Physics 103

Department of Physics
Princeton University

Precept Notes

S. Yost
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Harmonic Oscillator and Exam 3 Review Part 2

Announcements

There is an exam tomorrow.

- I strongly recommend working last year's exam. If you try it in 1 hour, how you do on it should be a good predictor of how you will do on the real exam. Also try 2005 Quiz 5 & 6, midterm problem 2.
- Make sure you can answer the Friday problems on your own. (Skip the galactic rotation curves.)
- Give enough details, and write clearly enough, to convince a skeptic that your answer is right.
- Don't waste time memorizing a lot of equations – you don't get full credit for that anyway.

After the exam, finish Chapter 14. Some notes will be posted.

SHM Example

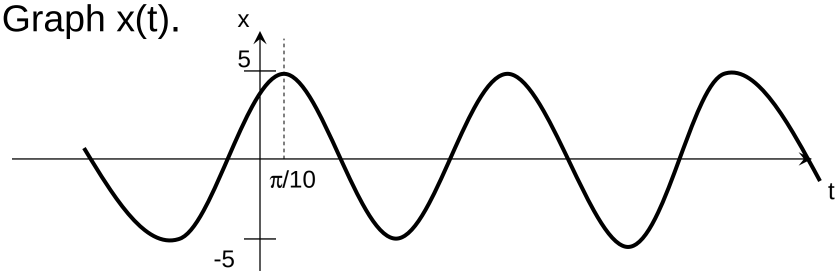
Let the position be in m, the time in s.

Suppose $x = 5\cos(2t - \pi/5)$.

- What are the amplitude and period?

  $A = 5\text{m}, \omega = 2\text{s}^{-1}$

- Graph x(t).



SHM Example

- What is the initial velocity?

  $\omega = 2\text{s}^{-1}$, r = 5 m, $v = r\omega$ = 10 m/s tangential

  $v_x(\theta = -\pi/5) = 10 \sin(\pi/5)$ (m/s)

- What is the maximum velocity?

  $v_{max} = v$ = 10 m/s.

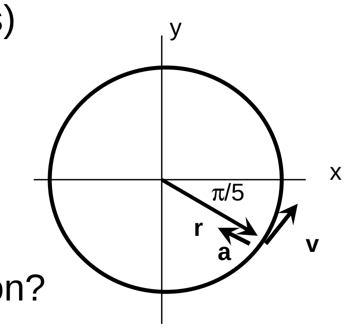
- What is the initial acceleration?

  $a = r\omega^2 = \omega v = 20\ \text{m/s}^2$ inward

  $a_x(\theta = -\pi/5) = -20\cos(\pi/5)$

- What is the maximum acceleration?

  $a_{max} = a$ = 20 m/s.



SHM Example

The velocity and acceleration can also be found using calculus:

$v = dx/dt = -10 \sin(2t - \pi/5),$

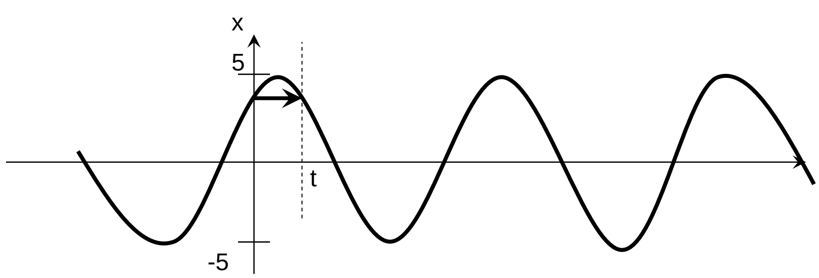
$v(t = 0) = 10 \sin(\pi/5)$ (m/s)

$a = dv/dt = -20 \cos(2t - \pi/5).$

$a(t = 0) = -20 \cos(\pi/5)$ (m/s^2)

SHM Example

How long does it take to get back to its initial position the first time?



SHM Example

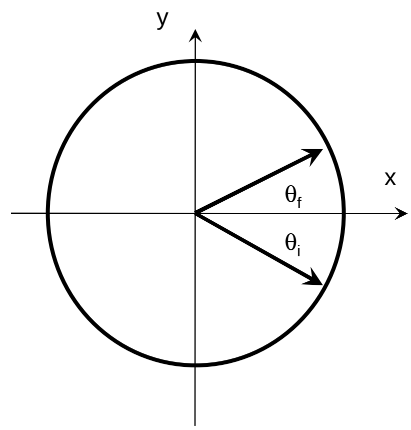
At t = 0, $x = 5\cos(-\pi/5)$,

$$\theta_i = -\pi/5.$$

Think in terms of the circle. It gets back to the same x when

$$\theta_f = +\pi/5$$

$T = (\theta_f - \theta_i)/\omega = \pi/5$ s



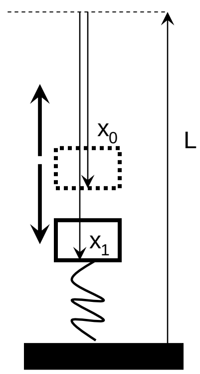
Spring Launcher

A mass m is set on a vertical spring of uncompressed length L. It sinks down a distance x_0 below L. What is k?

$$mg = kx_0$$

Suppose it is pushed down a distance x_1 below L.

How high does it rebound if it stays attached to the spring?



Spring Launcher

Let gravitational PE = 0 at the equilibrium position x_0. Measure x down.

Initially, $U = ½ kx_1^2 – mg(x_1 – x_0)$

At the high point,

$$U = ½ kx_2^2 – mg(x_2 – x_0).$$

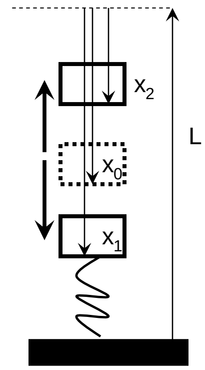
Subtract:

$$0 = ½ k(x_2^2 – x_1^2) + mg(x_1 – x_2)$$

$$= (x_2 – x_1)(½ k(x_2 + x_1) – mg).$$

The new solution is

$$x_2 = 2mg/k – x_1 = 2x_0 – x_1.$$



Spring Launcher

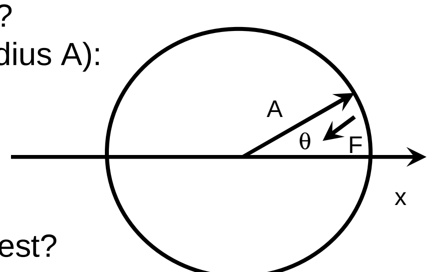
What is its oscillation frequency?

Projected circular motion (radius A):

$a_{max} = A\omega^2 = kA/m.$

$\omega = (k/m)^{1/2}$, $k = mg/x_0$

$\omega = (g/x_0)^{1/2}$



Where is the net force the greatest?

At the top and bottom: the **net forces** there are equal and opposite…

$F_{bottom} = k(x_1 – x_0) = kx_1 – mg$

$= mg(x_1 – x_0)$

$F_{top} = k(x_2 – x_0) = kx_2 – mg$

$= mg(x_0 – x_1)$

centripetal force
$F = (M+m)A\omega^2$

$F_x = (M+m)A\omega^2 \cos\theta$

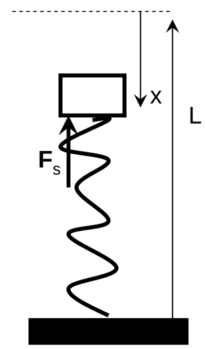
Spring Launcher

The force supplied by the spring alone is (for x down)

$F_s = kx$

so when $x < 0$, the spring holds up the mass, but for $x > 0$, the spring must pull it down.

If the spring is not attached to the mass, it can't pull down, and loses contact at $x = 0$.



Spring Launcher

If the spring is then released and the mass is not attached to it, how high will the mass go above its starting point (x_1 below L)?

Initial PE: $U_s = ½\, kx_1^2$

Final PE: $U_g = mgH$.

(I can pick x_1 as the point where gravitational PE is 0 now.)

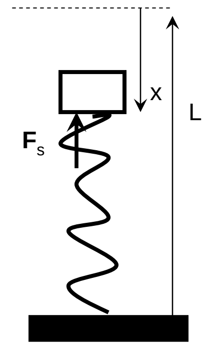
$H = kx_1^2/2mg = x_1^2/(2x_0)$.

Spring Launcher

- At what height does the mass lose contact with the spring?

  When $F_s = kx = 0$, no force due to the spring. This is the unstretched position, $x = 0$.
- Where is the object moving fastest?

  At its equilibrium point, $x = -x_0$, whether or not it stays attached to the spring.



Spring Launcher

If $x_1 > 2x_0$, but the spring is attached to the mass, it will pull up on the table when $x < 0$. What is the maximum force of the spring upward on the table?

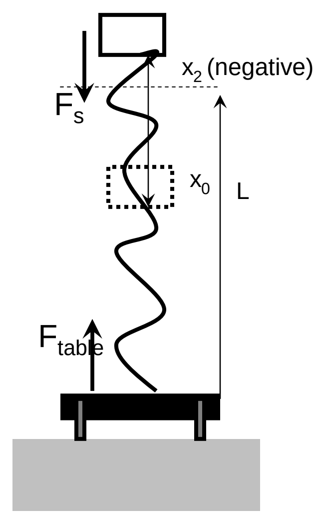
It is equal to the force on the mass at its highest point.

There, $F_s = kx_2 = k(2x_0 - x_1) < 0$

The force lifting up on the table is

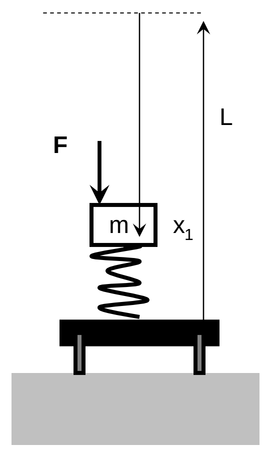
$$F_{table} = -F_s = k(x_1 - 2x_0).$$

Note that the weight of the object does not appear here.



Spring Launcher

If the table has mass M, for what x_1 can it be picked up off the ground by the spring? How much force would I need push down to compress the spring to this distance?



That requires $Mg = k(x_1 - 2x_0)$

$$x_1 = Mg/k + 2x_0 = (M/m + 2)x_0$$
$$= (M + 2m)g/k.$$

The equilibrium position is x_0, so the force needed to push down the spring is

$$F = k(x_1 - x_0) = k(M/m + 1)x_0$$
$$= (M + m)g.$$

Non-Uniform Rod

A rod of length L has a mass distribution that increases from the left to the right according to $\rho(x) = cx^2$.

- What is the position of the CM?
- What is the moment of inertia about $x = 0$?
- What is the moment of inertia about the CM?

Non-Uniform Rod

- What is the position of the CM?

$Mx_{cm} = \int_0^L x\,dm$ with $dm = \rho(x)dx = cx^2dx$.

$Mx_{cm} = \int_0^L cx^3dx = cL^4/4$

$M = \int_0^L dm = \int_0^L cx^2dx = cL^3/3$.

$x_{cm} = 3L/4$

Non-Uniform Rod

- What is the moment of inertia about x = 0?

$I = \int_0^L x^2dm = \int_0^L cx^4dx = cL^5/5$.

$M = cL^3/3$.

$I = (3/5)\ ML^2$.

- What is the moment of inertia about the CM?

Parallel axis theorem:

$I(x=0) = I_{cm} + Mx_{cm}^2$

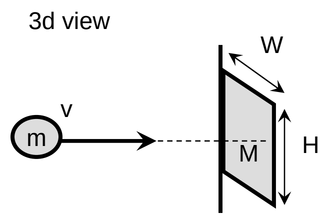
$I_{cm} = (3/5)\ ML^2 - (3/4)^2\ ML^2 = (3/80)\ ML^2$.

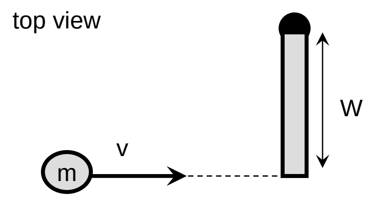
Rigid Body Collision

Sticky mud is thrown at door as shown and sticks at the outer edge. Find ω for the door if

$m = M/3$.

Angular momentum is conserved about the hinge, since the door cannot put a torque on it. (Neglect friction in the hinge.)





Rigid Body Collision

Initial: $L = mvW$.

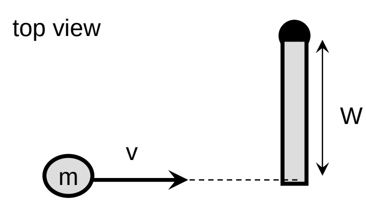
Final: $L = I\omega$.

$\omega = mvW/I$

$I = (MW^2/3) + mW^2$

$= 2mW^2$.

$\omega = v/2W$.



Is linear momentum conserved?

No – there is a horizontal force on the hinge of the door.

In a collision between two free bodies, both **p** and **L** are conserved about the CM or any other point at rest or moving with constant linear velocity.

Rigid Body Collision

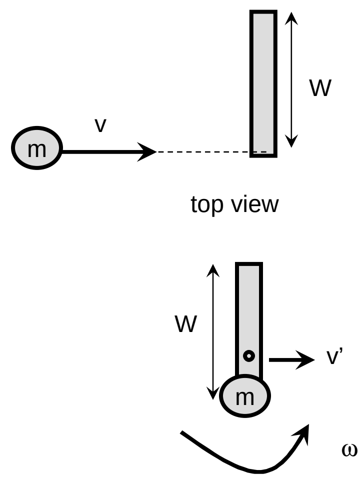
What would happen if the hinge weren't there?

Momentum is now conserved:

mv = (m + M)v' = 4mv'

v' = v/4.

The combined object would rotate about its cm at a rate given by angular momentum conservation.



Rigid Body Collision

CM is a distance

D = MW/2(M + m) = 3W/8 in from the outer edge.

Angular momentum:

L = mvD = 3mvW/8.

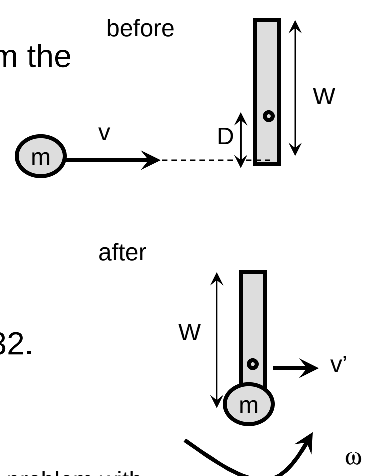
$= I_{cm}\ \omega$.

Parallel Axis Theorem:

$I_{cm} = MW^2/3 - MD^2 + mD^2$

$= mW^2 - 2m(3W/8)^2 = 23mW^2/32$.

$\omega = 12v / 23W$



This is slightly less than in the previous problem with the door attached to a hinge. It is about a different axis also.

Escape from Orbit

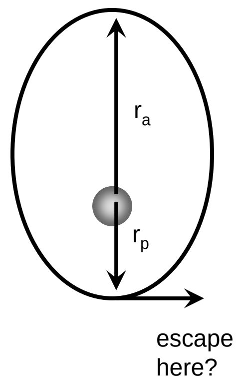
- If I am in an elliptical orbit about the earth with semimajor axis a and $r_a = 3\ r_p$, and want to escape to infinity, how much work must be done?

W = 0 – E = GMm/2a.

- Where should I burn the rocket to use the least fuel? Perigee or Apogee?

Competing effects: gravity is weaker at aphelion, but you are moving faster at perihelion, and may start out closer to escape velocity.

$r_p + r_a = 2a$, and $r_a = 3r_p$, so $r_p = a/2$, $r_a = 3a/2$



Escape from Orbit

Escape velocity determined by $\frac{1}{2} m v_e^2 - GMm/r = 0$.

$$v_e^2 = 2GM/r.$$

Orbital velocity determined by: $\frac{1}{2} m v^2 - GMm/r = - GMm/2a$.

$$v^2 = GM(2/r - 1/a) = v_e^2 - GM/a$$

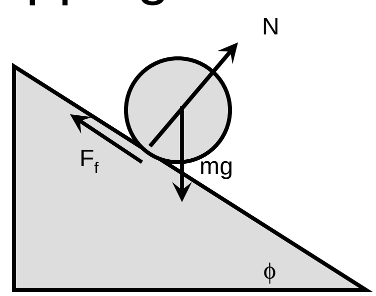
Perigee: $r_p = a/2$, $v_e^2 = 4GM/a$, $v_p^2 = 3GM/a$.

$\Delta v_p = (2 - 3^{1/2})GM/a = 0.268\ GM/a$

Apogee: $r_a = 3a/2$, $v_e^2 = 4GM/3a$, $v_a^2 = GM/3a$.

$\Delta v_a = GM/3^{1/2}a = 0.577\ GM/m = 2.15\ \Delta v_p$

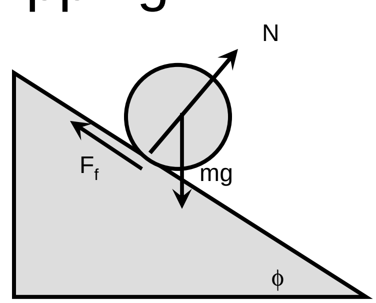
Rolling and Slipping



A wheel of moment of inertia $I = kmR^2$ rolls down an incline at angle ϕ, but it is so steep it slips.

- How long does it take to get to the bottom if the ramp length is L?

Rolling and Slipping



Linear:

$ma = mg \sin \phi - F_f$

Since it is slipping,

$F_f = \mu N = \mu mg \cos \phi$.

$L = ½ at^2$.

$a = g(\sin\phi - \mu\cos\phi)$.

$t = [2L/(g(\sin\phi - \mu\cos\phi))]^{1/2}$

Rolling and Slipping

- How many revolutions does the wheel make in this distance?

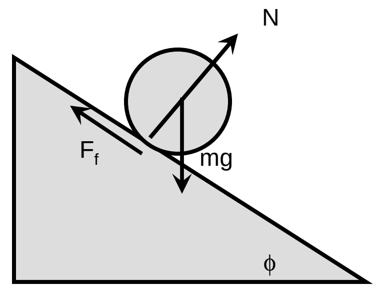
$\theta = \frac{1}{2}\alpha t^2$ with $\frac{1}{2}t^2 = L/a$.

So $\theta = L\alpha/a$.

$I\alpha = kmR^2\alpha = \tau = RF_f$.

$\alpha = F_f/kmR = \mu g \cos\phi/kR$.

revs. $= \theta/2\pi = \dfrac{\mu L}{2\pi kR(\tan\phi - \mu)}$



If not slipping,
revs = $L / 2\pi R$.

Rolling and Slipping

- Find the minimum μ_0 not to slip at this angle.

$a = R\alpha$ if not slipping.

$g(\sin\phi - \mu_0\cos\phi) =$

$\mu_0 g \cos\phi/k$

$k \sin\phi = \mu_0(1 + k)\cos\phi$

$\mu_0 = k\tan\phi / (1 + k)$.