Superstring Field Theory*

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Abstract

We describe the construction of a class of cubic gauge-invariant actions for superstring field theory, and the gauge-fixing of one representative. Fermion string fields are taken in the $-1/2$-picture and boson string fields in the 0-picture, which makes a picture-changing insertion carrying picture number -2 necessary. The construction of all such operators is outlined. We discuss the gauge $b_1 + b_{-1} = 0$, in which the action formally linearizes. Non-trivial scattering amplitudes are obtained by approaching this gauge as a limit.

1. INTRODUCTION

In this talk we wish to describe recent progress we have made in developing open superstring field theory.[1] To start out, it is perhaps worthwhile to make some general remarks on the motivation behind string field theory as well as alternatives to the string field approach.

As we all know, string theory was discovered and is still best understood as some kind of loop expansion of an as yet poorly understood fundamental theory. The formulation of this fundamental theory is the prime goal of most workers in this field. We think it is reasonable to search for this formulation in the framework of an action principle, but it is not clear from the beginning even what the fundamental dynamical variables should be. String field theory is the most straightforward way to discover this action principle, because it takes as dynamical variables fields associated with the particle states present in the theory at zero loops.

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But it is by no means necessary that the zero loop particle states be associated in such a direct way with the fundamental dynamical variables. A popular analogy to contemplate is the relation between the fundamental variables of QCD and the hadrons. This analogy is most persuasive in the context of 't Hooft's $1/N_{colors}$ expansion of QCD which resembles very closely the dual loop expansion.[2] However, if something like this is at work, a simple flat space quantum field theory like Yang-Mills theory can't possibly be the answer. For one thing, string theory contains gravity, so the putative fundamental theory must be generally covariant. For another thing, the short distance structure of string theory is ultrasoft, unlike the hard parton structure associated with the asymptotic freedom of nonabelian gauge theory. One might speculate that the answer is some generally covariant quantum field theory, but in how many dimensions? In fact, this kind of picture of string theory was tried over a decade ago in the context of a "wee parton" approximation to QCD.[3] Perhaps the "wee parton" assumption is linked to the requirement of general covariance; it certainly accounts for the softness of string theory.

Yet another possibility is a fundamental formulation in terms of the two dimensional world sheet in which topology change is taken as a dynamical variable. A version of this idea was proposed over a decade ago in the context of the light cone gauge.[4] Or maybe the framework for this idea should be universal moduli space.[5,6]

In any case, even if string field theory is not the ultimate formulation of string theory, it should be possible to develop an effective action principle which would at least be valid at the classical level. Wendt[7] discovered a flaw in Witten's initial extension of his action principle for the interacting open bosonic string to the superstring. It would be unsettling if this flaw were fatal. Our work, we believe, provides a new formulation of open superstring field theory which surmounts this difficulty. It does not touch on efforts to develop a superstring field theory based on the manifestly supersymmetric formalism,[8] nor does it deal with the problem of developing a string field theory which does not contain open strings.

2. String and Superstring Field Theory

We turn now to an introductory description of what string field theory is.[9] Ordinary field theory assigns a number (or finite collection of numbers) to each point in space-time. String field theory generalizes this concept by replacing the space of points in space-time by the space of paths in space-time. Thus the string field is a functional of curves: $A[x^\mu(\sigma), c(\sigma)]$. Because paths in space-time are not restricted to lie in equal-time hyperplanes, the theory must be a very special field theory for which it is possible to recover our customary notion of a unitary time evolution. We can implement this special feature by requiring the dynamics to be such that the domain of the string field can be consistently restricted to those paths which do lie in equal time hyperplanes. That is, it should be possible to formulate the theory on a submanifold of the full space of paths. In this sense we might describe such a theory as "topological" on path space. From this point of view, Witten's proposal that the string field action be a Chern-Simons three form on path space is very natural.

The basic ingredients of Witten's version of string field theory[10] are: string fields A; a derivation acting on string fields, Q, which is taken to be the first-quantized $BRST$ operator; an associative exterior product, $*$, for which Q is a derivation:

$$Q(A*B) = QA*B + (-)^A A*QB;$$

and a volume form on path space, $\int$, which assigns a number, $\int A$, to each string field. The ghost numbers assigned to these objects are $-1/2$ for A, $+1$ for Q, $+3/2$ for $*$, and $-3/2$ for $\int$. In terms of these quantities Witten's action for open bosonic string field theory takes the form

$$S = \frac{1}{2}\int A*QA + \frac{1}{3}\int A*A*A,$$

and its gauge invariance is just

$$\Delta A = Q\Lambda + A*\Lambda - \Lambda*A.$$

In order to extend these ideas to the spinning string, Witten introduced a fermionic partner Ψ for the bosonic string field A and took the natural generalizations to the spinning string of Q, $*$, and $\int$. These objects have total ghost number 1, 1/2, and $-1/2$, respectively. Unfortunately, there is no assignment of ghost number to A and Ψ which allows the construction of a Chern-Simons three form action without the use of picture-changing operators. In his initial formulation Witten assigned A ghost and picture number $-1/2$ and -1, respectively.

These assignments for Ψ were 0 and $-1/2$ respectively. Then his proposed action takes the form

$$S = \frac{1}{2}\int A*QA + \frac{1}{2}\int Y(\sigma=\frac{\pi}{2})\Psi*Q\Psi + \frac{1}{3}\int X(\sigma=\frac{\pi}{2})A*A*A + \int A*\Psi*\Psi.$$

In this expression X is the local operator introduced by Friedan, Martinec, and Shenker[11] which changes picture number by $+1$ unit. Y is the inverse picture changing operator which changes picture number by -1. Both are inserted into the action at the midpoint of the string singled out by the definition of $*$ and $\int$ and denoted by $\sigma = \frac{\pi}{2}$. These operators have the explicit representations[12,13]

$$Y(z) = c(z)\delta'(\gamma(z))$$
$$X(z) = \{Q, \Theta(\beta(z))\}$$

where δ is the Dirac delta function and Θ is the Heaviside step function satisfying $\Theta' = \delta$. Here c is the reparametrization ghost, γ the superghost, and β the superantighost. The argument of the fields is just $z = e^{i\sigma+\tau}$. X and Y satisfy the short distance product relation

$$X(z)Y(z') \underset{z'\to z}{\sim} I.$$

Now we can describe the flaw that Wendt[7] discovered in this version of superstring field theory. The source of the difficulty is that the picture changing operator $X(z)$ has a singular operator product with itself:

$$X(z)X(z') \underset{z'\to z}{\sim} \frac{\Omega}{(z-z')^2}.$$

This causes a difficulty with the proof of gauge invariance because the bosonic gauge transformation of A is

$$\Delta A = Q\Lambda + X(\sigma=\frac{\pi}{2})(A*\Lambda - \Lambda*A).$$

When one tries to check the nonlinear gauge invariance of the action, one finds two X's colliding at the same point. The hope that somehow this singularity is cancelled is dashed by Wendt's explicit calculation of the four string function, which gives the wrong result. One can try to fix the problem by adding terms to the action with higher powers of A with coefficients designed to cancel the discrepancy. This is less than satisfactory because (1) the coefficient of the A^4 term is infinite and this presumably is true also of higher terms and (2) it would destroy the very attractive Chern-Simons form of the action.

Our solution to the difficulty is based on the idea that the classical string field should be described in the 0-picture.[14] There are several motivations for this idea:

(1) The $SL(2)$ invariant string state is in the 0-picture sector.

(2) Working in this picture corresponds to the manifestly dual form of the Neveu-Schwarz dual resonance amplitudes for spinning strings.

(3) Most importantly, picture changing operators are in less dangerous configurations.

This last point is easily understood. In the 0-picture, A carries total ghost number $+1/2$. Thus QA and $A*A$ both carry ghost number $3/2$, so there is no need for a picture changing operator to balance ghost number. For example the field equations in this picture read:

$$\begin{aligned} QA + A*A + X[\Psi*\Psi] &= 0 \\ Q\Psi + A*\Psi + \Psi*A &= 0 \end{aligned}$$

and X appears only in the coupling of Ψ to A. Moreover the bosonic gauge transformation of A is now simply

$$\Delta A = Q\Lambda + A*\Lambda - \Lambda*A$$

so there is no collision of insertions in proving the gauge covariance of the field equations. One does need a new inverse picture changing operator Y_{-2} in order to balance ghost number in the action, however:

$$\begin{aligned} S = &\int Y_{-2}(\sigma = \frac{\pi}{2})[\frac{1}{2}A*QA + \frac{1}{3}A*A*A] \\ &+ \int Y(\sigma = \frac{\pi}{2})[\Psi*Q\Psi + A*\Psi*\Psi]. \end{aligned}$$

The operator Y_{-2} had not previously appeared in the literature so our first task was to construct it. It must be a world sheet operator attached to the midpoint of the string and so must be local or bilocal in the complex plane: $z = \pm i$ are both associated with the midpoint of the string. It must also change picture number P and ghost number each by -2 units. We actually employ the bilocal choice

$$Y_{-2} = Y(i)Y(-i) \equiv Y\bar{Y}$$

for our construction because it leads to the simplest and most manageable formulation. In the -1-picture bilocal insertions had been previously proposed by

Lechtenfeld and Samuel.[15] Since $\partial_z Y$ is BRST trivial, one can say that such a choice is in some sense equivalent to a completely local choice. However, there is a short distance singularity in $Y(i)Y(z)$ as $z \to i$, so there is a divergence in the trivial part.

We systematically searched for all local candidates for Y_{-2}. Since there is a 1-1 correspondence between local operators and Fock states $|Y_{-2}\rangle = Y(0)|0\rangle$ with $|0\rangle$ the $SL(2)$ invariant Fock state, we searched for a Fock state with picture number -2 and ghost number $-5/2$ (since $|0\rangle$ has ghost number $-1/2$) satisfying

a) Lorentz and scale invariance (in particular $\alpha_0 = 0$ and $L_0 = 0$);

b) $BRST$ invariance (*i.e.* it is annihilated by Q);

c) It is not $BRST$ trivial;

d) It is normalized so that $X(z)Y_{-2}(z) = Y(z)$.

Condition a) plus ghost and picture number constraints limit the choice to 15 candidates. Of these 10 turn out to be primary and among the primary ones 7 are $BRST$ invariant. However, the space of trivial states satisfying these conditions is found to be 10 dimensional so the coset space of candidates is only 5 dimensional. Only a 1 dimensional subspace of the coset space of nontrivial states is $BRST$ invariant, so finally Y_{-2} is unique up to $BRST$ equivalence. Although different choices are equivalent on-shell, they do lead to different off-shell actions. We work with the simple bilocal choice $Y\bar{Y}$.

3. New Gauge Invariances

For the fermionic kinetic term of the string field action, it has been pointed out[16,17] that the necessary inverse picture changing insertion gives rise to new gauge invariances in addition to those associated with BRST invariance. A similar feature applies to our new bosonic kinetic term. To understand this, notice that $Y(z) = c(z)\delta'(\gamma(z))$ is annihilated by $c(z)$ and by $\gamma^2(z)$. In fact, these new gauge invariances are needed to choose the gauge $\beta_0\Psi = 0$. Kugo and Terao achieve this gauge by first constructing a BRST invariant projection operator that annihilates the redundancies due to the new gauge invariances. They start by introducing a nonlocal version of X,

$$X_0 \equiv \{Q, \Theta(\beta_0)\}.$$

Then one can easily show that $YX_0Y = Y$ and $X_0YX_0 = X_0$, so that $\mathcal{P} = X_0Y$ is a projection operator. By virtue of these properties one can first restrict Ψ to satisfy $\Psi = \mathcal{P}\Psi$ and then use the Q gauge invariance to set $\beta_0\Psi = 0$. The resulting propagator is then $b_0X_0/L_0 = b_0\delta(\beta_0)/F_0$.

It is clear that identical considerations apply to our form of the bosonic kinetic term, only now all of the four operators $c(i), c(-i), \gamma^2(i), \gamma^2(-i)$ annihilate Y_{-2}. Thus we introduce two nonlocal X's

$$X_\pm = \{Q, \Theta(\beta_\pm)\},$$

where

$$\beta_\pm = \frac{1}{2}(e^{\pm\pi i/4}\beta_{-1/2} + e^{\mp\pi i/4}\beta_{1/2}).$$

Then $\mathcal{P} \equiv X_+X_-Y(i)Y(-i)$ is a BRST invariant projection operator which kills the new redundancies. One can then first restrict A to satisfy $A = \mathcal{P}A$, and then fix the Q invariance in a convenient way.

Before turning to gauge-fixing, we note that the interaction terms are separately invariant under the new gauge invariances because each of the four operators $c(i), c(-i), \gamma^2(i), \gamma^2(-i)$ have negative conformal weight. Insertion of a negative weight local primary field at the midpoint of a Witten vertex always vanishes because of the curvature singularity there.

4. Gauge-Fixing

We next consider a general class of Siegel-like gauges for the fixing of the Q gauge invariances. For the bosonic string this class of gauges is simply $v \cdot b\, A = 0$ where $v \cdot b = \sum_n v_n b_n$, and the v_n are any set of numbers. For general v the propagator following from this condition is

$$\Pi_v = \frac{b \cdot v}{L \cdot v} Q \frac{b \cdot I(v)}{L \cdot I(v)}$$

where $I(v)_{-n} = (-)^{n+1} v_n$. If $I(v) \propto v$, the propagator simplifies to $\frac{b \cdot v}{L \cdot v}$. This is true of the standard Siegel gauge $v \cdot b = b_0$, for example.

For the spinning string, it is awkward to attempt general gauges of exactly this form. Instead, we use v and the projection operator $\mathcal{P}$ to construct a new projection operator

$$\mathcal{P}_v \equiv \mathcal{P} \frac{b \cdot v}{L \cdot v} Q \mathcal{P}.$$

This construction works equally well in the fermionic and bosonic sectors, with $\mathcal{P}$ being the appropriate projector. The general class of gauges we consider then

are specified by

$$A = \mathcal{P}A = \mathcal{P}_v A,$$

and the corresponding propagators are just

$$\Pi_v^B = \mathcal{P}_v \frac{b \cdot v}{L \cdot v} X_+ X_- \mathcal{P}_v^T$$
$$\Pi_v^F = \mathcal{P}_v \frac{b \cdot v}{L \cdot v} X_0 \mathcal{P}_v^T$$

for the bosonic and fermionic components respectively, where the transpose is taken with respect to the string inner product. In general these propagators are prohibitively complicated. However, for special choices of v they become manageable. The fermionic propagator simplifies for $v \cdot b = b_0$ or $b_+ = \frac{1}{2}(b_1 + b_{-1})$. But the bosonic propagator only simplifies significantly for the latter choice, which is the one we studied most intensively.

5. b_+ Gauge

The gauge $v \cdot b = b_+$ is particularly simple because b_+ commutes with local operators located at $z = \pm i$. In fact, the simplification is spectacular: the gauge choice formally causes the cubic term in the action to vanish! The reason is that b_+ is a derivation of the $*$ algebra. For example, applying b_+ to the l.h.s. of the field equation for fields satisfying $b_+A = 0$ gives

$$0 = L_+A + b_+(A * A) = L_+A + (b_+A) * A - A * b_+A = L_+A,$$

linearizing the field equation. Similarly the cubic term in the action can be shown to formally vanish. One uses the fact that $b_+A = 0$ implies $A = b_+c_+A$ to replace one of the A's by b_+c_+A. Then one integrates by parts to throw the b_+ onto the remaining two A's, which then vanish. This gauge would have the same dramatic consequences for the bosonic string so as preparation we studied this case very carefully.

Of course, we know the interactions can't really be a gauge artifact. The resolution presumably lies in the fact that one can not reach this gauge for all field configurations. In fact, at the linearized level one can prove that for field configurations in the nonvanishing eigenspace of L_+, such a gauge can be reached. But this proof breaks down for fields in the kernel of L_+. This same caveat applies also to Siegel gauge $v \cdot b = b_0$, but there it is less problematic because a nontrivial kernel of L_0 exists only for special on-shell values of the space-time momentum p. In contrast, the kernel of L_+ is nontrivial for all p. Thus one can't regulate the problem by taking p slightly off-shell. Instead, we regulate our calculations by employing a nearby gauge $v \cdot b = b_+ + i\epsilon b_0$, letting $\epsilon \to 0$ at the end of the calculation. In this way we explicitly confirm that general tree amplitudes for the bosonic string and selected ones for the superstring come out correctly. The results of these calculations are independent of ϵ, as they should be. Since the vertex function is proportional to ϵ, this implies that there are compensating singularities in that amplitude.

The manner in which moduli space is covered in the limit $\epsilon \to 0$ is interesting. In Siegel gauge, all the diagrams of a cubic field theory provide an essential contribution. In particular, the six string tree diagram with three internal lines meeting at a cubic vertex is nonzero. We find that in the limit $\epsilon \to 0$ this diagram vanishes! The multiperipheral diagrams in which every vertex has at least one external line suffice to cover moduli space.

6. Conclusions

Our work strongly indicates that the difficulty discovered by Wendt is absent in our formulation of superstring field theory. The only qualification is that we haven't done a careful study of loop diagrams. But since the problem was initially present at tree level, it is significant that we have removed the difficulty at that level. Since the product of two Y's is just as singular as the product of two X's, the potential for a problem is present. However, in our scheme such singularities are prevented because of the nonlocal $X_\pm$'s that each propagator inserts between each pair of potentially colliding Y's.

In our work, we were led to consider a fascinating gauge choice which formally linearizes the theory. This gauge is somewhat analogous to the temporal or axial gauge choice in Chern-Simons gauge theories in $2+1$ dimensions. In that case one also linearizes the field equations, but nonlinearities do remain in the constraints due to gauge invariance. One might expect similar features in the case of superstring field theory. Linearization of the action does not guarantee linearization of constraints. The problem with checking this idea is that the canonical phase space of string field theory is not well understood. If one considers the path integral approach to Chern-Simons theories, one can see the analogy with our situation more clearly. In that case it is not really allowed to set A_0 to zero for all field configurations: one can only set the nonzero frequency modes to zero. The zero mode of A_0 then participates in the interactions, and in fact integration over it precisely imposes the constraints of the phase space approach. In the case of string field theory, the analogue of the zero frequency mode of A_0 would be the kernel of L_+.

To conclude, let us review the status of string field theory. Our work has provided us with a satisfactory formulation of spinning string field theory in the NSR formalism for *open* strings. Such a formulation does not implement space-time supersymmetry manifestly. For that one would have to employ the Green-Schwarz formalism, which is not yet developed sufficiently to apply to string field theory. There has been much recent activity in this direction along the lines of seeking linearizing gauge choices.[18] In spite of dramatic claims of progress, this approach is still problematic.[19] Another gap in our understanding is the absence of a satisfactory string field theory involving *only* closed strings. In this area there has been recent progress in finding at least a gauge fixed action which produces correct tree amplitudes.[20]

With our work, we now have an action principle for all the open string theories. Since it uses string fields as dynamical variables, it is an admittedly cumbersome formulation. Because of this, one hopes that a simpler action principle might be possible based on a different choice of variables. We have suggested other approaches in the introduction, and it is clearly important to explore these and other possibilities.

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