Review for Exam 1: French Chapters 1 – 6

Physics 1422 (Dr. Yost)

Exam 1 will cover chapters 1 through 6 in French's text (including the Prologue). The introductory material from Chapter 1 in Giancoli should be familiar as well. You may use any calculator for this exam. for this exam. You will be given any constants and conversion factors needed, as well as any equations needed from this review. You will not need to calculate a derivative or integral or use spherical coordinates on this exam. You should know the geometric interpretation of derivatives and integrals.

The essence of physics is learning to apply basic physical and mathematical concepts to analyzing new situations. It is important to get as much practice as possible with a wide variety of problems to develop your analytical skills. You will want to remember the basic equations presented in these notes, but the exam will primarily test reasoning, not memorization. Physical equations are often easy to remember once you truly understand them.

Giancoli Chapter 1: Introduction, Measuring, Estimating

Concepts: The nature of science, models, theories, and laws, units, measurement and uncertainty, scientific notation, estimating sizes

Equations: Unit conversions: You may convert units by inserting a factor of 1 in such a way that the unwanted units cancel. For example, if a quantity is given as x boods and you want it in terms of gobs, with a boods = 1 gob, then you can multiply x boods by the factor 1 = (1 gob/a boods) to get

$$x \text{ boods} = x \text{ boods} \left(\frac{1 \text{ gob}}{a \text{ boods}}\right) = \frac{x}{a} \text{ gobs.} \tag{1}$$

When the units are converted correctly, the unwanted unit (boods) cancels, leaving the desired unit (gobs).

Significant digits: A product or ratio of numbers has only as many significant figures as the least precise number being multiplied. (The number of significant figures in a sum or difference can change, however.) The general principle is that a calculated result cannot be more precise than the data going into it. Do not display unnecessary digits.

Units: meter (length), second (time), kilogram (mass) are fundamental SI units.

French Chapter 1: A Universe of Particles (with Prologue)

Concepts: A physicist's view of the universe, types of particles, size scales, estimating sizes

Equations: Chapter 1 and the Prologue are an introduction to the nature of physical law and the natural scientist's point of view of the universe, from the very small to the very large. The following equations from the Prologue are useful for estimating:

$$\begin{aligned}(1+x)^n &\approx 1+nx \quad \text{for} \quad x \ll 1\\ \sin\theta &\approx \theta \quad \text{for} \quad \theta \ll 1 \text{ (in radians)}\\ \cos\theta &\approx 1-\frac{\theta^2}{2} \quad \text{for} \quad \theta \ll 1 \text{ (in radians)}\\ \ln(1+x) &\approx x \quad \text{for} \quad x \ll 1\end{aligned}$$

Units: Numerous units appear in the Prologue and Chapter 1, especially in the problems. It is not necessary to memorize any of these. Any units needed in the problems will be defined.

French Chapter 2: Space, Time, and Motion

Concepts: frames of reference, coordinate systems, polar and spherical coordinates, vectors, vector addition, scalar product of vectors, average and instantaneous velocity and speed, relative velocity and relative motion

Equations:

For a two-dimensional vector $\mathbf{V}$ with components V_x, V_y, and angle θ measured counterclockwise from the x axis,

$$V_x = V\cos\theta, \qquad V_y = V\sin\theta, \tag{2}$$

$$V = \sqrt{V_x^2+V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}. \tag{3}$$

V is called the *magnitude* of the vector $\mathbf{V}$. When inverting the tangent to find the angle θ, remember to be sure to check what quadrant the answer should be in. The normal inverse tangent assumes $V_x > 0$. Otherwise, you must add 180° to the angle θ.

When vectors are added or subtracted, each component of the vector is added or subtracted independently: $\mathbf{A} \pm \mathbf{B}_i = A_i \pm B_i$.

The *scalar product* (dot product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$. The magnitude of a vector is related to its square using this product: $A = \sqrt{\mathbf{A}\cdot\mathbf{A}}$. The scalar product of two vectors may be written in terms of the magnitudes A and B and the angle θ between the vectors as $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$. If $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, note that $\mathbf{A}\cdot\mathbf{B} = 0$.

Average velocity: $\overline{\mathbf{v}}$ = displacement vector/time = $\Delta\mathbf{r}/\Delta t$.

Average speed = total distance traveled (along path)/time.

The notation for average speed and velocity are often similar, so you must distinguish them by context. However, the concepts are distinct: The average velocity of a trip that begins and ends at the same point is always zero, but the average speed is the length of the path taken divided by the time.

Instantaneous velocity: $\mathbf{v} = d\mathbf{r}/dt$.

Take the derivative of each component of the position vector independently to get the components of the velocity vector. In one dimension, the velocity is the slope of a graph of the position as a function of time, $v = dx/dt$.

The *relative velocity* $\mathbf{v}_{12}$ of object 1 relative to object 2 is the velocity of object 1 in the reference frame of object 2. Relative velocities add. If object 1 is moving with velocity $\mathbf{v}_{12}$ with respect to object 2, and object 2 is moving with velocity $\mathbf{v}_{23}$ with respect to object 3, then the velocity of object 1 with respect to object 3 is the vector sum $\mathbf{v}_{13} = \mathbf{v}_{12} + \mathbf{v}_{23}$.

Be careful to express each of these vectors in the same coordinate system. You cannot add vectors unless they are in the same coordinate system. Although the objects may be moving in different directions, they can still use the same coordinate axes.

If $\mathbf{R}$ is the relative separation between two objects moving with constant velociy, and $\mathbf{v}$ is their relative velocity, a collision will occur if $\mathbf{R}$ is parallel to $\mathbf{v}$. The collision will occur in a time t such that $\mathbf{R} = \mathbf{v}t$.

French Chapter 3: Accelerated Motions

Concepts: acceleration, straight-line motion, two-dimensional trajectories, projectile motion, falling objects, uniform circular motion, acceleration in polar coordinates

Equations:

Average acceleration = change in velocity/time = $\Delta\mathbf{v}/\Delta t$.

Instantaneous acceleration: $\mathbf{a} = d\mathbf{v}/dt$.

Take the derivative of each component of the velocity independently to calculate the components of the acceleration. In one dimension, the acceleration is the slope of a graph of the instantaneous velocity as a function of time: $a = dv/dt = d^2x/dt^2$.

For constant acceleration:

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{v}_0 + \mathbf{a}t \\ \mathbf{r}(t) &= \mathbf{r}_0 + \mathbf{v}_0 t + \mathbf{a}t_2/2 \\ v^2 &- {v_0}^2 = 2\mathbf{a}\cdot(\mathbf{r}-\mathbf{r}_0) \end{aligned} \tag{4}$$

Also, the average velocity is the average of the initial and final velocities when the acceleration is constant: $v = (\mathbf{v}_f + \mathbf{v}_i)/2$.

The acceleration due to gravity near the Earth's surface is $g = 9.80$ m/s^2 = 32.0 ft/s^2 directed downward. This is independent of the object.

In projectile motion, the x component is described by a constant velocity, and the y component is described by constant downward acceleration g, as for any falling body. The two components of motion are completely independent.

The position of an object in uniform circular motion can be described by a vector $\mathbf{r}(t) = \mathbf{i}r\cos(\omega t + \phi) + \mathbf{j}r\sin(\omega t + \phi)$ where ϕ specifies the initial position on the circle at $t = 0$. The radial unit vector $\mathbf{e}_r = \mathbf{r}(t)/r$ points in the direction of the moving object.

A perpendicular vector $\mathbf{e}_\theta = -\mathbf{i}\sin(\omega t + \phi) + \mathbf{j}\cos(\omega t + \phi)$ points in the direction of the motion, tangential to the circle. The velocity in uniform circular motion may be written $\mathbf{v}(t) = v\mathbf{e}_\theta$.

The period T of uniform circular motion is the time to make one revolution. The angular velocity is defined as $\omega = 2\pi/T$. The ordinary velocity may be expressed as $v = R\omega$.

In uniform circular motion, there is an inward-pointing acceleration vector with magnitude $a_c = v^2/R$ called the centripetal acceleration. The magnitude may also be expressed as $R\omega^2$.

French Chapter 4: Forces in Equilibrium

Concepts: forces, static equilibrium, forces as vectors, units of force, action and reaction, rotational equilibrium, torque, weight, pulleys and strings

Equations:

Forces add as vectors. An object is in static equilibrium if the sum of all forces on it add to zero, so there is no *net force* ($\sum \mathbf{F}_i = 0$), and there is no net influence which would cause it to rotate. (Rotations are considered below.)

Forces are measured in *Newtons* (N) or *pounds* (lb). Newtons are a derived unit: 1 N = 1 kg m/s^2. Pounds are related to Newtons according to 1 lb = 4.45 N.

Any time an object 1 exerts a force $\mathbf{F}_{12}$ on object 2, object 2 exerts an equal and opposite force on object 1: $\mathbf{F}_{21} = -\mathbf{F}_{12}$. This is true whether or not the forces on either object are in equilibrium.

Forces can cause an object to rotate if they are applied at different points. The tendency of a force to induce rotation is quantified by torque. If a force $\mathbf{F}$ acts at a point described by a vector $\mathbf{r}$ measured from pivot point P, the torque due to $\mathbf{F}$ about the point P has magnitude $\tau = rF \sin\phi$, where ϕ is the angle from $\mathbf{r}$ to $\mathbf{F}$. (This is Giancoli's notation from Chapter 10 – French uses the letter M, but I prefer τ since M is usually used for masses.) Torque is positive when it tends to create a counter-clockwise rotation, and negative for a clockwise rotation. Note that a force directed toward or away from P creates no torque about P. In two dimensions, the torque due to a force $\mathbf{F}$ acting at a point given by a vector $\mathbf{r}$ measured from a pivot point P may be expressed in components as $\tau = r_x F_y - r_y F_x$.

An object is in rotational equilibrium about a point P if the sum of all torques about P is zero: $\sum \tau_i = 0$. In static equilibrium, torques balance about any pivot point, but it is often best to choose one at one of the points where a force acts, so that there is no torque contribution from that force. Begin by drawing a diagram to be sure you are accounting for all of the forces acting on an object.

The tension on a string is the force pulling on it, which could be measured by attaching a spring balance to the end of the string. The tension always acts along the direction of the string. Pulleys can be used to change the direction of the string, and thereby redirect the force.

Weight is, defined operationally, the force measured when we use a scale. If we are at rest on earth, this force is the gravitational attraction of the object to the earth, and given by $W = mg$ where m is the object's mass and $g = 9.80$ N/kg. For an extended object, it may be considered to act at a point called the "center of gravity" which may be found by balancing the object.

Chapter 5: The Various Forces of Nature

Concepts: fundamental forces, electrostatic and gravitational forces, atomic forces, contact forces, friction

$$\text{Gravitational force between two masses:} \quad F = G\frac{m_1 m_2}{R_{12}^2}, \qquad (5)$$

$$\text{Electrostatic force between two charges:} \quad F = k\frac{q_1 q_2}{R_{12}^2}, \qquad (6)$$

where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$, $k = 8.99 \times 10^9\ Nm^2/C^2$. Electric charge is measured in Coulombs (C). An electron has charge $-e$ with $e = 1.60 \times 10^{-19}\ C$.

Modern physics recognizes four "fundamental" forces responsible for all interactions at the elementary particle level. These are, from strongest to weakest, the strong nuclear force, the electromagnetic force, the weak nuclear force, and gravity. The force is carried between objects by particles called "force carriers". The carrier of electromagnetism is the photon, the carrier of the strong nuclear force is the gluon, the carriers of the weak nuclear force are very massive particles called W and Z (around 80 – 100 proton masses), and the carrier of gravity is the (yet unobserved) graviton.

The weak nuclear force is responsible for radioactive decay. It is similar to electromagnetism, but the carriers are massive and unstable, so it acts only over very short distances. The strong nuclear force holds quarks together inside protons and neutrons. It acts between three kinds of charges called colors, and has the unique property that its strength increases with distance, makeing it impossible to observed combinations of quarks and gluons unless they are color-neutral.

When an object is in contact with another object, there is a force perpendicular to the contact area of the objects called the **normal force**. (Normal means perpendicular.)

The maximum frictional force by which an object can resist moving parallel to a surface is $F_f^{\max} = \mu_s F_N$, where μ_s is the **coefficient of static friction** and F_N is the normal force of the surface on the object.

When an object is in motion, the frictional force is proportional to the normal force: $F_f = \mu_k F_N$, where μ_k is the **coefficient of kinetic friction**.

turbulence at higher velocities. In general, it can be expressed as a velocity-dependent force $R(v) = Av + Bv^2$, where the linear term is due to viscosity and the quadratic term is due to turbulence. (The notation here is not universal.)

Chapter 6: Force, Inertia, and Motion

Concepts: inertia, inertial frames, inertial mass, Newton's law, impulse and work,

In an **inertial reference frame**, an object free of a net external force moves in a straight line at constant velocity.

Newton's Law: $\mathbf{F} = m\mathbf{a}$.

Work is the effect of applying a force over a distance: $W = Fx$. Work is measured in Joules, where $1J = 1Nm$. Work applied to a stationary object causes it to acquire **kinetic energy** $\frac{1}{2}mv^2$ equal to the amount of work:

$$Fx = max = \frac{1}{2}mv^2. \qquad (7)$$

Impulse is the effect of applying a force for a time interval: Ft. An impulse applied to a stationary object causes it to acquire **momentum** mv equal to the size of the impulse:

$$Ft = mat = mv. \qquad (8)$$

These equations for work, kinetic energy, impulse and momentum are just different ways of expressing the effect of Newton's laws. They are expressed above for constant forces and acceleration in one dimension, but will be generalized in later chapters.

Summary of Basic Calculus

On the first exam, calculus will be used only on a graphical level, but the following summary of basic calculus concepts may be helpful as we continue through the course. If you are taking calculus now, some of these concepts may not yet be familiar.

A *derivative* represent the instantaneous rate of change of a quantity. If a function $f(t)$ is graphed as a function of t, the derivative df/dt is the slope of the graph at point t. At "turning points" of the graph (local maxima or minima), $df/dt = 0$.

The second derivative of a function d^2f/dt^2 gives the rate of change of its slope. Geometrically, the second derivative of a function is positive when its graph curves upward, and negative when its graph curves downward.

A few useful derivatives are (you don't need to memorize them for the exam)

$$\frac{dt^n}{dt} = nt^{n-1},$$

$$\frac{d\sin(\omega t)}{dt} = \omega\cos(\omega t), \qquad \frac{d\cos(\omega t)}{dt} = -\omega\sin(\omega t) \tag{9}$$

$$\frac{de^{ct}}{dt} = ce^{ct}, \qquad \frac{d\ln(t)}{dt} = \frac{1}{t}. \tag{10}$$

In general, the derivative of an elementary function (any combination of polynomials, trigonometric functions, exponentials, and logarithms) is an elementary function.

The *chain rule* can be used to calculate the derivative of a function that depends on another function: If we have a function $f(x)$ and x in turn is a function of t, then

$$\frac{df}{dt} = \frac{df}{dx}\frac{dx}{dt}. \tag{11}$$

The *integral* of a function $f(t)$ from point t_1 to point t_2 is the area under the graph of the function f between the points t_1 and t_2. When $f(t)$ is negative, it makes a negative contribution to the area.

The *Fundamental Theorem of Calculus* states that integrals are the inverse of derivatives:

$$\int_{t_1}^{t_2} \frac{df}{dt}dt = f(t_2) - f(t_1). \tag{12}$$

The integral of an elementary function usually is not an elementary function. Many "special functions" are defined as integrals of elementary functions. When the integral does give an elementary function, finding it is a matter of working backwards to find what function's derivative is the function being integrated. This is something of an art, and is often done in practice by using integral tables. Integration techniques are the meat of most first courses in calculus. You will not be expected to do any challenging integrals on the exams in this course, but some may come up in homework.