place the tail of each vector at the midpoint, and be careful to get the directions right, but do not be concerned with the magnitudes. Zero acceleration may be marked by a heavy dot.

The acceleration is in the direction of motion in AB, since the car is speeding up, and against the direction of motion in EF, since it is slowing down. In segments BC and DE, the acceleration is perpendicular to the path (centripetal), since the speed is constant but the car is turning. There is no acceleration in segment CD.

(c) [2pt] How do the magnitudes of the acceleration compare in segments BC and DE?

A) The acceleration is greater in segment BC.

B) The acceleration is greater in segment DE.

C) The acceleration is zero in both segments.

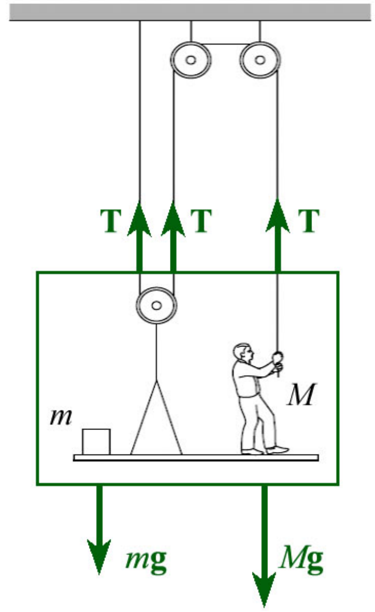
D) More information is needed to determine this.

The speeds are equal, so a smaller turning radius gives a bigger change in the direction of the velocity vector ($a = v^2/R$).

(d) [2pt] Draw the average velocity vector for the complete trip on the first figure, and label it $\vec{v}_{\rm avg}$. The direction should be shown carefully and the magnitude should make sense relative to the other velocity vectors in the diagram.

The average velocity is proportional to the net displacement vector, so it is along a line connecting point A to point F. The magnitude will be somewhat less than the speed between points B and D.

4. [9pt] A man of mass M on a light platform holds himself up by a rope attached as shown. The platform also holds a box of mass m. Assume the man and box balance so that the platform doesn't tip, and that all the pulleys are well-lubricated.



(a) [3pt] How much tension must the man put on the rope?

A) $(M+m)g$
B) $\frac{1}{2}(M+m)g$
C) $\frac{1}{3}(M+m)g$
D) $Mg+\frac{1}{2}mg$
E) $\frac{1}{2}Mg+mg$

Explain your answer.

Isolating the system shown inside the green box in the figure, the forces are the three tensions in the ropes pulling up, and the weights of the man and box pulling down. Since the rope runs over pulleys which are free to turn, the tensions are the same in each segment, giving a net upward force of $3T$. The net downward force is $(M+m)g$, so $T=\frac{1}{3}(M+m)g$.

(b) [3pt] On the figure of the man below, show all forces acting on just the man.

There is the tension T pulling up, the weight Mg pulling down, and the contact force of the platform pushing up on the man, F_c.

(c) [3pt] What is the contact force between the man and the platform?

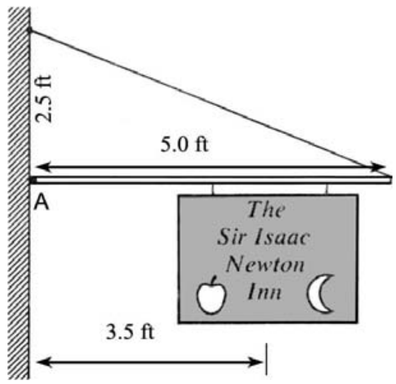
A) $\frac{1}{2}(M+m)g$
B) $\frac{1}{2}(M-m)g$
C) $\frac{1}{3}Mg+\frac{2}{3}mg$
D) $\frac{2}{3}Mg+\frac{1}{3}mg$
E) $\frac{1}{3}Mg-\frac{2}{3}mg$
F) $\frac{2}{3}Mg-\frac{1}{3}mg$

Explain your answer.

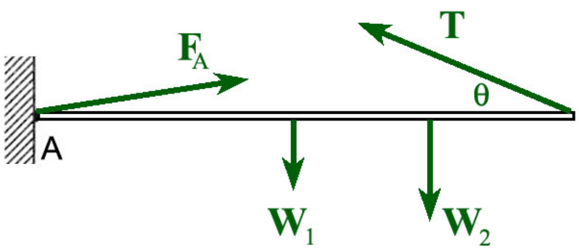
Balancing forces on the man gives $T+F_c=Mg$. Since $T=\frac{1}{3}(M+m)g$, we find

$$F_c = Mg - \frac{1}{3}(M+m)g = \frac{2}{3}Mg - \frac{1}{3}mg.$$

5. [12pt] A 76 lb sign is to be hung as shown in front of an inn. It's center is 3.5 feet from the wall. The supporting arm, of length 5.0 ft, weighs 44 lb and is attached to the wall at point A. The sign is to be supported by a guy wire attached to a bolt on the wall 2.5 ft above point A, as shown. We would like to know how much tension will be put on the guy wire, so we can choose the wire safely.



(a) [2pt] Show and label all forces acting on the support arm, at the location where they act. Use the diagram of the arm below.



(b) [2pt] What is the torque about point A due to the weight of the arm? (You can give the result in ft-lb's.)

The weight acts at the center of gravity, at the midpoint of the arm, so

$$\tau_1 = (2.5 \text{ ft})(44 \text{ lb}) = 110 \text{ ft-lb}.$$

(If you use the sign convention that clockwise torque is negative, this is a negative torque.)

(c) [2pt] What is the torque about point A due to the weight of the sign?

$$\tau_1 = (3.5 \text{ ft})(76 \text{ lb}) = 266 \text{ ft-lb}.$$

(If you use the sign convention that clockwise torque is negative, this is a negative torque.)

(d) [3pt] What is the "lever arm" for the torque about point A due to the tension in the guy wire? In other words, if the torque is expressed in the form LT, where T is the unknown tension, find the distance L.

The torque is (5.0 m) $\times T_y$, where $T_y = T \sin\theta$ with $\tan\theta = 2.5/5.0 = 0.5$, giving $\theta = 26.6°$ and $\sin\theta = 0.447$. Then the torque due to the tension in the guy wire is

$$\tau_3 = (5.0 \text{ m}) \times (0.447T) = (2.24 \text{ m}) \times T,$$

and the lever arm is 2.24 m. This length may be geometrically interpreted as the perpendicular distance from point A to the guy line, which gives another way to calculate it, if you wish.

(e) [3pt] What is the tension in the guy wire?

Balancing torques gives $\tau_3 = \tau_1 + \tau_2$, since τ_3 is the only counterclockwise torque about point A. The force of the wall contributes no torque since it acts at point A. Combining the previous results gives (2.24 m) $\times$ T = 376 ft-lb, so that $T = 168$ lb ≈ 170 lb.