Scott Yost – The Citadel

NEW RESULTS FROM THE HERWIRI PROJECT: VERSIONS 1 AND 2

with V. Halyo, M. Hejna, S. Joseph, S. Majhi, and B.F.L. Ward

INTRODUCTION

- The HERWIRI project is a multi-faceted collection of MC programs for Z and W production based on simultaneous YFS-inspired exponentiation of QCD and QED.
- The first phase, HERWIRI1.x, has been released, and is publicly available. I will concentrate on this version. It applies YFS-like exponentiation to the kernels for the QCD structure function evolution.
- The second phase, HERWIRI2.x, is still unreleased, but anticipated this summer. It applies exponentiated EWK corrections to Z/g production.
- This talk concentrates on version 1, but with some comments on version 2 at the end.

QCD⊗QED EXPONENTIATION

- The paradigm for the HERWIRI project is a simultaneous exponentiation of QCD and QED inspired by the YFS-exponentiation used in the LEP programs BHLUMI (Bhabha scattering) and KKMC (fermion pair production).
- Some modifications are required because of the gluon's self-couplings, but otherwise the general structure can be preserved.

QCD⊗QED EXPONENTIATION

- The general expression for the exponentiated hard process cross section can be expressed the form

$$d\hat{\sigma}_{\exp} = e^{Y_{\rm QCED}} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \frac{d^3 p_2}{p_2^0} \int \frac{d^3 q_2}{q_2^0} \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}^0} \prod_{j_1=1}^{m} \frac{d^3 k'_{j_2}}{k'^0_{j_2}}$$

$$\times \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot\left(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2}\right)+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}$$

- Incoming partons: p_1, q_1.
- Outgoing partons: p_2, q_2.
- n hard gluons k_j, m hard photons k_j'

QCD⊗QED EXPONENTIATION

- Restricted to QED, the previous expression is the standard exact YFS expansion which was the basis for the MC's BHLUMI and KKMC which were well tested in LEP1 and LEP2 physics applications.
- In QED, removal of the soft photon factors S is enough to render the beta functions IR-finite.
- This won't be true in QCD, but it is still possible to cancel IR divergences between the residuals, defining modified residuals to any finite order.

QCD EXPONENTIATON

- Schematically, in the QCD case, the elements are defined as

$$Y_{QCD} = 2\alpha_s (\mathrm{Re}\, B_{QCD} + \tilde{B}_{QCD})$$

$$2\alpha_s \tilde{B}_{QCD} = \int_{k>\varepsilon} \frac{d^3k}{k^0} \tilde{S}_{QCD}(k)$$

$$D_{QCD} = \int \frac{d^3k}{k^0} \tilde{S}_{QCD}(k) e^{ik\cdot y} - 2\alpha_s \tilde{B}_{QCD}$$

where YQCD is an IR-finite combination of virtual and soft real emission terms defined for an appropriate soft gluon factor $\tilde{S}_{QCD}$.

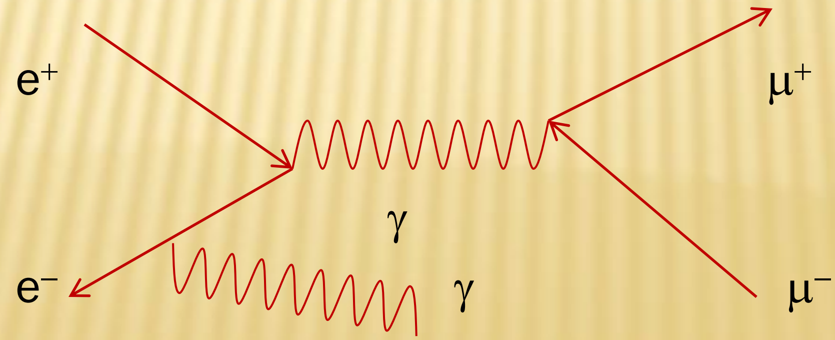
HERWIRI 1.X

- S. Joseph, S. Majhi, B.F.L. Ward, & S.A. Yost, Phys. Rev. D81, 076008 (arXiv:1001.1434)
- Current version 1.031 available at http://thep03.baylor.edu .
- Applies QCD exponentiation to the DGLAP kernels in the shower.
- HERWIG is the basis for the public version.
- An MC@NLO interface is also being tested.

HERWIRI1.X: BACKGROUND

- The motivation for HERWIRI1.x can be traced to LEP physics, where it was found that artifacts introduced at finite order by the soft photon cutoff ε were a significant limitation on the MC precision.
- For illustration, consider the QED cross section for and e^+e^- pair to produce a $\mu^+\mu^-$ pair with single-photon ISR:

$$e^+(p_1) + e^-(q_1) \rightarrow \mu^+(p_2) + \mu^-(q_2) + \gamma(k).$$



$$s = (p_1+q_1)^2$$
$$k^0 = (1-z)\sqrt{s}/2$$
$$zs = (p_2+q_2)^2$$

QED EXAMPLE

- The cross section can be separated into a hard real emission term and a soft + virtual term. At order α,

$$\sigma = \int_{z_{\min}}^{1} \rho(z)\sigma_{\text{Born}}(zs)$$

$$\rho(z) = \frac{\gamma(1+z^2)}{2(1-z)_+} + (1+\Delta_{S,V}^{(1)})\delta(1-z)$$

where the singular behavior of real emission in the soft limit at $\mathcal{O}(\alpha)$ is regulated by a "plus distribution" and

$$\gamma = \frac{2\alpha}{\pi}(L-1), \qquad \Delta_{S,V}^{(1)} = \frac{2\alpha}{\pi}\left(\frac{3}{2}L - 2 + \frac{\pi^2}{3}\right), \qquad L = \ln\left(\frac{s}{m_e^2}\right)$$

QED EXAMPLE

- The plus distribution may be represented using a soft-photon cut-off ε as

$$\frac{1}{(1-z)_+} = \frac{1}{1-z}\theta(1-z-\varepsilon) + \ln\varepsilon\,\delta(1-z), \quad \varepsilon \to 0$$

However, the limit is not taken in practice: ε will be a small parameter in the MC.

Experimentally, the cutoff depends on the detector threshold. At LEP, $z \approx .001$ corresponds to a threshold of about 100 MeV.

QED EXAMPLE

- While the plus-function correctly represents the $\mathcal{O}(\alpha)$ bremstrahlung contribution, the non-integrability may be thought of as a perturbative artifact, which can be removed by resummation.
- Resummation replaces the factor $(1-z)^{-1}$ by $(1-z)^{\gamma-1}$ for some $\gamma > 0$.
- This is a testable effect: measuring the cross section with the photon energy fraction cut off at ε should give a dependence of the form ε^{γ} if the exponentiated form is correct, rather than the $-\ln\varepsilon$ dependence of the $\mathcal{O}(\alpha)$ expression.

QED EXAMPLE

- YFS exponentiation replaces the distribution $\rho(z)$ containing the plus distribution by

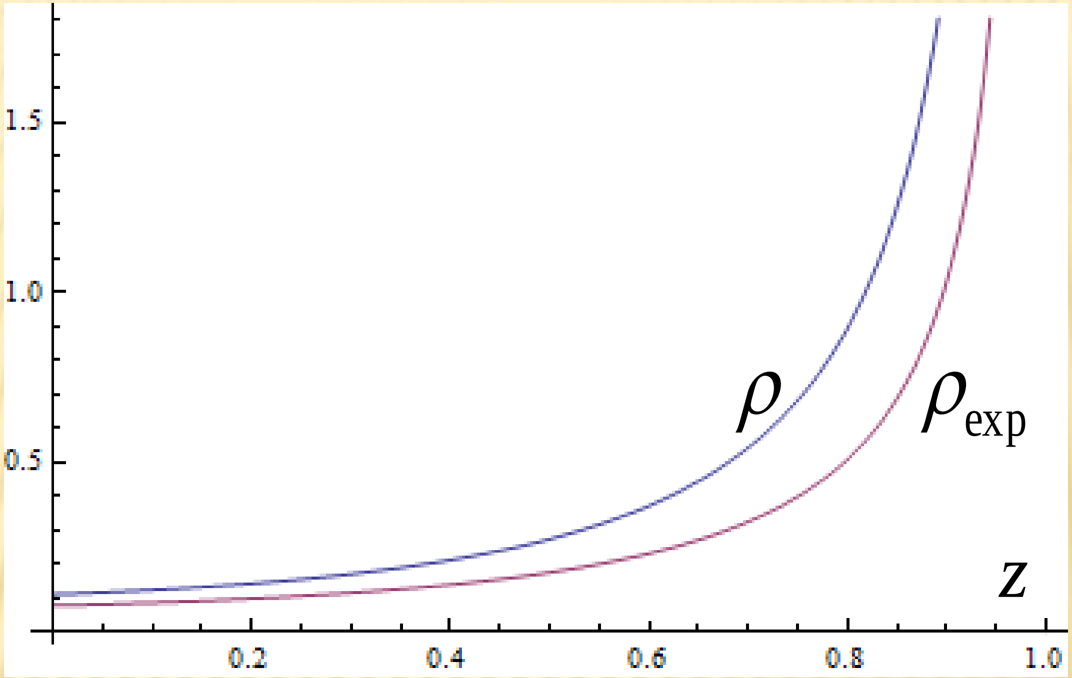
$$\rho_{\exp}(z) = \frac{1}{2} F_{YFS}(\gamma) e^{\delta_{YFS}} \gamma (1-z)^{\gamma-1} \left(1 + z^2 + \gamma\right),$$

$$\delta_{YFS} = \Delta^{(1)}_{S,V} - \frac{\gamma}{2}, \quad F_{YFS}\left(\gamma\right) = \frac{e^{-0.5772..\gamma}}{\Gamma(1+\gamma)}$$

- This no longer requires a cutoff for its definition, and has smooth behavior for $z \to 1$.

QED EXAMPLE

Plot for
s = 100 GeV,
ε = 0.001



softer photon

The exponentiated distribution is less sharply peaked for soft photons, and is integrable to $z = 1$. No cutoff is required to evaluate the resummed cross-section.

QCD CASE

- In the QCD case, an analogous situation occurs for gluon emission. Soft and collinear emission is included in the parton shower.
- An appropriate regularization is needed to properly define the structure functions, due to the divergence in the soft gluon limit. This leads to a plus function analogous to that found in the QED case.

QCD CASE

- In the QCD case, an analogous situation occurs for gluon emission. Here, the evolution of the parton density functions is governed by the DGLAP equations, which express them in terms of splitting kernels.
- Consider a scale μ relative to a reference scale μ_0. In terms of the momentum fraction x, the evolution of the non-singlet density function is given by

$$\frac{dq^{\mathrm{NS}}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\, q^{\mathrm{NS}}(y,t)\, P_{qq}\left(\frac{x}{y}\right)$$

where $t = \log\left(\frac{\mu^2}{\mu_0^2}\right)$ and the LO kernel is $P_{qq}(z) = C_F \frac{1+z^2}{1-z}$.

$(C_F = 4/3)$

QCD CASE

- P_{qq} has a singularity at $z = 1$, which can be remedied by replacing $(1-z)^{-1}$ by $(1-z)_+^{-1}$.
- Adding LO virtual corrections at $z = 1$ enforces the requirement that

$$\int_0^1 P_{qq}(z)dz = 0$$

giving

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(z-1)\right]$$

- The effect of the plus function is similar to the QED case: P_{qq} is set to zero right where it should be maximal.
- As in the QED example, an integrable, perhaps more physically reasonable kernel can be obtained by resummation.

RESUMMED KERNEL

- The YFS-inspired resummation produces a close analog of result found in QED:

$$P_{qq}^{\exp}(z) = C_F F_{YFS}(\gamma_q) e^{\delta_q/2}\left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\,\delta(z-1)\right]$$

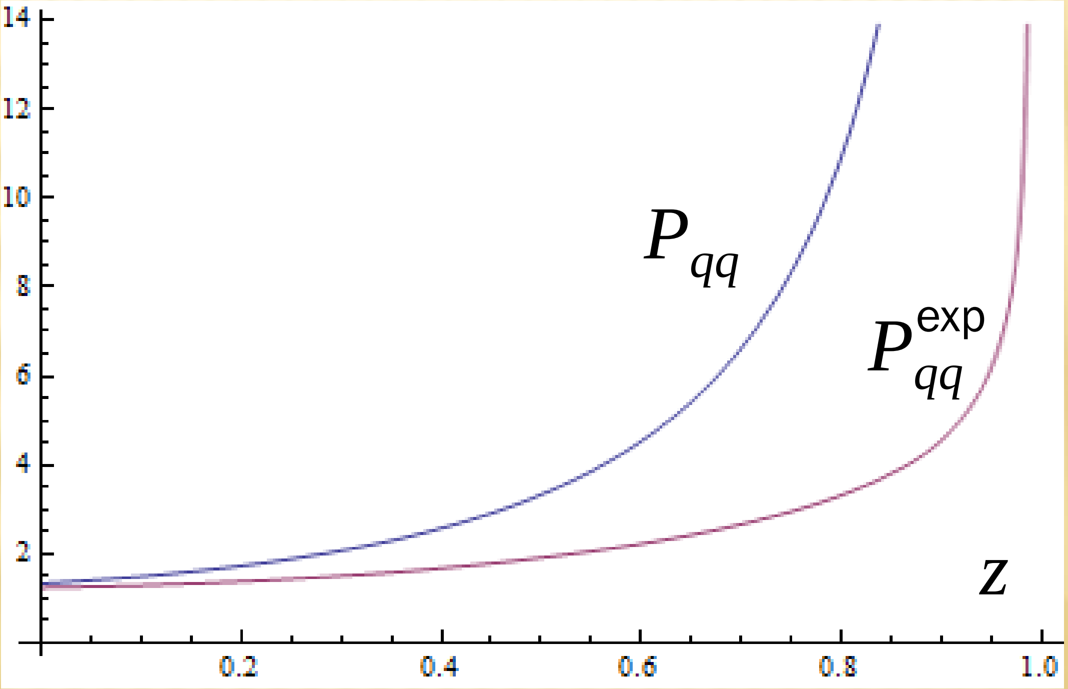
with

$$\gamma_q = \frac{C_F \alpha_s}{\pi} t = \frac{4C_F}{\beta_0}, \quad \beta_0 = 11 - \tfrac{2}{3} n_f , \quad \delta_q = \frac{\gamma_q}{2} + \frac{C_F \alpha_s}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$$

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q + 1} + \frac{1}{\gamma_q + 2}, \quad F_{YFS}(\gamma_q) = \frac{e^{-0.5772..\gamma_q}}{\Gamma(\gamma_q + 1)}$$

RESUMMED KERNEL

The exponentiated kernel has an integrable IR limit. This should give a more realistic behavior for $z \to 1$ than the plus distribution.



IR-IMPROVED DGLAP KERNELS

- Also, $P_{Gq}(z) = P_{qq}(1 - z)$ for $z < 1$:

$$P_{Gq}(z) = C_F \frac{1+(1-z)^2}{z}$$

which obey a quark momentum sum rule

$$\int_0^1 z dz \left(P_{Gq}(z) + P_{qq}(z)\right) = 0$$

- The remaining standard kernels at LO are

$$P_{GG}(z) = 2C_G\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

$$P_{qG}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right) \qquad C_G = N_c = 3$$

IR-IMPROVED DGLAP KERNELS

- The exponentiated forms of these kernels are

$$P_{Gq}^{\exp}(z) = C_F F_{YFS}(\gamma_q) e^{\delta_q/2} z^{\gamma_q - 1}\left\{1 + (1-z)^2\right\}$$

$$P_{GG}^{\exp}(z) = 2C_G F_{YFS}(\gamma_G)\, e^{\delta_G/2}\Big\{z^{\gamma_G - 1}(1-z) + z(1-z)^{\gamma_G - 1} + \tfrac{1}{2} z^{\gamma_G + 1}(1-z) + \tfrac{1}{2} z(1-z)^{\gamma_G + 1} + f_G(\gamma_G)\,\delta(1-z)\Big\}$$

$$P_{qG}^{\exp}(z) = 2C_G F_{YFS}(\gamma_G)\, e^{\delta_G/2}\left\{z^{\gamma_G}(1-z)^2 + z^2(1-z)^{\gamma_G}\right\}$$

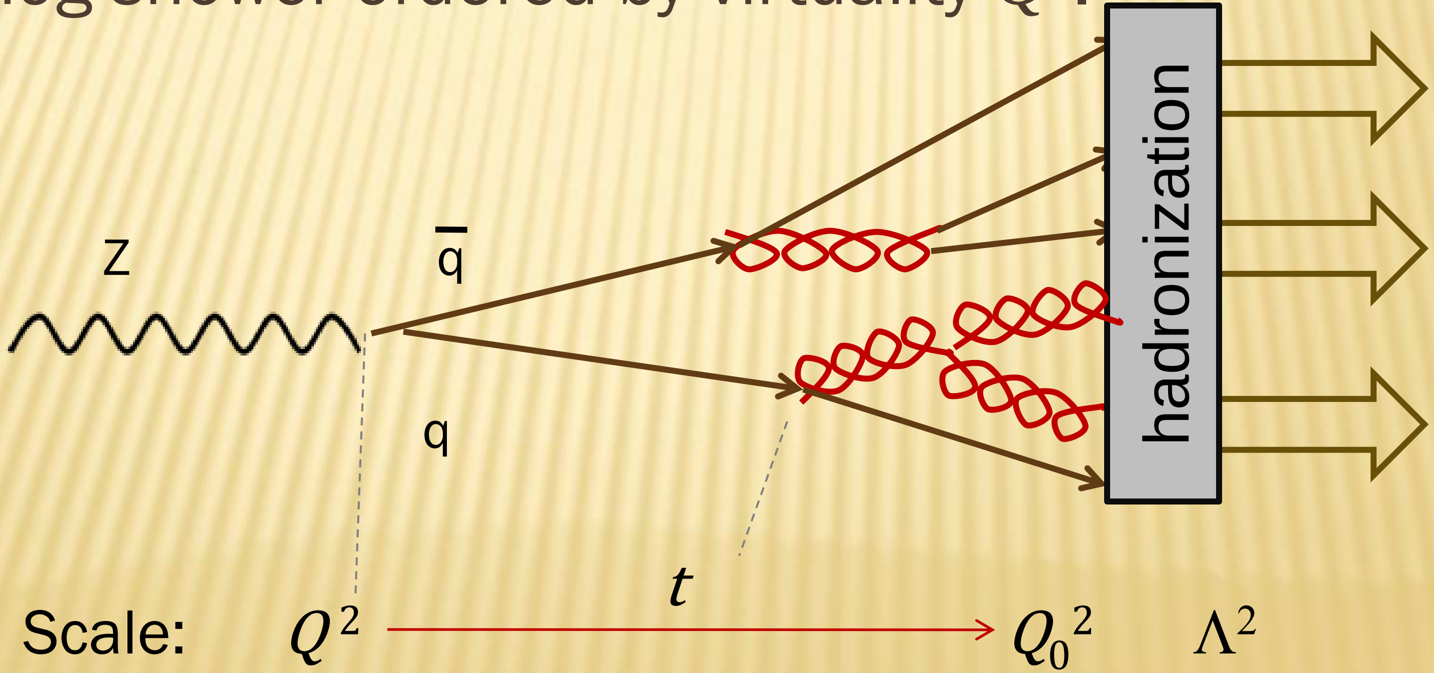
with

$$\gamma_G = \frac{C_G \alpha_s}{\pi} t = \frac{4C_G}{\beta_0}, \qquad \delta_G = \frac{\gamma_G}{2} + \frac{C_G \alpha_s}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$$

$$f_G(\gamma_G) = \frac{1}{(\gamma_G + 1)(\gamma_G + 2)}\left(\frac{n_f}{C_G}\frac{1}{(\gamma_G + 3)} + \frac{2}{\gamma_q} + 1\right) + \frac{1}{(\gamma_G + 3)(\gamma_G + 4)}\left(\frac{1}{(\gamma_G + 2)} + \frac{1}{2}\right)$$

EFFECT ON THE SHOWER

- The DGLAP evolution kernels determine the splitting probabilities in the shower. To see the effect of the new kernels, we consider a leading log shower ordered by virtuality Q^2.



EFFECT ON THE SHOWER

- Consider a parton splitting a → b + c.

The probability $\Delta_a(t, Q_0^{\,2})$ that the probability that parton a will not branch *below* virtuality t evolves according to the splitting function:

$$\frac{d\Delta_a(t,Q_0^2)}{dt} = -\frac{\Delta_a(t,Q_0^2)}{t}\sum_b \int_0^1 dz\,\frac{\alpha_s(t)}{2\pi}P_{ba}(z)$$

Which implies that

$$\log \Delta_a\left(Q^2, Q_0^2\right) = -\int_{Q_0^2}^{Q^2}\frac{dt}{t}\sum_b \int_0^1 dz\,\frac{\alpha_s(t)}{2\pi}P_{ba}(z)$$

EFFECT ON THE SHOWER

- The probability of not branching at virtuality $k_a{}^2$ greater than t is given by

$$\Delta_a\left(Q^2,t\right)=\frac{\Delta_a\left(Q^2,Q_0^2\right)}{\Delta_a\left(t,Q_0^2\right)}$$

- The probability $\Delta_a(Q^2,t)$ takes values in $[0,1]$, so the virtuality t of parton a can be generated with the correct distribution by generating a uniform random variable $R\in[0,1]$ and solving the equation $R=\Delta_a(Q^2,t)$ for t …

EFFECT ON THE SHOWER

- The momentum available after a $q\bar{q}$ split can be found using $\Delta_a\left(Q^2, Q_0^2\right) = e^{-I(Q_0^2 . Q^2)}$ with integral

$$I(Q_0^2 . Q^2) = \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int_0^1 dz \, \frac{\alpha_s(t)}{2\pi} P_{qG}(z)$$

Which can be evaluated using $\alpha_s(t) = \dfrac{2\pi}{\beta_0 \log(t/\Lambda)}$.

- The z integral gives

$$\int_0^1 dz \, \frac{\alpha_s(t)}{2\pi} P_{qG}(z) = \frac{\int_0^1 dz\left[z^2 + (1-z)^2\right]}{\beta_0 \log(t/\Lambda^2)} = \frac{2}{3\beta_0 \log(t/\Lambda^2)}$$

so

$$I = \frac{2}{3\beta_0} \log\left(\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right)$$

EFFECT ON THE SHOWER

Finally, this gives $\Delta_a\left(Q^2,t\right)=\left(\frac{\log(t/\Lambda^2)}{\log(Q^2/\Lambda^2)}\right)^{\frac{2}{3\beta_0}}$,

So solving for the virtuality using the generated value of $R\in[0,1]$ gives

$$t=\Lambda^2\left(\frac{Q^2}{\Lambda^2}\right)^{R^{3\beta_0/2}}$$

Here, $\beta_0=\frac{11}{3}n_c-\frac{2}{3}n_f$.

This is essentially the method used to generate branching in HERWIG, except that its shower is ordered by angle rather than virtuality.

SHOWER WITH EXPONENTIATED KERNELS

- The shower with exponentiated kernels can be generated by following the same steps using

$$P_{qG}^{\exp}(z) = 2C_G F_{YFS}(\gamma_G)\, e^{\delta_G/2} \left\{ z^{\gamma_G}(1-z)^2 + z^2(1-z)^{\gamma_G} \right\},$$

to obtain

$$\log \Delta_a(Q^2, t) = F(t) - F(Q^2)$$

$$\mathrm{Ei}(1,x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$$

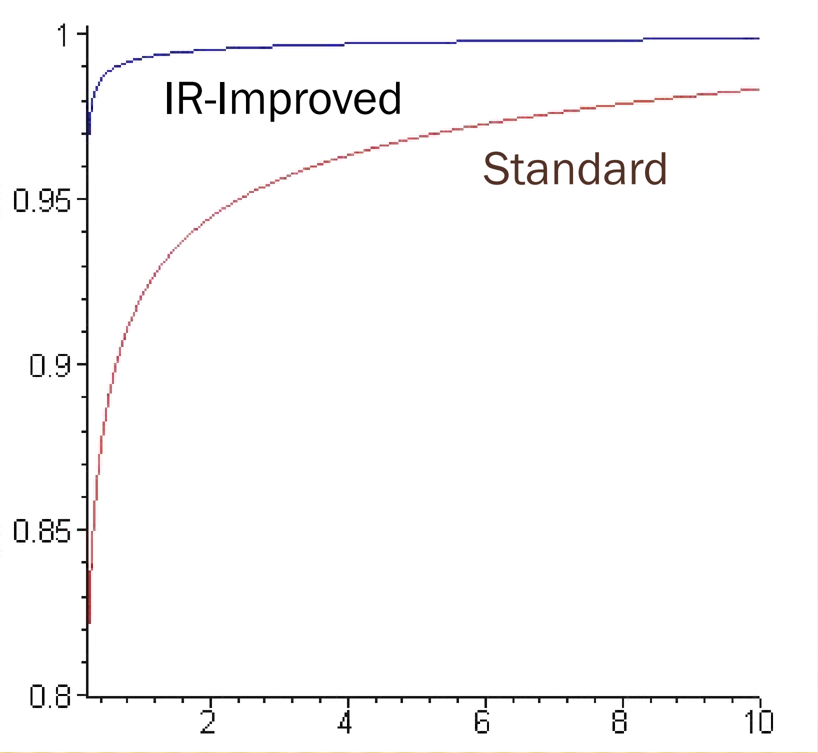
with

$$F(Q^2) = \frac{4F_{YFS}(\gamma_G)\, e^{\gamma_G/4}}{\beta_0(\gamma_G+1)(\gamma_G+2)(\gamma_G+3)} \; \mathrm{Ei}\left(1, \frac{8.369604402}{\beta_0 \log(Q^2/\Lambda^2)}\right)$$

Since this is a trancendental equation, Newton's method is used in HERWIRI to solve for *t*.

SHOWER WITH EXPONENTIATED KERNELS

- Comparison of $\Delta_a(Q^2, t)$ at $Q^2 = (25\ \mathrm{GeV})^2$:



t GeV^2

The two expressions agree to within a few percent over most of the range.

Experiment will decide which is better, when adequate precision is reached.

MC IMPLEMENTATION

- The improved Kernels were implemented in the HERWIG shower generator by modifying the routines where the kernels appear:

  HWBRAN, HWBSUD, HWBSU1 ,

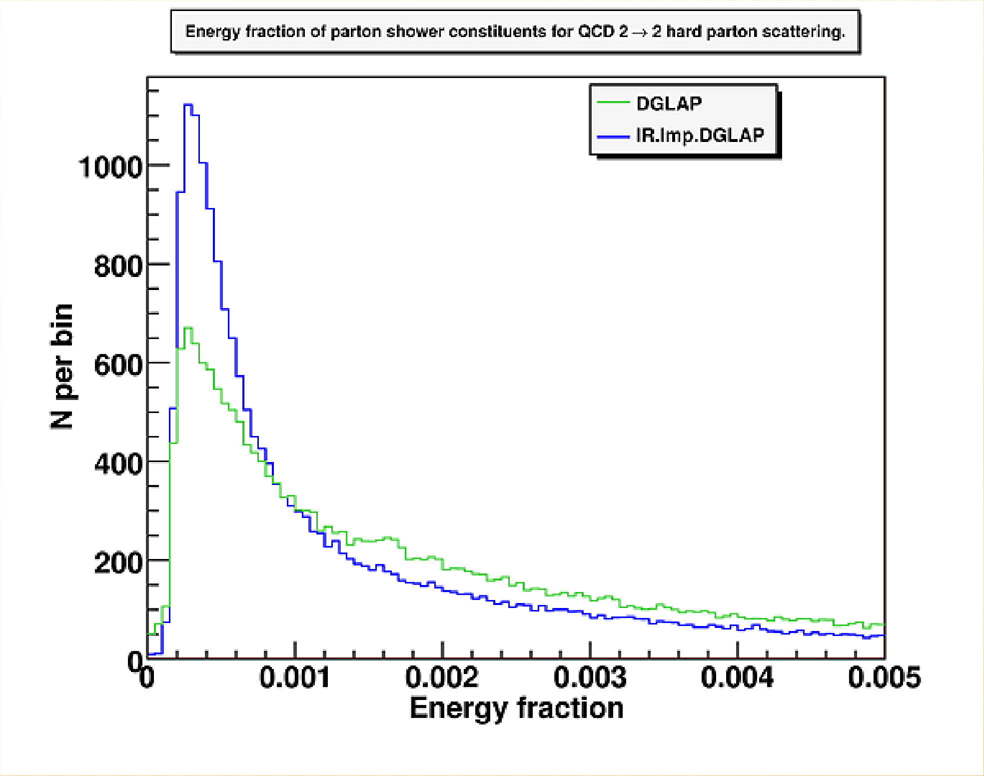
  HWBSU2, HWBSUG, HWIGIN,

  HWSGQQ, HWSFBR.

- The last two were not available in the initial release, but can be found in the current version.

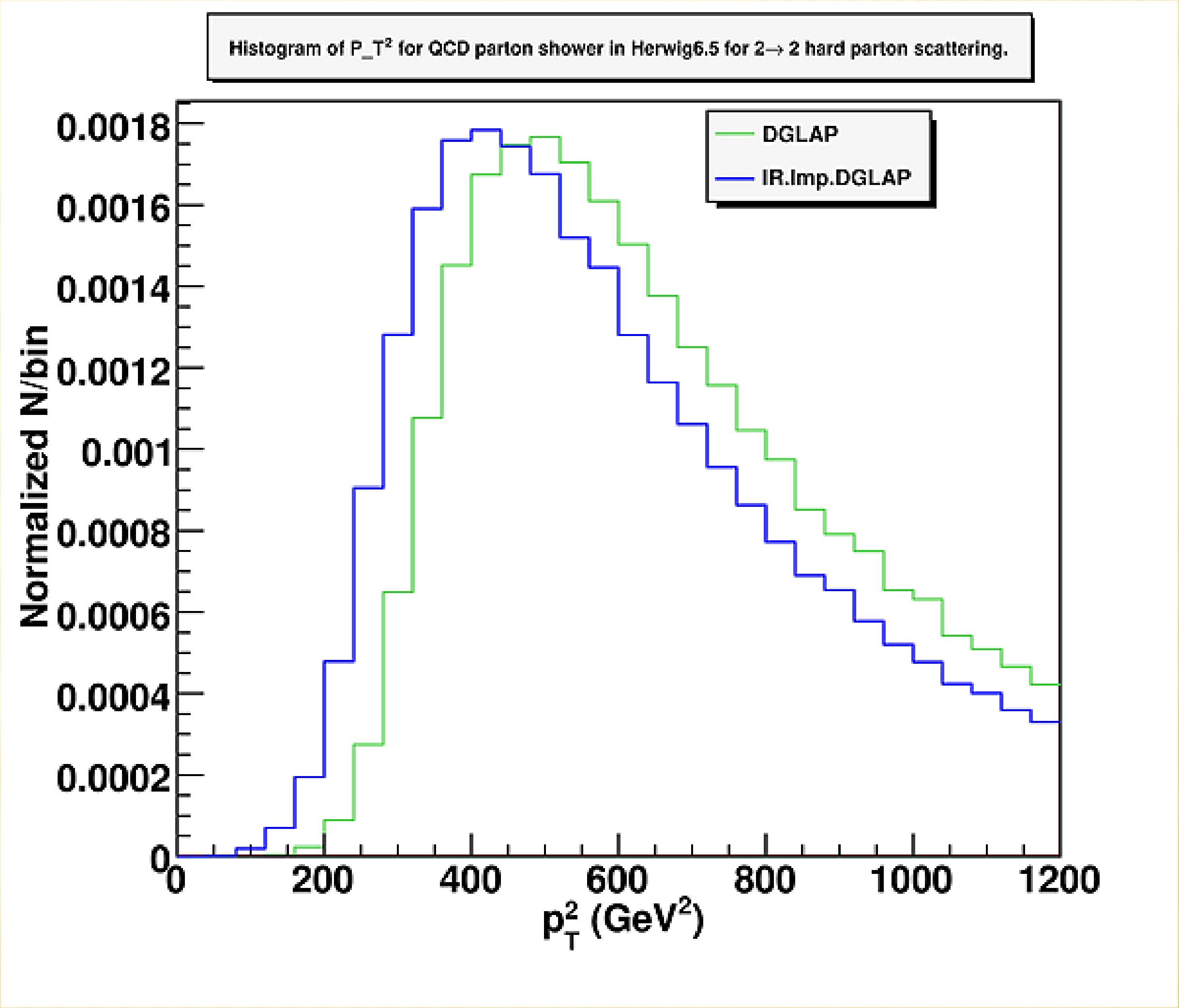
ENERGY FRACTION COMPARISON

- The distribution of $z = E_{\text{parton}}/E_{\text{beam}}$ for ISR is shown here for a 2⟶2 process at eventual LHC energies (14 TeV)
- This shows an increase in the generation of small energy fractions.



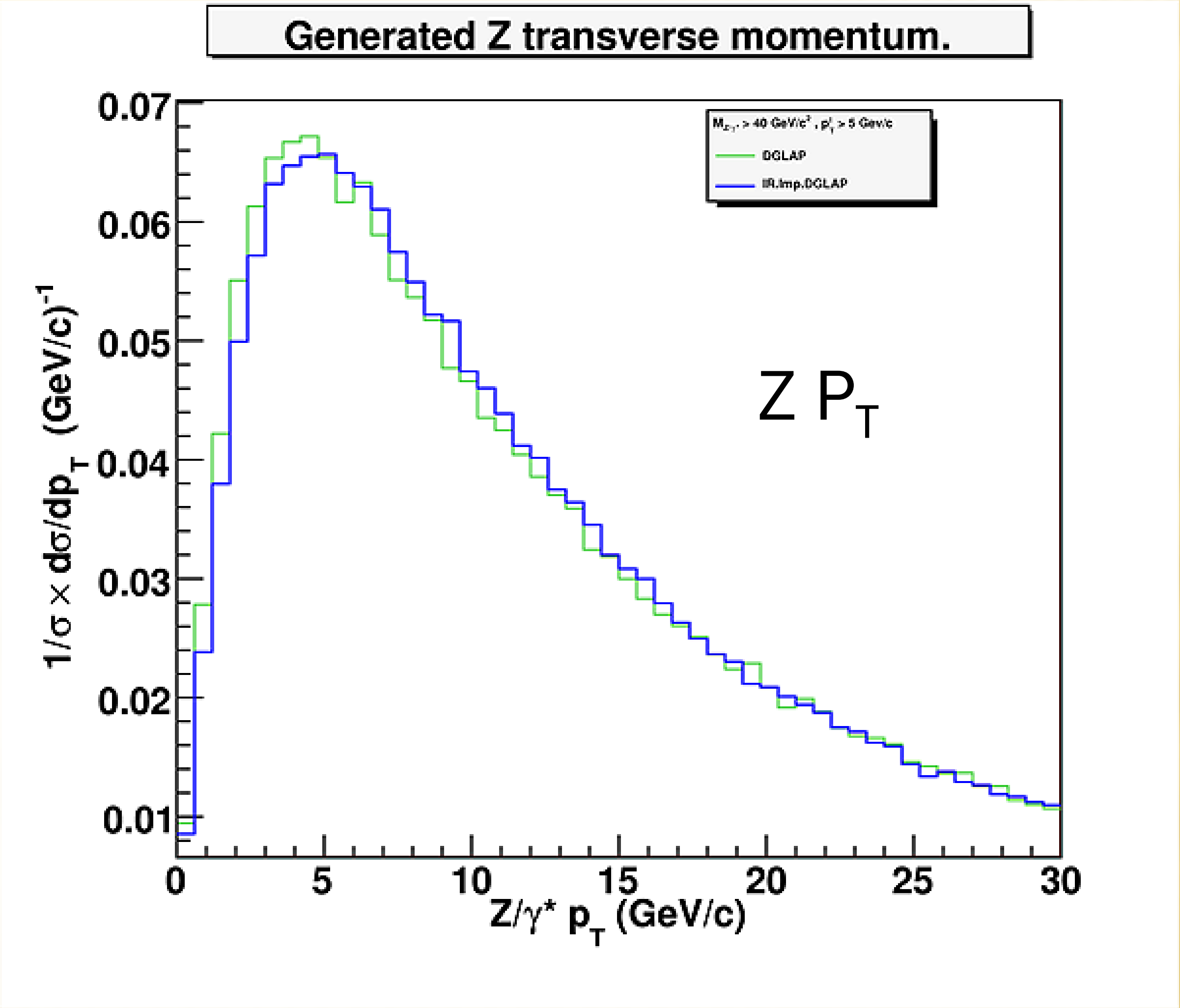
TRANSVERSE MOMENTUM COMPARISON

- This graph shows distribution of P_T^2 generated for the partons. There is a shift toward softer values.



Z PRODUCTION AT THE LHC

Here we consider $Z \to \mu^+\mu^-$ with an invariant mass cut $M_Z > 40$ GeV and muon rapidity cut $p_{T\mu} > 5$GeV.



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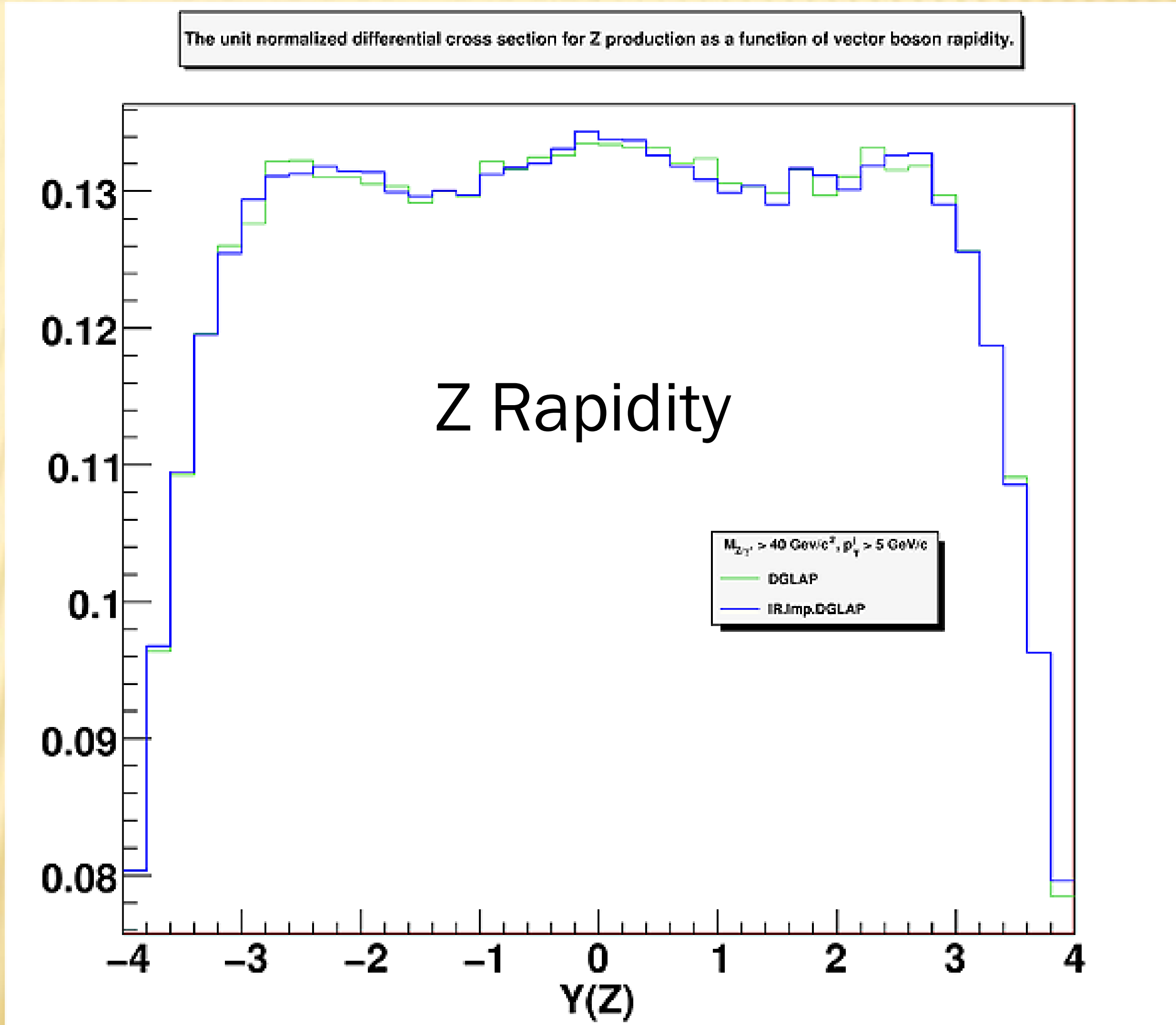
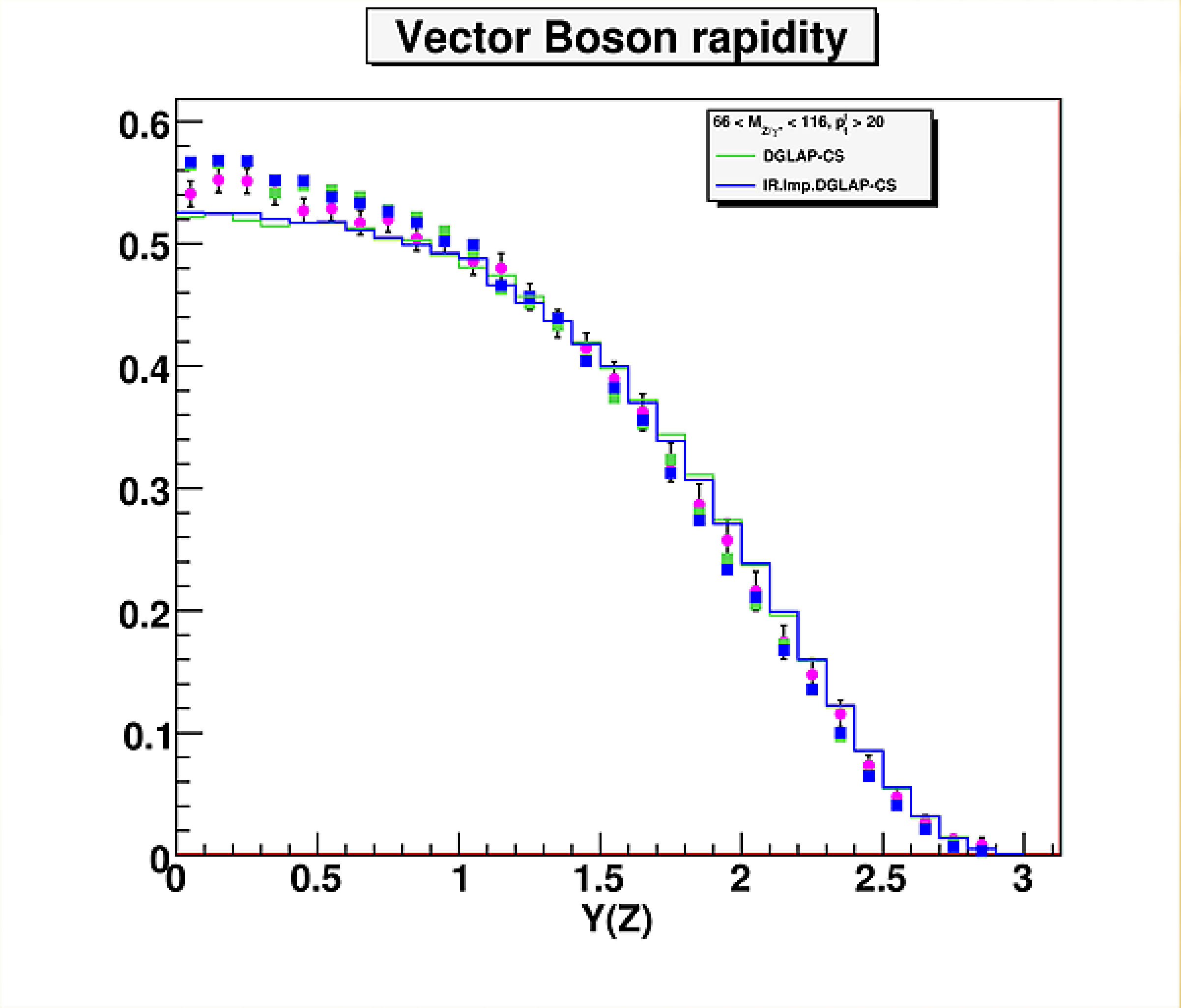


FNAL COMPARISONS

This plot compares Z boson rapidity data from CDF to HERWIG and HERWIRI1.031.

The squares show the effect of adding NLO via MC@NLO in each case.

Exponentiation has little effect. Adding NLO is significant.

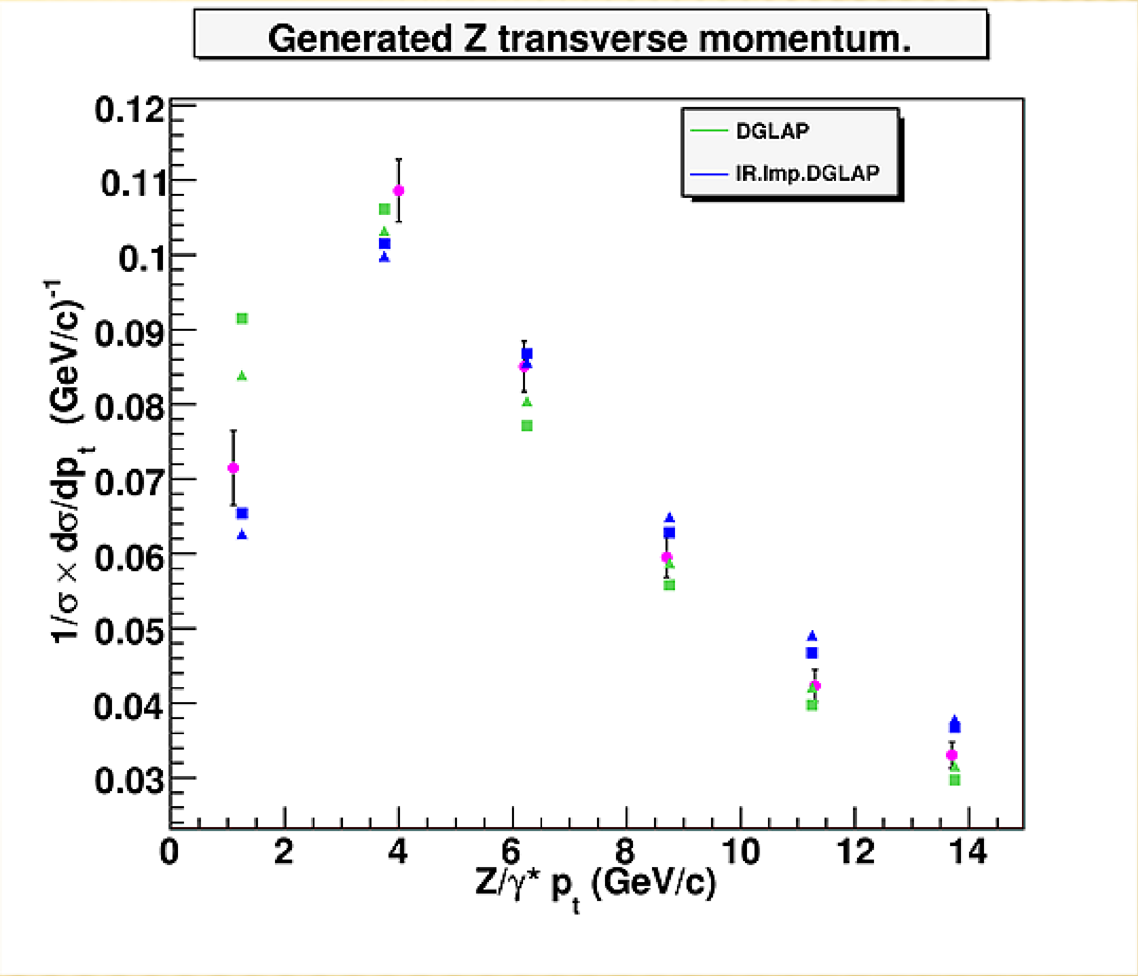


FNAL COMPARISONS

This graph compares D0 P_T data for Z production to HERWIG and HERWIRI.

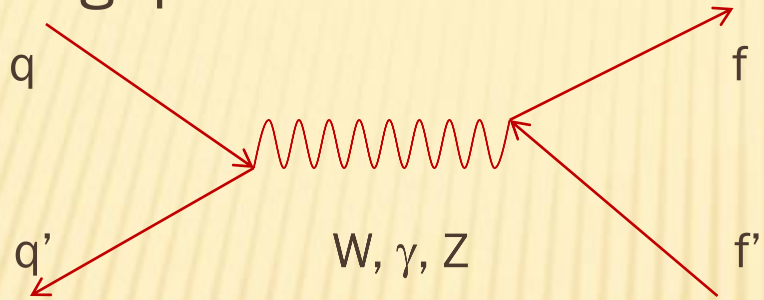
The effect of exponentiation is more notable in a P_T distribution, giving better agreement with data at low P_T as expected.

Here, triangles are LO, squares NLO.



HERWIRI2.X

- HERWIRI2.x is based on the LEP program KKMC, substituting quarks for the incoming e^+e^-.



- This gives the hard Drell-Yan process with radiative corrections from the charged lines.
- In the case of Z/γ production, the processes are essentially identical to LEP processes, with different masses and charges.
- This is incorporated into a shower generator – currently HERWIG.

KKMC

- The KK Monte Carlo is a program to calculate

$$e^+e^- \longrightarrow f\bar{f} + n\gamma$$

using YFS exponentiation to multi-photon radiative corrections with high precision.

The YFS residuals are calculated in a double expansion in powers of a and logarithms, as will be shown on the next slide, including 2 – 3 loops and up to 2 – 3 exact real photons (any number can be produced approximately).

Electroweak corrections are incorporated via the DIZET 6.21 library of D. Bardin, *et. al*. (also used in ZFITTER).

KKMC RADIATIVE CORRECTIONS

- KKMC uses a combined expansion in powers of α and big logarithms ($L = \ln s/m_f^2$):

	LL	NLL	NNLL	N^3LL
LO	1			
NLO	αL	α	← included	
NNLO	$\alpha^2 L^2$	$\alpha^2 L$	α^2	
N^3LO	$\alpha^3 L^3$	$\alpha^3 L^2$	$\alpha^3 L$	α^3

0.5 – 1 % | 0.1 – 0.5% | 0.01 – 0.05%

Errors shown are for LEP, with exponentiation. Without it, they are 2-5 times more.

PROGRAM STRUCTURE

As it exists now (as a work in progress), HERWIRI2.0 is implemented together with HERWIG.

- HERWIG generates the incoming hard parton pair, which are passed to KKMC, suitably adjusted to handle the charges and masses of the quarks.
- KKMC generates some number of ISR and FSR photons, which are merged into the event record.
- The weight generated by KKMC is used to reweight the HERWIG events appropriately.

SUMMARY

- HERWIRI1.x is the first step in a series of programs incorporating YFS-like nonabelian exponentiation into a hadronic shower for LHC physics.
- Effects are most notable in soft variables, such as the low P_{T} distributions. Experiments will show if the data favors exponentiation.
- HERWIRI2.x will be a full marriage of KKMC to a hadronic event generator. It is not quite ready for release, but should be producing results this summer. It will include both ISR and FSR multi-photon effects and the electroweak corrections of the DIZET library.
- Results from HERWIRI2.x are anticipated this summer.