Review for Exam 2: French Chapters 7 – 11

Physics 1422 (Dr. Yost)

Exam 2 will cover chapters 7 through 11 in French's text. You may use any calculator for this exam, but not notes. You will be given any constants and conversion factors needed, as well as a selection of equations which should be adequate if you understand the concepts. (Not every equation will be given on the exam – for example, you should be able to derive any orbital relations you need without remembering Kepler's laws.) Any derivatives needed will be given, but you should know how to use them.

The essence of physics is learning to apply basic physical and mathematical concepts to analyzing new situations. Every situation is different, so memorizing specific solution techniques is pointless. What matters is to get as much practice as possible with a wide variety of problems to develop your analytical skills. The exam will primarily test reasoning, not memorization. Physical equations are usually very easy to remember once you truly understand them. Drawing pictures often helps – remember this when you work the exam. If you see a situation you think is unfamiliar, try to draw an analogy with one you have seen before. The physical principles you have learned will apply in a wide variety of situations, but you need to recognize them.

Chapter 7: Using Newton's Law

Sections skipped: Circular Paths of Charged Particles in Uniform Magnetic Fields, Charged Particle in a Magnetic Field, Mass Spectrographs, Fracture of Rapidly Rotating Objects

Concepts: solving problems using Newton's law, centripetal force, curvilinear motion, resisted motion, simple harmonic motion

Problems are solved using a combination of Newton's laws and the kinematic equations governing velocity and acceleration studied in chapters 2 and 3. Remember especially the equations for constant acceleration,

$$v = v_0 + at, \qquad x = v_0 t + \frac{1}{2}at^2, \qquad v^2 - v_0^2 = 2ax, \tag{1}$$

and the equation for centripetal acceleration

$$a = v^2/R \tag{2}$$

in motion with radius of curvature R.

If there is no acceleration in a direction, the forces are balanced in that direction. This is true if the object is at rest or moving at constant velocity.

Always draw a diagram when solving force equations. Use **isolation diagrams** to clarify specifically which forces act on a component of interest. In problems with several components, you will need several isolation diagrams to solve all equations.

In addition to Newton's law, be sure to use any geometrical constraints in the problem.

The net force on any massless object must be zero. If a string is massless, the tension force is the same at either end (and any point in the middle). The tension on either side of a massless, frictionless pulley must be equal.

A scale measures the normal force pressing on it.

If an object is moving in a circle of radius R, the net force toward the center of the circle must be $F_c = mv^2/R$. This is called the **centripetal force** on the object. It is not an independent force somehow arising from the motion, but a consequence of all forces acting on the object.

There may also be acceleration along the path, tangential to the circle. In this case, the tangential acceleration is due to the tangential component of force on the object, $F_t = ma_t$. This acceleration (and force) is always perpendicular to the centripetal acceleration (and force). The equation for centripetal force applies whether or not the object is acclererating along its path, and whether or not the radius of curvature is changing.

When an object is forced to move against fluid resistance, it will acquire a limiting speed, or terminal velocity, determined by balancing the force F pushing the object (which would be mg for a falling object) against the resistive force $R(v)$ due to viscosity and turbulence: $F = Av + Bv^2$. Often only one of the A and B terms is significant. For air resistance on normal-sized objects, the B term dominates. For a spherical object of radius r moving through air, $A = (3.1 \times 10^{-4}\ \mathrm{Ns/m^2})r$ and $B = (0.87\ \mathrm{Ns^2/m^4})r^2$.

For an object moving under the influence of viscosity and a constant accelerating force only, the approach to terminal velocity has the form $v(t) = v_t(1 - e^{-t/\tau})$. The constants v_t and τ can be calculated using Newton's laws.

The force F exerted by a spring when it is extended a distance x is given by **Hooke's Law**: $F = -kx$, where k is the **spring constant** measured, typically, in N/m.

Simple harmonic motion: If an object on a spring is moved from its equilibrium position x_0 and then released, possibly with some initial velocity, it will oscillate about the point x_0 with frequency f, where

$$\omega = 2\pi f \quad \text{where} \quad \omega^2 = k/m. \tag{3}$$

The parameter ω is the **angular frequency** in radians per second. The frequency is the inverse of the period: $f = 1/T$. The equation of motion is

$$x - x_0 = A\sin(\omega t + \phi) \tag{4}$$

where A is the **amplitude**, ω and ϕ is an initial phase determined by the initial displacement and velocity. The velocity and acceleration are the first and second derivatives of the position:

$$v = A\omega\cos(\omega t + \phi), \qquad a = -A\omega^2\sin(\omega t + \phi). \tag{5}$$

There is no need to remember these if you can remember how to differentiate sines and cosines.

In simple harmonic motion, the maximum force occurs at the turning points, while the maximum speed occurs when the object crosses its equilibrium position.

Chapter 8: Universal Gravitation

Sections skipped: Finding the Distance to the Moon, The Gravitational Attraction of a Large Sphere, Finding the Distance to the Sun, The Discovery of Neptune.

Concepts: universal gravitation, Kepler's laws, mass and weight, weightlessness

Newton's Law of Universal Gravitation:

$$F = G\frac{m_1 m_2}{R^2}. \tag{6}$$

Circular orbits: the force of gravity is the centripetal force keeping the planet in orbit:

$$G\frac{m_1 m_2}{R^2} = \frac{m_2 v^2}{R} \tag{7}$$

if the mass m_2 is orbiting mass m_1.

Kepler's Third Law: The square of a planets orbital period is proportional the the third power of its orbital radius. Kepler's law is a consequence of the previous equations, using $T = 2\pi R/v$, so it doesn't need to be remembered independently, and will not be given explicitly on the exam. On any spherical planet of mass M and radius R, the acceleration of gravity on the surface is $g = GM/R^2$. For Earth, this gives $g = 9.8\ m/s^2$. An object's weight is mg, which depends both on its mass and on the strength of gravity. Kilograms are a unit of mass, but pounds are a unit of force, so the conversion between them depends on where you are. The MKS unit of weight is the Newton. An object inside a vessel which is falling with the acceleration of gravity will feel weightless.

Chapter 9: Collisions and Conservation Laws

Concepts: momentum, conservation of momentum, collisions, impulse, force exerted by a stream of particles, elastic collisions, inelastic collisions, center of mass frame

Sections skipped: Rocket Propulsion, Inelastic and Explosive Processes, The Pressure of a Gas

Equations:

Momentum is defined to be $\mathbf{p} = m\mathbf{v}$.

Newton's second law of motion may be written $\mathbf{F} = d\mathbf{p}/dt$.

Impulse is defined to be $\mathbf{J} = \int \mathbf{F} dt = \overline{\mathbf{F}} t$ where $\overline{\mathbf{F}}$ is the average force acting during the time t.

Impulse equals the change in momentum: $\mathbf{J} = \Delta \mathbf{P}$.

If particles of mass m travel at velocity v and transfer all their momentum to an object, the force on the object is $F = \mu v$, where μ is the mass per unit time striking the object. Total momentum is constant in any collision: $\mathbf{P}_i = \mathbf{P}_f$.

If the relative speed between two objects is the same before and after the collision, then the collision is said to be **elastic**. If the objects stick together, the collision is **inelastic**. In a one-dimensional elastic collision between two objects, the initial and final velocities are related by

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}. \qquad \text{(elastic 1-dim. collision)} \tag{8}$$

The kinetic energy is conserved in elastic collisions.

The location of the center of mass of a set of masses m_i at positions $\mathbf{r}_i$ is given by

$$\mathbf{r}_{\text{CM}} = \frac{\sum m_i \mathbf{r}_i}{M} \tag{9}$$

where $M = \sum m_i$ is the total mass. (The center of mass is the same as the center of gravity discussed in Chapter 4 when the gravitational field is constant.)

The **center of mass frame** is the reference frame of an observer traveling along with the center of mass of a set of objects. The velocity of the CM is given by $\mathbf{V}_{\text{CM}} = \mathbf{P}/M$ where $\mathbf{P}$ is the total momentum and M is the total mass of the system.

When a collision is viewed in the center of mass frame, the total momentum is always zero. For this reason, the center of mass frame is also called the **zero-momentum frame**. When a elastic collision of two objects is viewed in the center of mass frame, the incoming and outgoing speeds of each object are the same, but the direction of travel changes.

Chapter 10: Energy Conservation in Dynamics; Vibrational Motions

Concepts: Work, energy, power, potential energy, energy in one-dimensional motion, energy of simple harmonic oscillators.

Sections skipped: The Linear Oscillator as a Two-Body Problem, Collision Processes Involving Energy Storage, The Diatomic Molecule

Equations:

The work done by a constant one-dimensional force F over a distance x is

$$W = Fx, \tag{10}$$

If the force F is not constant, the work is an integral of the force $F(x)$ between the starting point x_1 and endpoint x_2

$$W = \int_{x_1}^{x_2} F(x)dx \tag{11}$$

A force does no work if there is no displacement.

The work-energy theorem states that the change in kinetic energy $K = \frac{1}{2}mv^2$ is equal to the net work done on the object by all forces:

$$\Delta K = W_{\text{net}}. \tag{12}$$

Power is the rate of doing work. Average power is the work done per unit time: $\overline{P} = W/t$. In one dimension, Instantaneous power is

$$P = \frac{dW}{dt} = Fv. \tag{13}$$

A force is conservative if the work it does on an object moving from point x_1 to point x_2 depends only on the two points, and not on the path between them. For a one-dimensional conservative force, potential energy $U(x)$ is as the work done against the force to get from x_1 to x_2:

$$U(x_2) - U(x_1) = -W = -\int_{x_1}^{x_2} F(x)dx \tag{14}$$

Potential energy is normally defined with respect to a reference point x_0 where $U(x_0)$ is **defined** to be zero. The choice of the zero point of potential energy is always arbitrary, since only changes in potential energy keep track of the work done.

Conservation of energy: The total mechanical energy $E = K + U$ is constant if there are no other forces acting on the object than the ones accounted for by the potential energy U. If there are other forces $\mathbf{F}_{\text{ext}}$ not included in U, which do work W_{ext} on the object, then the change in the total energy is given by

$$\Delta E = \Delta K + \Delta U = W_{\text{ext}}. \tag{15}$$

The other forces $\mathbf{F}_{ext}$ necessarily include any nonconservative forces, such as friction, since these cannot be expressed using a potential energy.

Gravitational potential energy at height h above Earth, with h small compared to Earth's radius:

$$U(h) = mgh. \tag{16}$$

Elastic potential energy for a spring with spring constant k compressed a distance x:

$$U(x) = \frac{1}{2}kx^2. \tag{17}$$

If a mass m is in simple harmonic motion, so that $x = A\cos(\omega t)$, then its total energy is $\frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$, where k is the spring constant, with $\omega = \sqrt{k/m}$. (Even if the harmonic motion is not due to an actual spring, an effective spring constant k can be defined by the relation $k = m\omega^2$).

In one dimension, the force on an object with potential energy $U(x)$ is given by a derivative of the potential energy function: $F(x) = -dU/dx$. The force is zero at a local minimum (stable equilibrium point) or local maximum (unstable equilibrium point) of the potential energy. Near a local minimum, the motion can be approximated as a harmonic oscillator. The effective spring constant is $k_{\text{eff}} = -F(x_0)/(x - x_0) = d^2U/dx^2$ at the equilibrium point x_0.

Properties of the motion of a particle acted on by a conservative force can be predicted from a graph of $U(x)$. If the particle has energy E, the motion is confined to a region where $E > U(x)$, and the kinetic energy at point x is $K(x) = E - U(x)$. When $E = U(x_0)$, the particle must stop: this is a *turning point* of the motion. If the particle moves between two turning points on either side of a local minimum of the potential, it will oscillate back and forth continuously. If the amplitude of the motion is small enough, the motion will be approximately simple-harmonic with $k = d^2U/dx^2$, evaluated at the equilibrium point.

Units: Work and energy are expressed in units of **Joules**, where 1 J = 1 Nm. Other common units are the calorie and electron volt. You do not need to remember the conversions, but 1 cal = 4.186 J and 1 eV = 1.602×10^{-19} J. English units are foot-pounds and British thermal units (Btu), where 1 ft·lb = 1.36 J and 1 Btu = 1054 J.

Power is expressed in **Watts**, where 1 W = 1J/s. Another common unit is the horse-power, with 1 hp = 746 W.

Chapter 11: Conservative Forces and Motion in Space

Concepts: Conservative forces, the pendulum, gravitational potential energy in space, escape velocity

Sections skipped: Gravitating Spherical Shell, A Gravitating Sphere, and everything after p. 457 (More About the Criteria for Conservative Forces, Fields, Equipotential Surfaces and the Gradient of Potential Energy, Motion in Conservative Fields, The Effect of Dissipative Forces, Gauss's Law, Applications of Gauss's Theorem)

Equations:

The concepts of work and energy can be extended to multi-dimensional systems using vectors:

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta, \tag{18}$$

where θ is the angle between the force and the displacement. Instantaneous power may be written

$$P = \frac{dW}{dt} = vF_{\text{parallel}} = \mathbf{F} \cdot \mathbf{v}. \tag{19}$$

where F_{parallel} is the component of the force parallel to the velocity.

The period of small oscillations of a pendulum with length L is $T = 2\pi\sqrt{L/g}$. For large oscillations, the period is somewhat longer than this. (A pendulum is only approximately a harmonic oscillator.)

Gravitational potential energy for an object of mass m a distance r from the center of an object of mass M:

$$U(r) = -\frac{GMm}{r}. \tag{20}$$

The constant in the potential energy is chosen so that $U(r) \to 0$ at $r \to \infty$.

The **escape velocity** is the initial speed needed for an object to escape a gravitational field (to $r = \infty$) starting from a given radius $r = R$ with no additional force. The condition is $K + U(R) = 0$ initially, which means the object gets to $r = \infty$ with zero kinetic energy. The required velocity is $v = \sqrt{2GM/R}$ for an object of mass m. This can also be written as $v = \sqrt{2gR}$, where g is the gravitational acceleration at radius R. You should remember how to derive these relations - they will not appear on the exam.

If a satellite is in a circular orbit at radius R, its gravitational and potential energy are related: $U(R) = -2K$. Its total energy is $E = K + U = \frac{1}{2}U(R)$. The relation between U and K comes from using "Kepler's Law" (universal gravitation and centripetal acceleration combined) to find a relation between the orbital velocity and radius.