Review for Exam 3: Chapters 10, 11, 13, 15, 16

Physics 1422 (Dr. Yost)

Exam 3 will cover chapters 10, 11, 13, 15, and 16 in the Giancoli text. You may use any calculator for this exam, but not notes. You will be given any constants you need, as well as any required moments of inertia. Any required equations from these notes will also be provided. You should remember basic algebraic and geometric relationships and trigonometric identities. Physics is cumulative, so you may need concepts from earlier chapters as well.

Chapter 10: Rotational Motion about a Fixed Axis

Sections skipped: Sec. 12

Concepts: Rotational motion, angular quantities, rolling, torque, right hand rules, moments of inertia, angular momentum, conservation of angular momentum

Equations:

Angular velocity and acceleration:

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt} \tag{1}$$

If R is the perpendicular distance from the rotational axis,

$$v = R\omega, \qquad a_{\tan} = R\alpha, \qquad a_{\text{rad}} = R\omega^2. \tag{2}$$

The frequency f, angular velocity ω, and period T are related by

$$f = \frac{1}{T} = \frac{\omega}{2\pi}. \tag{3}$$

If α is constant, all of the analogous formulas to one-dimensional motion hold for the corresponding rotational quantities:

$$\begin{aligned} \omega &= \omega_0 + \alpha t, & \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta, & \overline{\omega} &= \frac{1}{2}(\omega + \omega_0). \end{aligned} \tag{4}$$

If a force $\mathbf{F}$ acts at a displacement $\mathbf{R}$ from an axis, with an angle θ between $\mathbf{R}$ and $\mathbf{F}$, then the torque is defined by the cross product $\tau = \mathbf{R} \times \mathbf{F}$. The torque has magnitude

$$\tau = R_\perp F = RF_\perp = RF\sin\theta, \tag{5}$$

where $R_\perp$ is the perpendicular distance from the axis to the point where the force acts, and $F_\perp$ is the component of $\mathbf{F}$ perpendicular to $\mathbf{R}$. The direction of the torque is perpendicular to both $\mathbf{R}$ and $\mathbf{F}$, and given by a right hand rule. If place your fingers along $\mathbf{R}$ and curl them toward F, your thumb will point along the direction of the torque.

The total torque is related to the angular acceleration by

$$\tau = I\alpha \tag{6}$$

with I the moment of inertia, defined by

$$I = \sum_i m_i R_i^2 \quad \text{or} \quad I = \int dm R^2 \tag{7}$$

where each R is the perpendicular distance of a mass or mass element from the axis. Any moments of inertia needed will be given on the exam.

The moment of inertia depends on the choice of axis. If I is the moment of inertia about an arbitrary axis, and I_{CM} is the moment of inertia about a parallel axis through the center of mass, then the parallel axis theorem says that $I = I_{\text{CM}} + Mh^2$, where M is the mass of the object, and h is the distance between the axes. Note that it is important that one of the axes be through the center of mass.

Angular momentum is defined by

$$L = I\omega \tag{8}$$

If no external torques act on a system, angular momentum is conserved.

Rotational kinetic energy is defined by

$$K_r = \frac{1}{2} I\omega^2 \tag{9}$$

The rotational work is given by by $W = \int \tau d\theta$, and

$$W_r = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2. \tag{10}$$

The total kinetic energy of a rolling object is a sum of the translational kinetic energy $K_t = \frac{1}{2}mv^2$ and the rotational kinetic energy. If the object rolls without slipping, then $v = R\omega$, where R is the radius of the object. Otherwise, v and ω are not generally related.

Chapter 11: General Rotation

Sections skipped: Secs. 8 – 10.

Concepts: Rigid body rotation, conservation of angular momentum, vector nature of angular quantities, rotational imbalance, precession

Equations:

The angular velocity vector of a rotating object is taken to be along the axis of rotation, in a direction given by the "right-hand rule" that says if you curl your fingers in the direction of rotation, your thumb points along the axis in the direction of the angular velocity.

An object does not have to be rotating to have angular momentum about a point. If an object travels at constant speed v in a straight line whose distance of closest approach to the origin is $r_\perp$, the magnitude of its angular momentum about this origin is $L = mr_\perp v$. As a vector, the angular momentum is given by the cross product,

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{p}. \tag{11}$$

It is in a direction perpendicular to both $\mathbf{r}$ and $\mathbf{p}$ in a direction given by the "right hand rule."

The torque is the rate of change of angular momentum, $\boldsymbol{\tau} = d\mathbf{L}/dt$. when they are measured in an inertial frame, or about an origin at the CM. If there is no net torque on a system, the angular momentum is conserved about any inertial axis, or one through the center of mass (CM).

For a rigid body rotating about an axis of symmetry through the center of mass, the angular momentum is $L = I\omega$. For general asymmetric objects rotating about an arbitrary axis, both the angular momentum and angular velocity must be treated as vectors, and they need not point in the same direction. In general, the component of the angular momentum about the rotational axis will be given by $L_\omega = I_\omega \omega$, where the subscript ω is a reminder that this is just one component of the angular momentum vector, about the rotational axis, and I_ω is calculated about this axis. The existence of other components of $\mathbf{L}$ which do not point along the rotational axis, but normally rotate with the object, imply a rotating torque on the object. This is called "rotational imbalance."

The general motion of an object may be separated into motion of the center of mass, which is governed by Newton's law for a point object, $\mathbf{F} = m\mathbf{a}$, and rotation of the object about its center of mass, which is governed by $\tau = d\mathbf{L}/dt$ with the origin at the center of mass. The total angular momentum of an object about any axis is a sum of two terms: the "orbital" angular momentum of the CM of the object about the axis plus the "spin" angular momentum of the object about its CM.

In a collision, angular momentum is conserved about any axis in an inertial frame, which includes the CM frame of the system. The CM will move at a constant velocity throughout the collision, and the total angular momentum about the CM is conserved.

When a torque acts perpendicular to the angular momentum vector, it causes the angular momentum vector to rotate perpendicular to itself. If the torque remains constant, the angular momentum rotates at a rate $dL/dt = \tau$, making a circle of circumference $2\pi L$ in a time $T = 2\pi L/\tau$. This time is the precession period.

Chapter 13: Fluids

Sections skipped: Secs. 12 – 13 (and limited coverage of Sec. 10)

Concepts: Density, specific gravity, pressure, Pascal's principle, barometers, buoyancy, Archimedes' principle, fluids in motion, flow rate, equation of continuity, Bernoulli's principle, viscosity

Equations:

Density is mass per unit volume: $\rho = m/V$. Specific gravity is the ratio $\rho/\rho_{\mathrm{H_2O}}$ of the density to the density of water, $\rho_{\mathrm{H_2O}} = 1\ \mathrm{g/cm^3} = 1000\ \mathrm{kg/m^3}$.

Pressure is force per unit area, $P = F/A$. It is measured in pascals, with 1 Pa = 1 N/m^2. Atmospheric pressure is approximately $1.01 \times 10^5 Pa$. Gauge pressure is the difference between pressure and atmospheric pressure.

The pressure under a depth h of fluid of density ρ is $P = \rho g h$. This is just the force per unit area due to the weight of the fluid. If the fluid is compressible, the density may change with height, but at any height, $dP/dy = -\rho g$. The pressure in a faucet due to water in a storage tank is given by $P = \rho g h$ with ρ the density of water, and h the height of the top of the water in the tank above the faucet. The height h is called the *pressure head*.

Pascal's principle states that applying pressure to a confined fluid increases the pressure throughout by the same amount. This is used to construct hydraulic lifts. If force F_1 is applied to a piston of area A_1, the force on a second piston of area $A_2 > A_1$ will be $F_2 = F_1 A_2/A_1 > F_1$. The mechanical advantage of a hydraulic lift is $F_2/F_1 = A_2/A_1$.

A barometer can be made by placing an evacuated tube vertically in a reservoir of fluid. The fluid will rise in the tube until the weight of fluid in the tube equals the atmospheric pressure. No amount of suction can draw the fluid higher than this up a tube, since the pressure in the tube can never be less than zero. (Suction does not pull the liquid up the tube, but allows the atmospheric pressure to push it up.)

Archimedes' principle states that the buoyant force on an object is equal to the weight of water it displaces. If the fluid has density ρ_f and the object has volume V, the buoyant force is $F_B = \rho_f V g$. An object will float in a fluid if the buoyant force on the object is less than the objects weight when fully submerged, *i.e.* if its density is less than the density of the fluid. The fraction of the volume submerged is given by the ratio of the densities ρ_O/ρ_f of the object to the fluid.

Laminar flow occurs when a fluid flows in smooth paths with definite streamlines. Turbulent flow occurs when erratic eddies form, and the flow is chaotic, without regular streamlines. In this chapter, we are concerned only with laminar flow.

The volume rate of flow of a fluid through a pipe of cross section A is $Q = Av$ where v is the velocity of the fluid. This measures the volume per unit time of fluid flowing through the pipe:

$$Q = \frac{dV}{dt} = \frac{Adl}{dt} = Av. \tag{12}$$

The equation of continuity states that Q is the same at any two different points in the pipe, so that

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \tag{13}$$

Bernoulli's principle is a statement of the work-energy theorem for fluids, neglecting viscocity, which is the analog of friction for fluids. It states that the quantity

$$P + \frac{1}{2}\rho v^2 + \rho g h \tag{14}$$

is conserved. The pressure term represents the work per unit volume ΔV done on the moving fluid:

$$W = Fv\Delta t = F\Delta l = PA\Delta l = P\Delta V. \tag{15}$$

The term $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume, and $\rho g h$ is the potential energy per unit volume. Bernoulli's principle applies to both fluids and gasses. The main trick in applying it is knowing how to set the various variables in terms of the physics of the problem, so it is essential to review the examples and homework problems.

The flow rate of a viscous fluid through a pipe of length L and radius R with a pressure of P_1 and P_2 at the two ends is given by Poiseuille's equation:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \tag{16}$$

where $Q = Av$ is in m^3/s, as above, and η is the "coefficient of viscosity" which measures the amount of resistance to the flow. The important thing to remember about this is how it scales with pipe radius: as the fourth power. **Units:**

Pressure is measured in Pascals, 1 Pa = 1 $\mathrm{N/m^2}$.

Chapter 15: Wave Motion

Sections skipped: Sec. 15-5, 15-10, 15-11, anything relating to elastic moduli

Concepts: pulses, waves, amplitude, wavelength, frequency, period, longitudinal waves, transverse waves, wave velocity, energy transmitted by waves, principle of superposition, reflection and transmission, standing waves, resonance, harmonics

Equations:

Any wave or pulse can be represented by a *displacement function* $D(x,t)$ showing how far (D) a particle at position x will be displaced from equilibrium at time t. If the wave or pulse travels with speed v and holds its shape (is "non-dispersive"), then $D(x,t) = D(x - vt)$. When v is positive, the wave moves to the right. For a sine wave, the wave velocity is related to the frequency and wavelength by $v = f\lambda = \omega/k$, where the wave number is defined to be $k = 2\pi/\lambda$. A sine wave of amplitude D_M can be represented as $D(x,t) = D_M \sin(kx - \omega t)$. If F_T is the tension on a cord, and μ is the linear mass density (kg/m), then the velocity of transverse waves on the cord is $v = \sqrt{F_T/\mu}$.

The sum of two waves with speed v is also a wave with speed v. The sum of waves is called the *superposition* of the waves. If the amplitude of the sum is greater, the waves are said to *interfere constructively.* If it is less, then they interfere *destructively.*

A wave is reflected from a free end without a phase change, but is reflected from a fixed end with a 180^o phase shift (which means it is inverted).

A standing wave vibrates in place. If the ends of a string are fixed so that the displacement is $y = 0$ at $x = 0$ and $x = L$, then the vibrations can occur at wavelengths $\lambda_n = 2L/n$, $n = 1, 2, 3, \ldots$. The frequency of these vibrations is $f_n = nv/2L$, where v is the speed of wave propagation on the string. The frequency f_n is called the n^{th} *harmonic*, and f_1 is called the *fundamental frequency.*

The power carried by a wave is

$$P = \frac{1}{2} v A \rho \omega^2 D_M^2 \tag{17}$$

where $\omega = 2\pi f$ is the angular frequency and D_M is the amplitude, ρ is the density of the medium, v is the wave speed, and A is the cross-sectional area. The *intensity* is the average power per unit area: $I = \overline{P}/A$. The most important features of the power relation are that the intensity is proportional to the square of the frequency and the square of the amplitude.

The intensity of a spherical wave variea according to the *inverse square law.* At two distances r_1 and r_2, the intensities are related by the inverse square law, $I_2/I_1 = (r_1/r_2)^2$.

The frequency of a wave is measured in Hertz, 1 Hz = 1 s^{-1}. The intensity of a wave is measured in W/m^2.

Chapter 16: Sound

Sections skipped: 16-7, 16-8, 16-9 (and the derivations in 16-2)

Concepts: loudness, pitch, audible range, pressure waves, decibels, string instruments, wind instruments, overtones, harmonics, open pipes, closed pipes, interference, beats

Equations:

The speed of sound in air is approximately $v \approx (331 + 0.60\ T)$ m/s, with T in Celcius degrees. At room temperature, 20°C, the speed of sound is 343 m/s. You don't need to remember this.

The **pitch** of a sound is determined by its frequency. Higher musical notes have higher frequency. Music is divided into octaves, with each octave corresponding to a doubling of frequency. The range of normal human hearing is approximately 20 Hz to 20 kHz.

The **sound level** in decibels is given by

$$\beta(\text{in dB}) = 10 \log_{10} \frac{I}{I_0} \tag{18}$$

where I is the intensity of the sound wave in W/m^2 and I_0 is a reference intensity, normally taken to be the threshold of human hearing, or 1.0×10^{-12} W/m^2. An increase of 10 dB corresponds to a factor of 10 in intensity. A doubling of power corresponds to about 3 dB. The threshold of pain is about 120 dB, which corrseponds to an intensity of 1 W/m^2.

The fundamental vibrational mode of a stringed instrument is $\lambda = 2L$, where L is the length of the string. The overtones or harmonics are at wavelengths $\lambda_n = 2L/n$, for $n = 1, 2, 3, \ldots$.

A tube open on both ends also produces wavelengths $\lambda_n = 2L/n$. A tube closed on one end produces wavelengths $\lambda_{2n-1} = 4L/(2n-1)$. An open tube produces all integer multiples of the fundamental frequency $f_1 = v/2L$, while a tube closed on one end produces only the odd integer multiples of $f_1 = v/4L$, so that $f_n = nf_1$ with $n = 1, 3, 5, \ldots$. Whether the tube is open or closed, the integer $n = f_n/f_1$ is called the *order* the harmonic, and the difference in frequency between two successive harmonics is $\Delta f = v/2L$.

If two sounds are produced simultaneously with frequencies f_1 and f_2, they will interfere and produce "beats" with frequency $|f_1 - f_2|$.

Units:

Decibels (dB) are used to measure sound intensity logarithmically.