Review for Exam 4: Chapters 10, 11, 13

Physics 1422 (Dr. Yost)

Exam 4 will cover chapters 10, 11, and 13 in the Giancoli text. You may use any calculator for this exam, but not notes. You will be given any constants you need, as well as any required moments of inertia, unless the point of the problem is to derive it. You should remember basic algebraic and geometric relationships and trigonometric identities. Physics is cumulative, so you may need concepts from earlier chapters as well.

Chapter 10: Rotational Motion about a Fixed Axis

Sections skipped: none

Concepts: Rotational motion, angular quantities, rolling, torque, right hand rules, moments of inertia, angular momentum, conservation of angular momentum

Equations:

Angular velocity and acceleration:

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt} \tag{1}$$

If R is the perpendicular distance from the rotational axis,

$$v = R\omega, \qquad a_{\rm tan} = R\alpha, \qquad a_{\rm rad} = R\omega^2. \tag{2}$$

The frequency f, angular velocity ω, and period T are related by

$$f = \frac{1}{T} = \frac{\omega}{2\pi}. \tag{3}$$

If α is constant, all of the analogous formulas to one-dimensional motion hold for the corresponding rotational quantities:

$$\begin{aligned} \omega &= \omega_0 + \alpha t, & \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta, & \overline{\omega} &= \frac{1}{2}(\omega + \omega_0). \end{aligned} \tag{4}$$

If a force F acts a distance R from an axis, with an angle θ between $\vec{R}$ and $\vec{F}$, then the torque is defined to be

$$\tau = R_\perp F = RF_\perp = RF\sin\theta, \tag{5}$$

where $R_\perp$ is the perpendicular projection of R onto F, and $F_\perp$ is the perpendicular projection of F onto R. (The projection forms are often easier to use when the vectors are resolved into cartesian coordinates.) The direction of the torque is perpendicular to both R and F, and given by a right hand rule. If place your fingers along R and curl them toward F, your thumb will point along the direction of τ.

The total torque is related to the angular acceleration by

$$\tau = I\alpha \tag{6}$$

with I the moment of inertia, defined by

$$I = \sum_i m_i R_i^2 \quad \text{or} \quad I = \int dm(R(m))^2 \tag{7}$$

where each R is the perpendicular distance of a mass or mass element from the axis.

The moment of inertia depends on the choice of axis. If I is the moment of inertia about an arbitrary axis, and I_{CM} is the moment of inertia about a parallel axis through the center of mass, then the parallel axis theorem says that $I = I_{\text{CM}} + Mh^2$, where M is the mass of the object, and h is the distance between the axes. Note that it is important that one of the axes be through the center of mass.

Angular momentum is defined by

$$L = I\omega \tag{8}$$

If no external torques act on a system, angular momentum is conserved.

Rotational kinetic energy is defined by

$$K = \frac{1}{2}I\omega^2 \tag{9}$$

There is a rotational analog of the work-energy theorem: The work is given by $W = \int \tau d\theta$, and the chain rule implies that

$$W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2. \tag{10}$$

The total kinetic energy of a rolling object is a sum of the translational kinetic energy $\frac{1}{2}mv^2$ and the rotational kinetic energy. If the object rolls without slipping, then $v = R\omega$, where R is the radius of the object.

Chapter 11: General Rotation

Sections skipped: 6 and 8 – 10.

Concepts: Rigid body rotation, conservation of angular momentum (The vector aspects were included in the reading for conceptual reasons, but will not be on the exam and are omitted from this review.)

Equations:

An object does not have to be rotating to have angular momentum about a point. If an object travels at constant speed v in a straight line whose distance of closest approach to the origin is r, the magnitude of its angular momentum about this origin is $L = mrv$.

For a rigid body rotating about an axis of symmetry through the center of mass, the angular momentum is $L = I\omega$. For general asymmetric objects rotating about an arbitrary axis, both the angular momentum and angular velocity must be treated as vectors, and they need not point in the same direction. We did not consider such cases in this class, but you should be aware not to over-generalize the results we have used.

The torque and angular momentum are related by

$$\vec{\tau} = \frac{d\vec{L}}{dt}. \tag{11}$$

when they are measured in an inertial frame, or about an origin at If there is no net torque on a system, the angular momentum is conserved about any inertial axis, or one through the center of mass (CM).

The general motion of an object may be separated into motion of the center of mass, which is governed by Newton's law for a point object, $\vec{F} = m\vec{a}$, and rotation of the object about its center of mass, which is governed by $\vec{\tau} = d\vec{L}/dt$ with the origin at the center of mass. The total angular momentum of an object about any axis is a sum of two terms: the "orbital" angular momentum of the CM of the object about the axis plus the "spin" angular momentum of the object about its CM.

In a collision, angular momentum is conserved about any axis in an inertial frame, which includes the CM frame of the system. The CM will move at a constant velocity throughout the collision, and the total angular momentum about the CM is conserved.

Chapter 13: Fluids

Sections skipped: 10 – 13

Concepts: Density, specific gravity, pressure, Pascal's principle, barometers, buoyancy, Archimedes' principle, fluids in motion, flow rate, equation of continuity, Bernoulli's principle, airflow, lift

Equations:

Density is mass per unit volume: $\rho = m/V$. Specific gravity is the ratio $\rho/\rho_{\mathrm{H_2O}}$ of the density to the density of water, $\rho_{\mathrm{H_2O}} = 1\ \mathrm{g/cm^3} = 1000\ \mathrm{kg/m^3}$.

Pressure is force per unit area, $P = F/A$. It is measured in pascals, with 1 Pa = 1 $\mathrm{N/m^2}$. Atmospheric pressure is approximately $1.01 \times 10^5 Pa$. Gauge pressure is the difference between pressure and atmospheric pressure.

The pressure under a depth h of fluid of density ρ is $P = \rho gh$. This is just the force per unit area due to the weight of the fluid. If the fluid is compressible, the density may change with height, but at any height, $dP/dy = -\rho g$. The pressure in a faucet due to water in a storage tank is given by $P = \rho gh$ with ρ the density of water, and h the height of the top of the water in the tank above the faucet. The height h is called the *pressure head*.

Pascal's principle states that applying pressure to a confined fluid increases the pressure throughout by the same amount. This is used to construct hydraulic lifts. If force F_1 is applied to a piston of area A_1, the force on a second piston of area $A_2 > A_1$ will be $F_2 = F_1 A_2/A_1 > F_1$. The mechanical advantage of a hydraulic lift is $F_2/F_1 = A_2/A_1$.

A barometer can be made by placing an evacuated tube vertically in a reservoir of fluid. The fluid will rise in the tube until the weight of fluid in the tube equals the atmospheric pressure. No amount of suction can draw the fluid higher than this up a tube, since the pressure in the tube can never be less than zero.

Archimedes' principle states that the buoyant force on an object is equal to the weight of water it displaces. If the fluid has density ρ_f and the object has volume V, the buoyant force is $F_B = \rho_f V g$. An object will float in a fluid if the buoyant force on the object is less than the objects weight when fully submerged, *i.e.* if its density is less than the density of the fluid. The fraction of the volume submerged is given by the ratio of the densities ρ_O/ρ_f of the object to the fluid.

Laminar flow occurs when a fluid flows in smooth paths with definite streamlines. Turbulent flow occurs when erratic eddies form, and the flow is chaotic, without regular streamlines. In this chapter, we are concerned only with laminar flow.

The volume rate of flow of a fluid through a pipe of cross section A is $Q = Av$ where v is the velocity of the fluid. This measures the volume per unit time of fluid flowing through the pipe:

$$\frac{dV}{dt} = \frac{Adl}{dt} = Av. \tag{12}$$

The equation of continuity states that at any two different points in the pipe,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \tag{13}$$

Bernoulli's principle is a statement of the work-energy theorem for fluids, neglecting viscocity, which is the analog of friction for fluids. It states that the quantity

$$P + \frac{1}{2}\rho v^2 + \rho gh \tag{14}$$

is conserved. The pressure term represents the work per unit volume ΔV done on the moving fluid:

$$W = Fv\Delta t = F\Delta l = PA\Delta l = P\Delta V. \tag{15}$$

The term $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume, and $\rho g h$ is the potential energy per unit volume. Bernoulli's principle applies to both fluids and gasses. The main trick in applying it is knowing how to set the various variables in terms of the physics of the problem, so it is essential to review the examples and homework problems.

An example of Bernoulli's principle is the lift on an airfoil. An airfoil is designed so that the velocity v_1 of air flowing above the airfoil is greater than the velocity v_2 below. Bernoulli's principle then says that there will be a pressure difference, $P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$, which creates a net *lift* force $F_L = A(P_2 - P_1)$, where A is the area of the wing. (This analysis is somewhat incomplete, since turbulence also plays a major role in lift.)

Viscosity is a retarding force due to the friction between adjacent layers of a fluid. It is defined as follows. Imagine two plates of area A, separated by a fluid layer of thickness l. If the top plate is moved at a speed v parallel to the bottom plate (so that the thickness l is constant), the force on the plate is $F = \eta A v / l$, where η is the *coefficient of viscosity*. "Thicker" fluids have a higher coefficient of viscosity.

The flow rate of a viscous fluid through a pipe of length L and radius R with a pressure of P_1 and P_2 at the two ends is given by Poiseuille's equation:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L} \tag{16}$$

where $Q = Av$ is in m^3/s, as above. The important thing to remember about this is how it scales with radius: as the fourth power.

Units:

Pressure is measured in Pascals, 1 Pa = 1 N/m^2.

Viscosity is measured in Poise (P), 1 P = 1 Ns/m^2 = 1 Pas.