Physics 103

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Precept Notes

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December 12, 2007

Special Relativity

(Introduction)

Announcements

- Prof. Staggs substituted for this precept, so I don't have notes on it. Here are some things from her notes, with some additions to put things more how I might think of it.
- Next Monday is on a Thursday schedule: no precept.
- Next Tuesday is the 4th exam, but it's here, not in the lecture.
- The last topic for the semester is Ch. 36, Special Relativity, but due to limited time, we will be concentrating on just the modifications to the concepts of space and time, not on the modifications of Newtonian dynamics, or coordinate transformations (Lorentz transformations).

Galilean Invariance

- Newtonian mechanics is based on Galilean invariance, which says the laws of physics are the same for any inertial observer.
- This isn't wrong, but appear to be in serious conflict with the laws of electricity and magnetism formulated in the 19th century. These laws predict that light moves at speed

  $c = 3 \times 10^8$ m/s

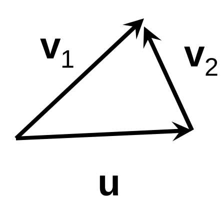
  regardless of the observer.
- It took Einstein's theory of special relativity to reconcile these facts, which are both true!

Galilean Transformations

- What's the problem?

Remember that early in the course, we said that if someone at rest measures the velocity of an object as $\mathbf{v}_1$, and someone moving at velocity $\mathbf{u}$ with respect to the first also measures this objects velocity, and calls it $\mathbf{v}_2$, the relation between the two velocities is

$$\mathbf{v}_2 = \mathbf{v}_1 - \mathbf{u}.$$



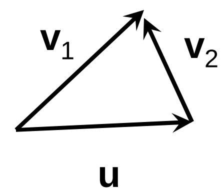
Galilean Transformations

This transformation of velocities when measured by different observers seems obvious, since we normally assume everyone would measure distances and times the same way.

But it is completely inconsistent with the verified fact that the speed of light is the same for any observer.

Not only does c not transform, but it is impossible to add *any* two velocities in such a way that the sum is greater than c!

This requires a modification of our concepts of space and time.

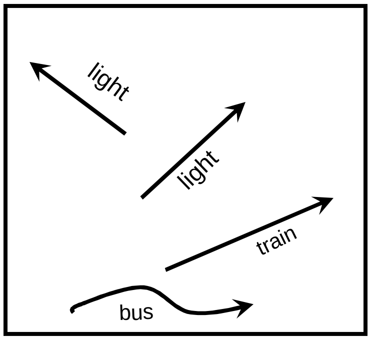


What can we trust?

- Einstein realized the deep significance of the fact that the laws of physics were the same to all inertial observers. This must be preserved.
- A consequence is the fact that the laws of physics acting in a frame of reference moving at constant velocity must be identical to those acting at any other constant velocity.
- In particular, everyone can trust their own watch, which is moving along with them, but it is not clear that they can trust anyone else's.
- Similarly, they can carry a meter stick along with them and use it to measure things at rest relative to them, but there is no guarantee that a moving person will make the same measurements.
- These two points dispense with the idea, assumed by Newton and just about everyone else before Einstein, that distances or times are something absolute, that can be measured without regard to how one is moving.

Space-Time

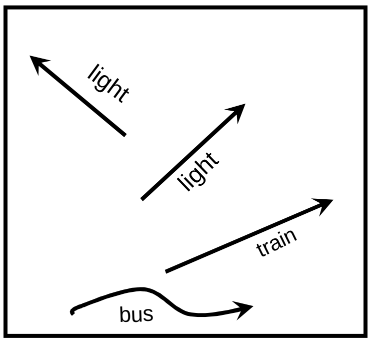
- We used space-time diagrams to describe motion before, but we always labeled the axis in a way we expected to be universal.
- This is no longer possible. Space-time is made up of events occurring at a particular place and particular time, but measuring these places and times depends on a particular observer, each of whom may choose his own coordinates to describe space-time. We can't even say for sure which way "time" points.
- The one thing we can be sure of, is that light moves on lines of slope c, and anything else moves on lines with slope less than c. The speed of light is a "speed limit" for the universe. The direction of time has to be steeper than the light-paths, but how steep can vary.



In this picture of space-time, c = 1.
Light rays slope upward at 45º.

Space-Time

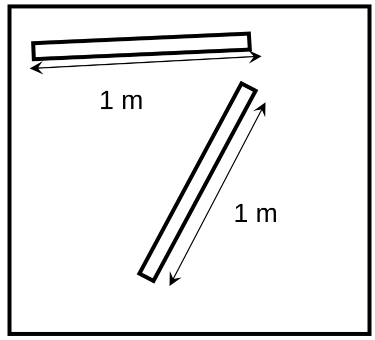
- Is the train at rest?
- Who knows – that depends on the observer. It is meaningless to say something is absolutely at rest, and the picture doesn't give a way to tell. But it does have a constant velocity less than c.
- Is the bus at rest?
- No – there is no constant velocity from which a curved path through space and time can be at rest, either from Galileo or Einstein's view. This is an accelerating object.



In this picture of space-time, c = 1.
Light rays slope upward at 45º.

Space

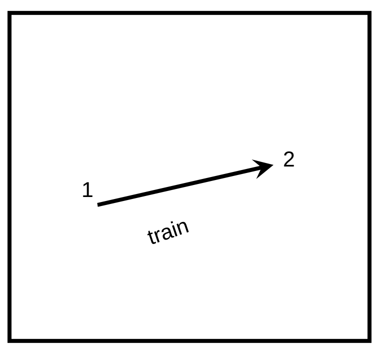
- How can we measure anything in this apparently formless spacetime?
- Think about space. It doesn't come with axes drawn on it either, but we can say how long a meter stick is, no matter which way it points.
- The trick is just to measure the stick along the stick. Don't try to measure it in some crazy direction, and you will always get 1 m.



space alone

Space-Time

- The points in space-time are events, and if they are along a line with slope < c, we can find a velocity to move at that will take us from one event to the other. In that reference frame, the events occur at the same location, and we measure the time according to a clock in the same frame.
- This is the analog of measuring a stick along the stick: no matter how you move, you measure your time along your own path through spacetime.
- All observers can agree on that definition of time, because it is specified without giving any "coordinate system" or external reference frame – it just depends on your own motion.

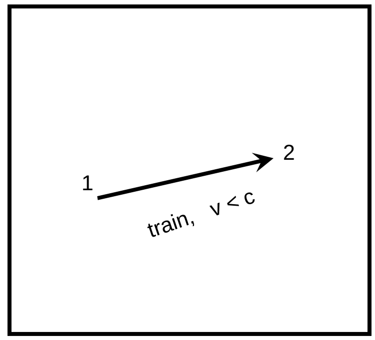


Space-Time

The points 1, 2 are two events along the path of the train through space-time.

In the rest-frame of the train, they are at the same position, but different times.

A clock on the train can be used to measure this time difference, called the **proper time** between events 1 and 2.



Proper Time and Length

- We call the time measured in the rest frame of the observer **proper time**. If everyone carried around an identical watch, all of them would be measuring this time in their own reference frames.
- We can also define a **proper distance**, by measuring the distance between two points at the same time in our own reference frame. If we were holding a meter stick in our hands, this would be 1 m. It is the size of the size of an object from its own point of view, assuming it is small enough to measure itself in a negligible amount of time.

Proper Time and Length

The key points to remember are that

- to measure a proper time, you should measure it at the same place, in your reference frame.
- to measure a proper distance, both ends of the object should be measured at the same time.

All observers can agree, for example, on the proper time it takes an atomic clock to go through a certain number of cycles, or the size of an atom, as long as it is measured according to these rules.

Time and Distance

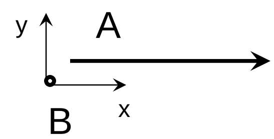
It is actually not necessary to specify how to measure times and distance independently: since light moves the same rate for everyone, we can measure distances in units of how far light goes in a second on our clock. In fact, this is the current standard:

The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

Comparing Times

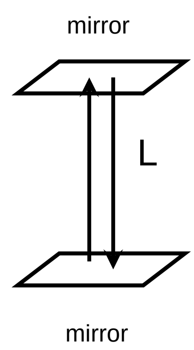
The fun starts when different observers start comparing each other's watches, or measuring times for events that occur away from themselves, in a different place.

- To make such comparisons, we need a process everyone can agree on. Fortunately, there is such a process.
- Assume observer A is moving at velocity v_x in a frame where B is fixed at $x = y = 0$.
- A and B are at rest relative to each other in the y direction, and can agree on distances along the y axis, but not necessarily along the x axis.



Light Clock

- To compare times, they constuct a **light clock** that uses a beam of light they can both see. The clock reflects the beam back and forth between two mirrors a proper distance L apart, taking a time $t_0 = 2L/c$ to go make one "tick". Both observers agree that this clock works as expected in their own reference frame, no matter how they move, and is perfectly synchronized with any other watches they may be carrying along.
- But then, they decide to compare each other's light clocks.



Light Clock

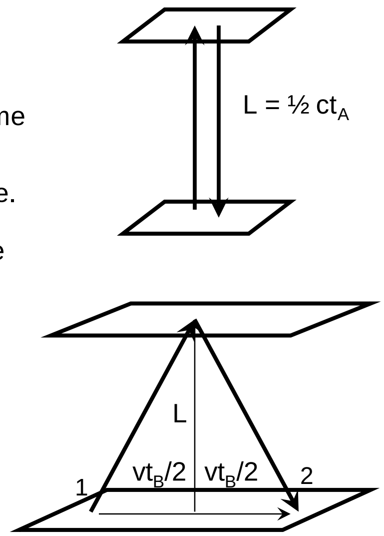
- A sees his clock measure proper time t_0= 2L/c in one tick.
- B sees A's clock measure a different time t_B, because the light has to go farther. This is the time between two events 1 and 2 at different points: not proper time.
- In time t_B, B sees A's clock move a distance $x = vt_B$ in the x direction, so the total path length is $2(L^2 + (vt_B/2)^2)^{1/2}$. Thus, he sees a tick of A's clock take time

$$t_B = 2[(ct_0/2)^2 + (vt_B/2)^2]^{1/2} / c$$

Solve for t_A, finding

$$t_0 = t_B[1 - (v/c)^2]^{1/2}. \quad \text{Or,}$$

$$t_B = \gamma t_0 \text{ with } \gamma = [1 - (v/c)^2]^{-1/2}.$$



Comparing Times

- The proper time t_0 in a frame moving at speed v is related to the time t_B measured by an observer at rest by $t_B = \gamma t_0$ with **time dilation** factor $\gamma = [1 - (v/c)^2]^{-1/2}$.

  γ is always greater than 1, and called the time dilation factor. γ becomes infinite when v = c.
- The **proper time** between two given events is the *shortest* time that can be observed between these events by any moving observer.

Time Dilation

Does this mean I will actually age at a different rate if I am traveling at a speed very close to c?

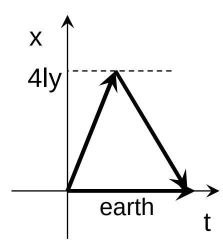
- No. I will see myself age at the same rate as when I am at rest. This is determined by physical processes, which we know must work the same in any inertial frame.

Can I use this as a time machine to visit the Earth in a distant future 1,000,000 years from now to see what people have evolved into?

- Yes, if you can figure out how to go fast enough. At speeds very close to the speed of light, people on earth would see your clocks and aging move very slowly. Generations would be born and die, people would evolve into something else, and you would continue aging slowly until you came back to earth. (But you could never go back.)

Example

The nearest star is about 4 light-years away. Suppose I build a very fast ship and get there in 10 years, according to my watch. Then I come home. How much time has passed on Earth? By my watch, a proper time t = 20 years pass between the events of my leaving and returning. In the rest frame of Earth, this corresponds to a time $t_E = \gamma t$, with $\gamma = (1 - (v/c)^2)^{-1/2}$ for v/c = 0.4.

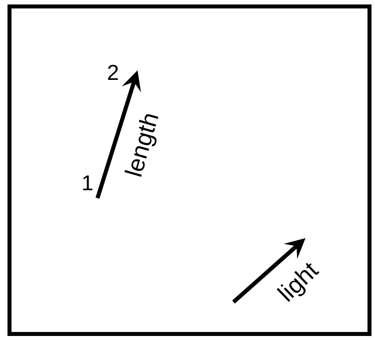


Then γ = 1.09 and t_E = 22 years.

Measuring Lengths

Anyone can find a good way to measure lengths in their own rest frame: measure the positions of the ends at the same time according to your own clock. That is the **proper length** or **rest length**.

It is possible to define a distance between any two events in space-time if the slope of a line between them is greater than c. In that case, they will be simultaneous for some observer moving at an appropriate speed.



Length of Rocket

Suppose a rocket zooms past the earthat speed v, and its occupants would measure its length to be L_0 from their own point of view. This is its proper length. What is its length from the point of view of an earth-bound measurer?

Keep track of events! The length must be measured between simultaneous positions.

Length of Rocket

Stand in one place and observe the rocket.

The rocket will take a proper time t_0 to pass. (Proper because you haven't moved)

You can infer the length to be $L = vt_0$.

A person in the rocket would see you pass the rocket in time γt_0. You would travel a distance L_0 in that time at speed v. Thus, $L_0 = \gamma v t_0$. Comparing lengths implies that $L = L_0/\gamma$.

Therefore, you measure the rocket to be shorter than its proper length by a factor of $1/\gamma$. For v = c, the observed length goes to zero.

This shrinking of the lengths of a moving object is called a Lorentz contraction.

Lorentz Contraction

Can I really change my shape by traveling in a fast rocket?

- Not from my point of view. My size is determined by the parts that make me up, and these don't change when I go fast.

What is the Earth's radius, from the point of view of the rocket, in terms of the radius r_0 we measure on Earth?

- The rocket would also see the Earth as Lorentz contracted: $r = r_0/\gamma$.

The rest length of an object is always its longest length. A moving observer will measure a shorter length.