General Physics I-A

Final Exam

Version 2

May 8, 2007

Acceleration of Gravity: $g = 9.8\,\mathrm{m/s^2}$
Gravitational Force Constant: $G = 6.67 \times 10^{-11}\,\mathrm{N{\cdot}m/^2/kg^2}$
Mass of Earth $= 5.97 \times 10^{24}\,\mathrm{kg}$
Radius of Earth $= 6380000\,\mathrm{m}$
1 liter $= 1000\ \mathrm{cm^3} = 1 \times 10^{-3}\,\mathrm{m^3}$
1 atm $= 1.013 \times 10^5\,\mathrm{N/m^2}$
Density of water: 1.00 g/cm^3 = 1000 kg/m^3
Density of air: 1.25 kg/m^3
Speed of sound in air: 343 m/s (at 20°C)
1 cal = 4.186 J
Specific heat of water: 1.00 cal/g
Heat of fusion of water: 79.7 cal/g
Heat of vaporization of water: 539 cal/g
Avogadro's number: $N_A = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Absolute zero: 0 K = -273.15°C
Boltzmann's Constant: $k = 1.38 \times 10^{-23}$ J/K
Ideal Gas Constant: $R = N_A k = 8.315$ J/(mol K)
Stefan-Boltzmann Constant: $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2{\cdot}K^4}$
Moments of Inertia about CM:

Thin Hoop: MR^2, Solid Cylinder: $\frac{1}{2}MR^2$,
Solid Sphere: $2MR^2/5$, Rod: $ML^2/12$.

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1. [1pt] Water entering a house flows with a speed of 0.20 m/s through a pipe of 2.0 cm inside diameter. What is the speed of the water at a point where the pipe tapers to a diameter of 5.0 mm?

A) 3.2 m/s
B) 2.2 m/s
C) 1.8 m/s
D) 8.2 m/s

Explanation:
The equation of continuity for the volume rate of flow through the pipe says that the area times the velocity of the water is constant. Therefore, ${d_1}^2\, v_1 = {d_2}^2\, v_2$, where $d_1 = 2.0\,\mathrm{cm}$ and $v_1 = 0.20\,\mathrm{m/s}$ are the diameter and water velocity where the pipe enters the house, and $d_2 = 0.50\,\mathrm{cm}$ and v_2 are the diameter and velocity at the point of interest inside the house. Therefore, $v_2 = 16\ v_1 = 3.2\,\mathrm{m/s}$.

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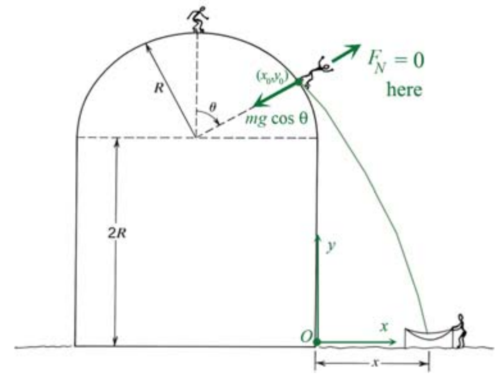
2. [1pt] A daredevil astronomer stands at the top of his observatory dome (see figure) wearing roller skates, and starts with negligable velocity to coast down over the dome surface. The radius of the dome is $R = 9.87\,\mathrm{m}$.



Neglecting friction, at what angle θ does he leave the dome's surface?

A) 45.0°
B) 41.4°
C) 70.5°
D) 48.2°

Explanation:
We can find the speed of the skater at angle θ using energy conservation. The change in potential from the top of the dome to angle θ is

$$\Delta U = mgR \cos\theta - mgR = mgR\,(\cos\theta - 1).$$

Since the skater starts at rest, his kinetic energy at angle θ is

$$K = \tfrac{1}{2}\, mv^2 = -\Delta U = mgR\,(1 - \cos\theta).$$

As long as the skater stays on the dome, his centripetal acceleration is $a_c = v^2/R$ toward the center of the dome. Newton's law states that the centripetal force must be the net radial force on the skater, which is the normal force F_{N} directed radially outward, and the radial component of his weight, $mg \cos\theta$ directed inward. Therefore,

$$mg \cos\theta - F_{\mathrm{N}} = ma_c = mv^2/R.$$

When the skater just loses contact with the dome, $F_{\mathrm{N}} = 0$, and $g \cos\theta = v^2/R$. This implies that

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\, mgR \cos\theta.$$

Setting this expression for the kinetic energy at angle θ equal to the one found previously using energy conservation gives

$$\tfrac{1}{2} \cos\theta = 1 - \cos\theta$$

which implies that $\cos\theta = 2/3$, or $\theta = 48.2°$. Note that the result is independent of the mass of the skater, the radius of the dome, and the strength of gravity (as long as these are all nonzero).

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3. [1pt] A railroad car traveling at 8.3 mph collides with an another car weighing twice as much traveling at 5.0 mph in the same direction and hitches onto it. How fast will the two cars be moving after they are joined?

A) 0.6 mph
B) 6.1 mph
C) 5.8 mph
D) 7.2 mph

Explanation:

The final speed must be the same, v_f. Let $v_1 = 8.3$ mph be the speed of the lighter train and $v_2 = 5.0$ mph be the speed of the heavier train. Momentum conservation implies that

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f.$$

Since $m_2 = 2m_1$, we find that $v_1 + 2v_2 = 3v_f$, which implies that $v_f = 6.1$ mph.

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4. [1pt] A solid, uniform disk and a ring roll down an incline. Let M denote the inertial mass and R the radius. The ring is slower than the disk if

A) $M_{\text{ring}} = M_{\text{disk}}$.
B) The ring is always slower regardless of the relative values of M and R.
C) $R_{\text{ring}} = R_{\text{disk}}$.
D) $M_{\text{ring}} = M_{\text{disk}}$ and $R_{\text{ring}} = R_{\text{disk}}$.

Explanation:

The wheel with the fastest linear velocity will get to the bottom the quickest. Energy conservation implies that if the ramp has height h, the kinetic energy at the bottom is $K_{\text{t}} + K_{\text{r}} = Mhh$, where $K_{\text{t}} = \frac{1}{2}Mv^2$ and $K_{\text{r}} = \frac{1}{2}I\omega^2 = \frac{1}{2}Iv^2/R^2$, using the rolling condition, $v = R\omega$. Then, dividing the energy conservation equation by M, we find that

$$\frac{1}{2}v^2\left(1 + \frac{I}{mr^2}\right) = gh. \tag{1}$$

This implies that the largest speed is attained when I/MR^2 is the smallest. For a solid, uniform disk, $I = \frac{1}{2}MR^2$, and for a ring, $I = MR^2$. The value of I/MR^2 is therefore $\frac{1}{2}$ for the solid disk and 1 for the ring, regardless of M and R. This means the ring is always slower than the disk, regardless of mass and size.

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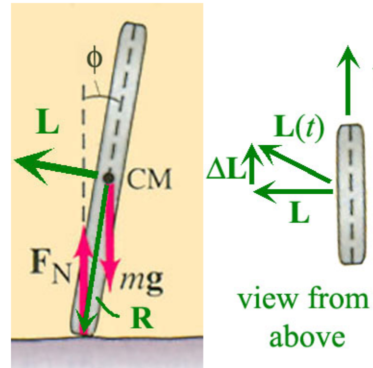
5. [1pt] A tire rolls away from you while leaning slightly toward the right, as shown.



Which statement about the angular momentum of the tire and the torque on it is correct?

A) The angular momentum points toward the right and the torque points into the page.
B) The angular momentum points toward the left and the torque points into the page.
C) The angular momentum points toward the right and the torque points out of the page.
D) The angular momentum points toward the left and the torque points out of the page.

Explanation:

If the wheel in the figure is moving away from the observer, its angular velocity vector and angular momentum vector will both point to the left of the wheel and perpendicular to it, by the right-hand rule, pointing toward the left, and slightly upward. The torque vector $\boldsymbol{\tau} = \mathbf{R}\times\mathbf{F}$ will point in the direction of the wheel's motion. Therefore, the change $\Delta\mathbf{L} = \boldsymbol{\tau}t$ in the angular momentum will be in the direction of the wheel's motion, into the page.

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6. [1pt] It is proposed to put up an earth satellite in a circular orbit with a period of 4.48 hr. How high above the earth's surface would it have to be?

A) 5.2 km
B) 3.7 km
C) 7.4 km
D) 14.8 km

Explanation:

The centripetal acceleration of the satellite must be the acceleration of gravity at a distance $R = R_{\text{E}} + h$ from the center of the earth, where $R_{\text{E}} = 6380$ km is the radius of the earth and h is the height of the orbit above the earth's surface. Gravity follows an inverse square law, so the acceleration of gravity a distance R from the center of the earth is $g(R_{\text{E}}/R)^2$, since it is $g = 9.8\,\text{m/s}^2$ at the surface. The centripetal acceleration is v^2/R, so

$$v^2 = gR_E{}^2/R.$$

The orbital velocity at radius R is related to the period T by $v = 2\pi R/T$. Then the previous relation can be rewritten as

$$4\pi^2 R^3 = gR_E{}^2 T^2,$$

where all terms have been brought into the numerator. The proposed period is $T = 4.48\,\text{hr} = 16128\,\text{s}$, so

$$R^3 = g(R_E T/\pi)^2/4.$$

Evaluating this and taking the cube root gives $R = 13800\,\text{m}$, for a height of $h = R - R_E = 7.4\,\text{km}$ above the earth's surface.

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7. [1pt] Consider two identical glasses. Glass 1 contains only water. Glass 2 contains water and several floating plastic balls. The levels of water in the two glasses are identical. Which glass is heavier?

A) Glass 1.

B) They have the same weight.

C) It is impossible to say without more information.

D) Glass 2.

Explanation:
The mass of a floating plastic ball is equal to the mass of water it displaces. This is equal to the mass of water that would occupy the space containing the underwater part of the plastic ball if it weren't there. This means that when the water levels are the same, the mass of plastic balls plus water is the same as the mass of just the water.

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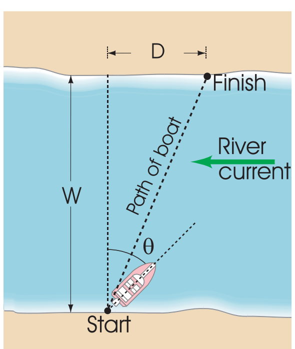
8. [1pt] A boat, whose speed in still water is 2.20 m/s, must cross a $W = 280\,\text{m}$ wide river and arrive at a point $D = 106\,\text{m}$ upstream from where it starts, as shown in the figure below.



To do so, the pilot must head the boat at a $\theta = 37.0°$ upstream angle. What is the speed of the river's current?

A) 0.7 m/s

B) 1.3 m/s

C) 2.0 m/s

D) 2.3 m/s

Explanation:
The boat's speed in still water is $v_{BW} = 2.20\,\text{m/s}$. The boat is heading at an angle $\theta = 37.0°$ measured upstream relative to the direction straight across the river, so its velocity vector is $\mathbf{v}_{BW} = v_{BW}\,(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)$, where $\mathbf{i}$ is a unit vector pointing across the stream, and $\mathbf{j}$ is a unit vector pointing upstream. The velocity of the water with respect to ground is $\mathbf{v}_{WG} = -v_{WG}\mathbf{i}$, where v_{WG} is the unknown speed of the river current. The sum of these velocities is the velocity of the boat relative to the ground:

$$\begin{aligned} \mathbf{v}_{BG} &= \mathbf{v}_{BW} + \mathbf{v}_{WG} \\ &= \mathbf{i}v_{BW}\cos\theta + \mathbf{j}(v_{BW}\sin\theta - v_{WG}). \end{aligned} \tag{2}$$

The ratio of the distance traveled upstream to the distance traveled across the stream is the ratio of the velocity components in those two directions:

$$\frac{D}{W} = \frac{v_{BW}\sin\theta - v_{WG}}{v_{BW}\cos\theta}, \tag{3}$$

which may be solved for the unknown v_{WG} to give

$$v_{WG} = v_{BW}\left(\sin\theta - \frac{D}{W}\cos\theta\right) = 0.7\ m/s. \tag{4}$$

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9. [1pt] A particular organ pipe can resonate at 166 Hz, 249 Hz, and 332 Hz, but not at any other intermediate frequencies. The organ pipe is

A) open on both ends with fundamental frequency 42 Hz.

B) open on one end with fundamental frequency 42 Hz.

C) open on one end with fundamental frequency 83 Hz.

D) open on both ends with fundamental frequency 83 Hz.

Explanation:
In terms of the fundamental frequency f_1, the harmonics of an open pipe are $f_n = n\,f_1$, while the harmonics of a closed pipe are $f_n = (2n - 1)\,f_1$. For the open pipe, the difference between successive harmonics is $\Delta f = f_1$, while for the closed pipe, the difference between successive harmonics is $2f_1$. To find the fundamental frequency f_1, we first must decide whether the pipe is open or closed. To decide, we can form the ratios $f_n/\Delta f$ for successive harmonics f_n. If these ratios are consecutive integers, the pipe is open, while if they are consecutive half-odd-integers, the pipe is closed on one end. For the harmonics given, $\Delta f = 83\,\text{Hz}$, and the values of $f_n/\Delta f$ are $166/83 = 2$, $249/166 = 3$, and $332/166 = 4$. Any of these suffices to show that the pipe is open on both ends, and that the second, third, and fourth harmonics were given, so that the fundamental frequency is $\Delta f = 83\,\text{Hz}$.

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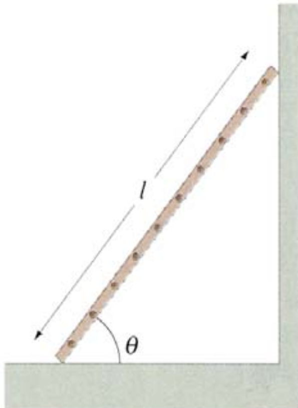
10. [1pt] A uniform ladder of mass 17.8 kg and length 4.6 m leans at an angle θ against a frictionless wall.



If the coefficient of static friction between the ladder and the ground is 0.639, what is the minimum angle at which the ladder will not slip?

A) 38°
B) 57°
C) 18°
D) 33°

Explanation:

The sum of torques about the point where the ladder touches the wall is

$$\tau = \frac{1}{2}mgx \text{ - } F_N x + F_f y$$

where $x = L \cos\theta$ is the distance of the bottom of the ladder from the wall, and $y = L \sin\theta$ is the height of the top of the ladder. The normal force is $F_N = mg$, and the maximum frictional force is $F_f = \mu mg$. Then at the maximum angle θ,

$$\mu mgy = \frac{1}{2}mgx$$

Since $y/x = \tan\theta$, the maximum angle is $\theta = \tan^{-1}(1/(2\mu)) = 38.0°$.

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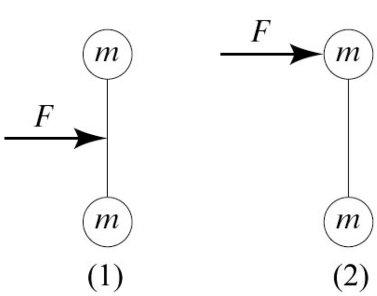
11. [1pt] A force F is applied to a dumbbell for a time interval t, first as in figure (1) and then as in figure (2).



In which case does the dumbbell acquire the greater center-of-mass speed?

A) figure 2
B) The answer depends on the rotational inertia of the dumbbell.
C) no difference
D) figure 1

Explanation:

The force times the time (impulse) gives the change in linear momentum of the center of mass of the dumbbell. Where the force is applied has no influence on the center-of-mass motion.

12. [1pt] In which case does the dumbbell acquire the greater energy?

A) figure 2
B) The answer depends on the rotational inertia of the dumbbell.
C) no difference
D) figure 1

Explanation:

The energy is the sum of the translational and rotational energy of the dumbell. Since the linear momentum is the same in each case, the translational energy is the same as well. But the dumbell in case (2) also acquires rotational energy, since there is a torque on the dumbell. In case (1), there is no torque, and therefore no rotation. This means that the total energy is greater in case (2).

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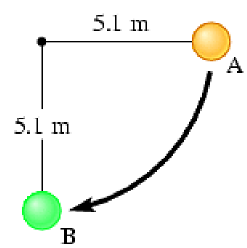
13. [1pt] Two pendulum bobs have equal masses and lengths (5.1 m). Bob A is initially held horizontally while bob B hangs vertically at rest (the figure below). Bob A is released and collides elastically with bob B. How fast is bob B moving immediately after the collision?



A) 20 m/s
B) 10 m/s
C) 5 m/s
D) 15 m/s

Explanation:

By energy conservation, the speed of the bob when it falls to the bottom is determined by $\frac{1}{2}mv^2 = mgh$, with $h = 5.1\,\mathrm{m}$. The factors of m cancel, giving $v = 10\,\mathrm{m/s}$ when the first bob reaches the bottom of its swing, just before striking the stationary bob. Since the two bobs have the same mass, momentum and energy are conserved when the first bob comes to rest, transferring all its momentum to the stationary one, giving it an initial speed of $10\,\mathrm{m/s}$.

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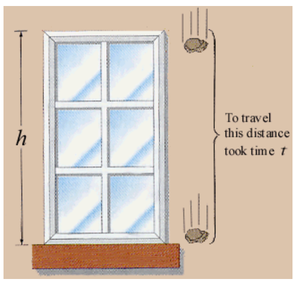
14. [1pt] A falling stone takes time t = 0.23 s to travel past a window of height $h = 3.5\,\mathrm{m}$.



From what height above the top of the window did the stone fall?

A) 8.8 m
B) 12.2 m
C) 17.3 m
D) 6.1 m

Explanation:

The stone took a time $t = 0.23\,\mathrm{s}$ to cross the window of height $h = 3.5\,\mathrm{m}$, so $h = v_0 + \frac{1}{2}gt^2$, where v_0 is the speed of the stone at the top of the window. This speed is unknown, but related to the height y above the window from which the stone was dropped by ${v_0}^2 = 2gy$. Solving the previous equation for v_0 gives

$$v_0 = h/t + \tfrac{1}{2}gt = 15.5\,\mathrm{m/s}.$$

Solving the second equation for y then gives

$$y = {v_0}^2/(2g) = 12.2\,\mathrm{m}.$$

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15. [1pt] A spring that obeys Hooke's Law both in extension and compression is extended by 20 cm whan a mass of 5.40 kg is hung from it. What is oscillation period if the mass is pulled down further and let go?

A) 0.90 s
B) 0.13 s
C) 0.02 s
D) 0.14 s

Explanation:

The spring constant is the force applied divided by the distance the spring stretches:

$$k = mg/\Delta x = (5.40\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\ /\ 0.200\,\mathrm{m} = 265\,\mathrm{N/m}.$$

The angular frequency of oscillation when the spring is pulled down further is $\omega = (k/m)^{1/2} = 7.00\,\mathrm{s}^{-1}$, giving a period of $T = 2\pi/\omega = 0.90\,\mathrm{s}^{-1}$.

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16. [1pt] A spring of negligable mass exerts a restoring force given by

$$F(x) = \text{-}k_1x + k_2x^2\ .$$

Calculate the potential energy stored in the spring for displacement x as a function of k_1 and k_2, taking $U(x) = 0$ at $x = 0$.

A) k_1x^2 - k_2x^3
B) $2k_1x^2$ - $3k_2x^3$
C) k_1 - $2k_2x$
D) $k_1x^2/2$ - $k_2x^3/3$

Explanation:

The potential can be obtained by integrating the force over distance:

$$U(x) = \text{-}\int F(x) = k_1x^2/2 - k_2x^3/3.$$

The integration constant is set so that $U(0) = 0$.

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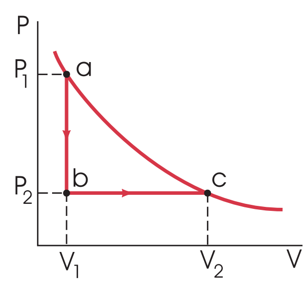
17. [1pt] Consider the following cyclic process. An ideal gas enclosed in a piston starts at point a and is cooled at constant volume so that its pressure drops from P_1 to P_2 at point b. Then the gas expands at constant pressure, from a volume of V_1 to V_2, where the temperature reaches its original value at point c. Then the gas is compressed at constant temperature back to its original pressure P_1 and volume P_2, returning to point a.



For which of the following processes is the absolute value of work done the greatest (without concern for the sign)?

A) bc
B) ab
C) ca
D) abca (complete cycle)

Explanation:
The work done in any process on a PV diagram is the area under the curve representing the process. The more area under the curve, the greater the magnitude of work done. In this case, In this case, the absolute value of work done is greatest for process ca, less for bc, and zero for ab. The net work done in the cycle abca is less than in process ca, because it is the difference between the absolute values of work done from b to c and from c to a.

18. [1pt] Suppose that in the diagram for the previous problem, $P_1 = 2.28\,\mathrm{atm}$, $P_2 = 1.23\,\mathrm{atm}$, $V_1 = 6.79\,\mathrm{L}$, and $V_2 = 12.6\,\mathrm{L}$. How much work is done by the gas in going from point a to c?

A) 724 J
B) -724 J
C) 971 J
D) -971 J

Explanation:
The work done when the gas expands at constant temperature is $W = nRT\ \ln(V_2/V_1)$. The ideal gas law implies that $nRT = P_1V_1 = P_2V_2 = 15\,\mathrm{atm{\cdot}\,L}$. The units can be converted to Joules using $1\,\mathrm{atm} = 1.013\times10^5\,\mathrm{N/\,m^2}$ and $1\,\mathrm{L} = 1\times10^{-3}\,\mathrm{m^3}$, which implies that $1\,\mathrm{atm{\cdot}\,L} = 101.3\,\mathrm{J}$. Then $nRT = 1570\,\mathrm{J}$. Also, $\ln(V_2/V_1) = 0.618$, so $W = 971\,\mathrm{J}$.

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19. [1pt] An ideal refrigerator removes heat at a rate of 0.10 kW from its interior (+2.0°C) and exhausts heat at 40.0°C. How much electrical power is used?

A) 20 W
B) 14 W
C) 13 W
D) 15 W

Explanation:
The Carnot efficiency for a refrigerator is e_C = 1 - $T_\mathrm{C}/T_\mathrm{H}$, where the absolute temperatures inside and outside the refrigerator are, respectively, $T_\mathrm{C} = 275.15\,\mathrm{K}$ and $T_\mathrm{H} = 313.15\,\mathrm{K}$. This gives $e_\mathrm{C} = 0.121$. The efficiency may also be written as $e_\mathrm{C} = W/Q_\mathrm{H}$, where $Q_\mathrm{H} = W + Q_\mathrm{C}$, with Q_H and Q_C the heat released into the room and removed from the interior, and W the electrical power used. Then

$$W = e_\mathrm{C}(W + Q_\mathrm{C})$$

and solving for W gives

$$W = \frac{Q_C}{e^{-1} - 1} = \frac{0.10\,\mathrm{kW}}{7.24} = 13.8\ \mathrm{W}. \tag{5}$$

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20. [1pt] On a cold winter day, the outside temperature is -15.0°C. Inside the house the temperature is +20.0°C. Heat flows out of the house through a window at a rate of 220.0 W. At what rate is the entropy of the universe changing due to this heat conduction through the window?

A) 0.112 J/(K · s)
B) 0.122 J/(K · s)
C) 0.109 J/(K · s)
D) 0.102 J/(K · s)

Explanation:
The entropy change due to heat flowing out of the room is

$$\Delta S_1 = \text{-}Q/T_\mathrm{H} = \text{-}220.0\,\mathrm{W}/293.1\,\mathrm{K} = \text{-}0.750\,\mathrm{W/\,K}.$$

The entropy change due to heat flowing into the exterior is

$$\Delta S_2 = Q/T_\mathrm{C} = 220.0\,\mathrm{W}/258.1\,\mathrm{K} = 0.852\,\mathrm{W/\,K}.$$

The total entropy change in this process is then

$$\Delta S = \Delta S_1 + \Delta S_2 = 0.102\,\mathrm{W/\,K}.$$

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21. [1pt] If one jet engine at a certain distance has a loudness of 100 dB, how loud will the plane be if if all four of its engines are turned on?

A) 106 dB
B) 112 dB
C) 300 dB
D) 130 dB

Explanation:
Each factor of 2 in intensity increases the loudness by 3 dB. Therefore, four engines will be 6 dB louder than one, giving a loudness of 126 dB.

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22. [1pt] How high can you suck water up a straw? The pressure in the lungs can be reduced to about 10 kPa below atmospheric pressure.

A) 0.7 m
B) 1 m
C) 2 m
D) 0.5 m

Explanation:
The water in the straw is lifted by the pressure difference between the bottom and top of the straw. The height h the water can be lifted is determined by $\rho gh = \Delta P = 1.00\times10^4\,\mathrm{N/m^2}$, where $\rho = 1000\ \ \mathrm{kg/m^3}$ is the density of water and $g = 9.8\ \mathrm{m/s^2}$. This gives a maximum height of $h = 1.0\,\mathrm{m}$.

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23. [1pt] A 3.5 kg rock is suspended by a massless string from one end of a 1-m measuring stick.

What is the mass of the measuring stick if it is balanced by a support force at the 0.25-m mark?

A) 1.8 kg
B) 2.4 kg
C) 3.5 kg
D) 2.7 kg

Explanation:

The weight of the rock acts 0.25 m to the left of the support. The weight of the measuring stick acts at its center of gravity, which will be at the center, assuming the stick is uniform. The center of the meter stick is 0.25 m to the right of the support. Since this is the same distance as the distance to the rock, the stick must weigh as much as the rock.

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24. [1pt] A string 2.0 m long is held fixed at both ends. If a sharp blow is applied to the string at its center, it takes 0.050 s for the pulse to travel to the ends of the string and return to the middle. What is the fundamental frequency of oscillation for this string?

A) 12 Hz
B) 6 Hz
C) 10 Hz
D) 15 Hz

Explanation:

The wavelength of the fundamental vibrational mode is twice the length of the string, or $\lambda = 2 \times 2.0\,\text{m} = 4.0\,\text{m}$. The fundamental frequency is $f = v/\lambda$, where the wave velocity is the same as the pulse velocity. Since the pulse travelse one string length in 0.050 s, the wave velocity is 40.00 m/s. This gives $f = 10.0\,\text{Hz}$.

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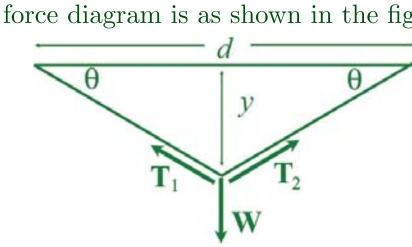
25. [1pt] The two trees in the figure are 7.20 m apart.

Calculate the force **F** a backpacker must exert to hold a 18.5 kg backpack so that the rope sags at its midpoint by 1.40 m.

A) 466 N
B) 250 N
C) 500 N
D) 932 N

Explanation:

Neglecting any friction in the pulley, the magnitude of the force **F** applied by the backpacker is the same as the magnitude of the tension in the rope. The tension on the rope can be found by balancing forces at the point where the backpack hangs. The force diagram is as shown in the figure.



The condition for the forces to be balanced is that the vector sum of the forces must vanish: $\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = 0$. This equation has two components. Only the vertical component is really needed in this problem. The sum of the upward tensions must equal the weight: $T_1^y + T_2^y = W$, where $W = mg = 181\,\text{N}$ is the weight of the backpack. Since the backpack is hanging in the middle of the rope, the problem is symmetric, and the magnitudes of the two tensions must be the same: $T_1 = T_2 = F$ is the same as the force applied by the backpacker to hold the pack up. Therefore,

$$W = 2F \sin\theta.$$

The angle θ can be found from the fact that the distance d between the trees is $d = 7.2\,\text{m}$, and the rope sags a distance $y = 1.40\,\text{m}$. This implies that $\tan\theta = 2y/d = 0.389$, so that $\theta = 21.3°$. Then

$$F = W\ /\ (2\sin\theta) = 250\,\text{N}.$$

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26. [1pt] Incandescent lightbulbs are filled with an inert gas to lengthen the filament life. With the current off (at $T = 20.0°$C), the gas inside a lightbulb has a pressure of 115 kPa. When the bulb is burning, the temperature rises to 70.0°C. What is the pressure at the higher temperature?

A) 147 kPa
B) 135 kPa
C) 138 kPa
D) 142 kPa

Explanation:
The ideal gass law says that P/T is unchanged when the bulb is heated, since the volume and amount of gas inside are fixed. The initial temperature is 20°C = 293.15 K, and the final temperature after the bulb is hot is 70.0°C = 343.1 K. Therefore, the pressure inside the hot bulb is

$$P = 115.0\,\text{kPa}\;(343.1/293.15) = 134.6\,\text{kPa}.$$

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27. [1pt] What mass of water at 36°C must be added to 1.00 kg of ice at 0°C to yield liquid water at 12.0°C?

A) 0.26 kg
B) 3.82 kg
C) 9.55 kg
D) 2.04 kg

Explanation:
An unknown mass m_w of water at tmperature $T_1 = 36°$C is added to a mass $m_\mathrm{i} = 1.00$ kg of ice at the freezing point 0°C, yielding liquid water at temperature $T_2 = 12°$C. The latent heat of fusion for ice is $L_\mathrm{f} = 79.7$ cal/g, so energy conservation gives

$$m_\mathrm{i} L_\mathrm{f} + m_\mathrm{i} c_\mathrm{w} (T_2 \text{ - } 0°\text{C}) = m_\mathrm{w} c_\mathrm{w} (T_1 \text{ - } T_2)\ .$$

The specific heat of water is $c_\mathrm{w} = 1.00$ cal/g. Therefore, solving for the mass of water gives

$$\begin{aligned} m_w &= m_i \left[\frac{L_f + c_w T_2}{c_w (T_1 - T_2)}\right] \\ &= 1.00\,\text{kg}\ \left[\frac{79.7 + 12}{24}\right] = 3.82\,\text{kg}. \end{aligned} \tag{6}$$

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28. [1pt] To contain some unruly demonstrators, the riot squad approaches with fire hoses. Suppose that the rate of flow of water through a fire hose is 24 kg/s and the stream of water from the hose moves at 17 m/s. What force is exerted by such a stream on a person in the crowd? Assume that the water comes to a dead stop against the demonstrator's chest.

A) 410 N
B) 620 N
C) 320 N
D) 200 N

Explanation:
The mass of water being brought to a stop each second is $\Delta m = 24$ kg. The water leaves the hose at speed $v = 17$ m/s, so the change in momentum, or impulse, each second is $\Delta mv = 410$ kg m/s. Dividing this by $t = 1$ s gives the impulse per second, which is equal to the force, $F = \Delta mv/t = 410$ N.

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29. [1pt] Two satellites A and B of the same mass are going around Earth in concentric orbits. The distance of satellite B from Earth's center is twice that of satellite A. What is the ratio of the centripetal force acting on B to that acting on A?

A) $1/\sqrt{2}$
B) 1/8
C) 1
D) 1/2
E) 1/4

Explanation:
The centripetal acceleration is equal to the acceleration due to gravity at the height of the satellite. By the inverse square law of Newtonian gravity, the gravitational acceleration at twice the distance is one quarter as great. Therefore, the centripetal acceleration is also one quarter as great.

30. [1pt] What is the ratio of the tangential speed of satellite B to that of A?

A) $\sqrt{2}$
B) 2
C) 1
D) $1/\sqrt{2}$
E) 1/2

Explanation:
The centripetal acceleration is v^2/r at a distance r from the Earth's center. By the inverse square law, the centripetal acceleration for satellite B is 1/4 that of /satellite A, since it is twice as far away. Therefore, ${v_B}^2/r_B = {v_A}^2/(4r_A)$. Rearranging this gives

$${v_B}^2\ /\ {v_A}^2 = r_B/(4r_A) = 1/2.$$

Therefore, $v_B/v_A = 1/\sqrt{2}$.