LOOP CORRECTIONS TO CONFORMAL INVARIANCE FOR TYPE-I SUPERSTRINGS

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BRST anomalies arising from modular integrations can be cancelled between different genus world-sheets at the price of moving the world-sheet σ-model away from its conformal fixed point. We have fully implemented this procedure for the lowest-loop corrections to the type-I superstring. The associated conformal invariance violations are responsible for important corrections to the background field equations including both the coupling of the gauge field to gravity and the anomaly-cancelling interactions of the antisymmetric tensor field.

String loop amplitudes have singularities at points in moduli space where the topology of the world-sheet changes [1]. These singularities typically produce BRST anomalies [2], which we propose to cancel between different genus world-sheets by moving the σ-model away from its conformal fixed point. This procedure generates quantum corrections to the string field equations [3], but despite partial successes [4–6] its consistency has been doubted [7]. In this letter we present further strong evidence in its favor, based on a comprehensive study of a soluble model with no pathologies: the O(32) superstring in a constant background gauge field.

All anomalies of interest to us derive from the disappearance of zero-mass bosons into the vacuum. For the closed sector of the type-I superstring these bosons are the metric perturbation $h_{\mu\nu}=g_{\mu\nu}-\delta_{\mu\nu}$, the ghost dilaton Φ [8], and the antisymmetric tensor field strength $H_{\mu\nu\rho}$. In the presence of condensates of these fields, the world-sheet σ-model is summarized by an operator

$$\Psi(x)=\int \mathrm{d}^{10}k\,\{\tfrac{1}{2}h_{\mu\nu}(k)V^{\mu}_{-1}\tilde{V}^{\nu}_{-1}$$
$$+\tfrac{1}{2}\Phi(k)[V^{b}_{-1}\tilde{V}^{c}_{-1}-V^{c}_{-1}\tilde{V}^{b}_{-1}]$$
$$+2^{-1/2}\not{H}(k)_{AB}V^{A}_{-1/2}\tilde{V}^{B}_{-1/2}\} \tag{1}$$

built out of the background fields and a set of left-moving vertex operators [9,10]

$$V^{\mu}_{-1}=\psi^{\mu}\exp(-\phi)c\exp(\mathrm{i}k\cdot x_{+}) ,$$
$$V^{b}_{-1}=2\partial\xi\exp(-2\phi)c\exp(\mathrm{i}k\cdot x_{+}) ,$$
$$V^{c}_{-1}=\tfrac{1}{2}\eta c\exp(\mathrm{i}k\cdot x_{+}) ,$$
$$V^{A}_{-1/2}=S^{A}\exp(-\phi/2)c\exp(\mathrm{i}k\cdot x_{+}) , \tag{2}$$

plus corresponding right-moving vertex opertors $\tilde{V}$. In (2), the subscript indicates the superghost picture (a convenient choice has been made), the superghosts have been bosonized, and S^{A} is the positive chirality spin field. The antisymmetric tensor term is a product of left and right fermion vertices [9,10], joined by the wave function

$$\not{H}(k)_{AB}\equiv[\tfrac{1}{2}(1+\gamma_{11})\gamma^{\mu\nu\rho}C]_{AB}H_{\mu\nu\rho}(k) , \tag{3}$$

where C_{AB} is the O(10) spinor metric [11] and $\gamma^{\mu\nu\rho}$ is an antisymmetrized γ-matrix product. The structure of (3) is fixed [12,13] by the fact that like-chirality SO(10) spinors can only couple to odd-rank tensors and that, on a non-orientable world-sheet, Ψ must be odd under the interchange $V\Leftrightarrow\tilde{V}$. This leads to the requirement that the fermion wave function

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(3) must be odd in $\not{A}B$ which, in turn, uniquely selects the third-rank antisymmetric tensor.

Ψ can be regarded as a σ-model lagrangian, and conformal invariance conditions are derived by requiring its commutator with the BRST charge $Q+\tilde{Q}$ to vanish. The action of Q on the vertices (2) is [10]

$$[Q, V]_\pm = \tfrac{1}{2}k^2\partial c V + [Q_1, V]_\pm \ , \tag{4}$$

$$\{Q_1, V^\mu_{-1}\} = k^\mu V^c_{-1} \ ,$$

$$[Q_1, V^c_{-1}] = 0 \ ,$$

$$[Q_1, V^b_{-1}] = k_\mu V^\mu_{-1} \ ,$$

$$[Q_1, V^A_{-1/2}] = -U^B_\eta (\not{k})_B{}^A \ , \tag{5}$$

where

$$U^B_\eta \equiv -2^{-3/2}\eta \dot{S}^B \exp(\phi/2) c \exp(\mathrm{i}k\cdot x)_+ \tag{6}$$

is an auxiliary quantity built out of the negative chirality spin field, $\dot{S}$, and the peculiar statistics in (5) are due to the c-ghost factors in the vertices. The full BRST anomaly of the σ-model is easily seen to be

$$[(Q+\tilde{Q}), \Psi(x)] = \{-\tfrac{1}{2}(\partial c + \bar{\partial}\tilde{c})\Box\Psi(x)$$
$$-\tfrac{1}{2}\mathrm{i}(h_{\mu\nu,\nu} - \Phi_{,\mu})[V^c_{-1}\tilde{V}^\mu_{-1} - V^\mu_{-1}\tilde{V}^c_{-1}]$$
$$-\tfrac{1}{2}\mathrm{i}\sqrt{2}[\partial_\lambda H_{\mu\nu\rho}\gamma^{\lambda\mu\nu\rho} + 3\partial^\mu H_{\mu\nu\rho}\gamma^{\nu\rho}\gamma_{11}]_A{}^B$$
$$\times(U^A_\eta \tilde{V}_{B,-1/2} + V^A_{-1/2}\tilde{U}_{B,\eta})\} \ . \tag{7}$$

Setting it to zero gives linearized gauge-fixed wave equations for the massless fields which amount to a first-order approximation to the usual σ-model beta functions. The complete beta funtions can be recoverd by including further BRST anomalies associated with "collisions" of multiple insertions of Ψ on the world-sheet. We will not pursue this point as we want to study a new class of BRST anomalies arising, not from σ-model perturbation theory on a given world-sheet, but rather from changing the topology of the world-sheet by adding holes and crosscaps. Our strategy will be to cancel these "topological" anomalies against a non-zero value of (7), thus breaking the conformal invariance of the σ-model. We of course have to show that complete cancellation is in fact possible and that the resulting background field equations are consistent and physically correct.

Let us first discuss the mechanics of adding a boundary to a given world-sheet. In its simplest incarnation, a boundary is a place where free boundary conditions are imposed on the world-sheet fields. Type-I superstrings have a massless gauge multiplet which couples to boundaries so that a background gauge field is represented by a boundary action (essentially a Wilson line) rather than a σ-model interaction term [14]. We will study this more general case, in which free boundary conditions are augmented by a boundary action, in order to fully explore the physics of spacetime condensates of massless bosons. Inserting a boundary B is equivalent to creating a particular closed string state $|B; F\rangle$ (F denotes the background gauge field) which is joined to the original world-sheet by a closed string propagator Π (representing an integration over the radius of the hole cut into the world-sheet to create the boundary). The stringy generalization of Maxwell's equation is just the condition that the boundary state be annihilated by the BRST charge $(Q+\tilde{Q})$. The propagator may be written as

$$\Pi = \left(\frac{b_0+\tilde{b}_0}{2}\right)\int_{-\infty}^{0} \mathrm{d}\tau \exp[\tau(L_0+\tilde{L}_0)] \ , \tag{8}$$

where the antighost factor is the Teichmüller insertion associated with a modular integration. If the boundary state has a zero-mass piece (zero eigenvalue of $L_0+\tilde{L}_0$), the τ integration will diverge at $\tau\to-\infty$, corresponding to the insertion of a zero-radius hole. By a familiar argument [5], this divergence is associated with a BRST anomaly: the action of the BRST charge on the combined propagator and boundary state insertion is a total derivative in τ, leading to a net anomaly equal to the insertion at a point on the original world-sheet of the operator corresponding to the zero-mass piece of $|B; F\rangle$. To resolve 0/0 ambiguities it will be convenient to regulate the divergence by allowing a small momentum k to cross the boundary and, once the anomaly has been calculated, to let $k\to0$. The crosscap is a loop correction of the same order as the hole and its anomaly must be calculated as well. Essentially the same procedure may be used, with the simplification that, since a crosscap is not a boundary, and cannot be coupled to the gauge field, its anomaly is F-independent [5].

We now proceed to the calculation of the zero-mass piece of the boundary state for non-zero, but spacetime constant, background gauge field. (Exact results are not available for non-constant gauge field and the constant case has enough structure for our purposes.) For the superstring we have to deal with the complications of NS versus R sectors and superghost pictures. At a boundary, only the sum s_L+s_R of left and right superghost charges is conserved and the boundary state will contain all values of s_L-s_R [15]. If there is only one boundary in an amplitude, however, we can project out a particular s_L and s_R piece of the boundary state by specifying s_L and s_R for all the other vertices. We will use this freedom to compute the boundary state in the simplest pictures. The zero-mass piece of the boundary state will have a projection on both the NS–NS sector (graviton and dilaton) and the R–R sector (antisymmetric tensor). The former is fairly easy to construct by lagrangian path-integral arguments while the latter, since it involves spin fields and fermion vertices is more delicate to construct. We will infer the R–R sector answer from the NS–NS sector by spacetime supersymmetry arguments. All of our results will be expressed in terms of the vertices used to construct the σ-model lagrangian (1).

The zero-mass piece of the NS–NS sector boundary state, in the $(s_L, s_R)=(-1,-1)$ picture, can be shown to be

$$|B;F\rangle_{NS}=\kappa\,\mathrm{tr}[\det(1+F)]^{1/2}$$
$$\times\{[(1-F)/(1+F)]_{\mu\nu}V^{\mu}_{-1}\tilde{V}^{\nu}_{-1}$$
$$+V^{c}_{-1}\tilde{V}^{b}_{-1}-V^{b}_{-1}\tilde{V}^{c}_{-1}\}[(\partial c+\bar{\partial}\tilde{c})/2]|\Omega\rangle\,, \quad (9)$$

where κ is the open string coupling squared, $|\Omega\rangle$ is the SL_2 invariant vacuum and the trace (coming from the Wilson line) is over the gauge group. This expression is exact for constant non-abelian F taking values in an abelian subgroup of the gauge group (which is automatic for a constant gauge field strength by virtue of the Yang–Mills equations [16]). A complete proof of this formula will be given elsewhere [13], but its essential features are easily checked. For $F_{\mu\nu}=0$, it reduces (after eliminating the bosonized superghost fields in favor of the original β and γ superghosts) to our previously obtained superstring result [5], which is little more than an expression of OSp(10|2) invariance. For $F_{\mu\nu}\neq 0$, picture-changing to s_L, $\tilde{s}_R=0$ reproduces our previous results for the bosonic string in a background gauge field [4]. The F-dependence of the overall normalization is crucial and can be checked by functional integration [13].

The corresponding result for the crosscap (which does not couple to F) is just [5,12] the negative of (9) for an O(32) gauge group evaluated at $F_{\mu\nu}=0$. Thus, if the gauge group *is* O(32) the sum of the hole and crosscap anomalies in the NS–NS sector vanishes at $F=0$, but not otherwise.

Since $F_{\mu\nu}$ lies in an abelian subgroup, $[(1-F)/(1+F)]_{\mu\nu}$ is an orthogonal matrix. Thus the efect in (8) of turning on $F_{\mu\nu}$ is just an F-dependent Lorentz rotation of the right-moving vector fields relative to the left-moving ones plus an overall rescaling by $[\det(1+F)]^{1/2}$. This algorithm is quite general and we will eventually want to use it to construct the F-dependent R–R sector (antisymmetric tensor) hole anomaly from its $F=0$ value, which we will in turn obtain from supersymmetry considerations. Since the R–R sector is built out of Lorentz spinor operators, S^A, we will need to know the spinor representation, $M(F)$, of the above orthogonal matrix. This is easily calculated if we take $F_{\mu\nu}$ in the Cartan subalgebra of O(10):

$$F_{\mu\nu}=\bigoplus_{j=1}^{5}\begin{pmatrix} O & f_j \\ -f_j & O \end{pmatrix}. \quad (10)$$

Then $(1-F)/(1+F)$ corresponds to rotation through angles $\vartheta_j=-2\tan^{-1}f_j$, and the spinor representation is

$$M(F)_A{}^B=\delta_A^B\prod_{j=1}^{5}(1+f_j^2)^{-1/2}[1-\mathrm{i}\sigma_3^{(j)}f_j]\,. \quad (11)$$

Although a background gauge field breaks Lorentz invariance, a fixed rotation of the right-moving fields merely redefines the Lorentz generators. This redefinition affects the hole, but not the crosscap and the violation only occurs when hole and crosscap are combined, because their Lorentz symmetries are incompatible. The same remark applies to spacetime supersymmetry.

The action of the spacetime supersymmetry charge on the vertices (2) is [9]

$$\{A^A_{1/2}, V^{\mu}_{-1}\}=\tfrac{1}{2}V^{B}_{-1/2}(\gamma^{\mu}\not{k})_B{}^A\,,$$
$$[A^A_{1/2}, V^{b}_{-1}]=0\,,$$
$$[A^A_{1/2}, V^{c}_{-1}]=-U^B_{\eta}(\not{k})_B{}^A\,,$$
$$[A^A_{-1/2}, V^B_{-1/2}]=-(\mathrm{i}/\sqrt{2})(C\gamma_{\mu})^{AB}V^{\mu}_{-1}\,. \quad (12)$$

Similar formulae are true in other pictures up to BRST transforms which can be gauged away [13]. The regularization of allowing a non-zero momentum k to flow in through the boundary (thus attributing a momentum k to the vertices) is crucial since otherwise supersymmetry would annihilate the NS–NS boundary state. It is also necessary to regulate the left- and right-moving vertices differently with $k_L^\mu = -\tilde{k}_R^\mu$. This is quite reasonable since it preserves the standard bosonic boundary conditon $\partial x^\mu = -\bar{\partial} x^\mu$ [5].

Acting on (9) at $F_{\mu\nu}=0$, the closed string supersymmetry generator $\Lambda^A + \tilde{\Lambda}^A$ will create a spacetime fermion in the NS–R+R–NS sector. Since supersymmetry is exact in the absence of a background gauge field, this spacetime fermionic piece must be cancelled by $\Lambda^A + \tilde{\Lambda}^A$ acting on the zero-mass piece of the R–R hole state. We write the latter in the general form

$$|B; 0\rangle_R = V^A_{-1/2} L_{AB} \tilde{V}^B_{-1/2} [(\partial c + \bar{\partial}\tilde{c})/2] |\Omega\rangle \quad (13)$$

and use the cancellation condition to determine the wave function L_{AB}. The result (which is picture independent if one is careful to gauge away BRST transforms turns out to be

$$L_{AB} = i\kappa (\not{k}_L C)_{AB}/\sqrt{2}\,. \quad (14)$$

The result for $F \neq 0$ will be obtained by the procedure indicated earlier: carry out an F-dependent Lorentz rotation, using the spinor representation (11), of the left-moving vertices relative to the right-movers and then renormalize by a factor of $\sqrt{\det(1+F)}$. Because $k_L = -\tilde{k}_R$, (14) has the left–right antisymmetry appropriate to fermion wave functions on non-orientable world-sheets even though it only contains one γ-matrix, unlike (3). We of course do not identify k_L with any physical momentum. It is just a regulator, and will be set to zero after the BRST operator has acted, and before any comparison is made with the lower genus surface. Experience has convinced us that this strange procedure is correct.

After multiplying the zero-mass hole state by the closed string propagator (8), we apply $Q + \tilde{Q}$ to calculate the BRST anomaly [5]. The NS–NS sector result is

$$\begin{aligned}(Q+\tilde{Q})\Pi|B; F\rangle_{NS} &= \tfrac{1}{2}\kappa\, \mathrm{tr}[\det(1+F)]^{1/2} \\ &\times (\partial c + \bar{\partial}\tilde{c})\{V^\mu_{-1}[(1-F)/(1+F)]_{\mu\nu}\tilde{V}^\nu_{-1} \\ &+ V^c_{-1}\tilde{V}^b_{-1} - V^b_{-1}\tilde{V}^c_{-1}\}|\Omega\rangle\,. \end{aligned} \quad (15)$$

This came from applying (4), (5) to (8) and (9), using $k_L = -\tilde{k}_R$. In the R–R sector we first calculate the BRST anomaly for zero-field using (5), (13) and (14), and then switch on $F_{\mu\nu}$, using the previously described Lorentz rotation algorithm, to get

$$\begin{aligned}(Q+\tilde{Q})\Pi|B; F\rangle_R &= 2^{-3/2} i\kappa\, \mathrm{tr}[\det(1+F)]^{1/2} M(F)_A{}^B \\ &\times \{U^A_\eta \tilde{V}_{B,-1/2} + V^A_{-1/2}\tilde{U}_{B,\eta}\}|\Omega\rangle\,. \end{aligned} \quad (16)$$

In both (15) and (16) we have taken the limit of vanishing regulator momentum k. The crucial point is that singular factors of k in the propagator cancel against vanishing factors of k in the BRST operator and in the boundary state itself to give a finite limit. Now we observe that $\prod_{j=1}^5 (1+f_j^2)^{-1/2}$ in (11) cancels the determinant in (16), leaving $\prod_{j=1}^5 [1 - i\sigma_3^{(j)} f_j]$, which is a fifth-order polynomial $F_{\mu\nu}$. A Lorentz covariant version of this polynomial is obtained by expanding $\exp(-\frac{1}{2}\gamma^\mu\gamma^\nu F_{\mu\nu})$ and omitting all terms with repeated Lorentz indices. The overall gauge trace cancels odd powers of $F_{\mu\nu}$ and the complete F-dependence of (16) is contained in the expression

$$\begin{aligned}\mathrm{tr}[\det(1+F)]^{1/2} M(F)_A{}^B &= \mathrm{tr}\{1 + \tfrac{1}{2}(F\wedge F)_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma} \\ &+ \tfrac{1}{24} i^*(F\wedge F\wedge F\wedge F)_{\mu\nu}\gamma^{\mu\nu}\gamma_{11}\}_A{}^B\,. \end{aligned} \quad (17)$$

For zero field, the hole and crosscap cancel for O(32) [5,12]. Since the crosscap is F-independent, its inclusion just subtracts the $F_{\mu\nu}=0$ values from (15) and (16). Because of the determinant cancellation in (17), the zero-form (constant) term then disappears for any F. This is very fortunate since it is an unphysical state whose coupling would cause difficulty with the hexagon anomaly [12]. The extra uncancelled pieces of (17), generated from the zero-form by the $F_{\mu\nu}$ rotation, are important physical states.

The simple polynomial dependence on F of the R–R-sector anomaly was derived from properties of O(10) rotation matrices, but we have an independent proof of this crucial determinant cancellation. A fermionic functional integral calculation of the boundary state [13] gives a result proportional to

$$\prod_{m>0} \det(1+F)\,, \quad (18)$$

where m are the oscillator frequencies. This is formally the inverse of the determinant from the bosonic functional integral and one might expect the two to cancel. By the ζ-function formulae

$$\sum_{m=1,2,\ldots} 1=\zeta(0)=-\tfrac{1}{2}\,,$$

$$\sum_{m=1/2,3/2,\ldots} 1=(2^0-1)\zeta(0)=0\,, \qquad (19)$$

the two determinants do indeed cancel in the R–R sector (where boson and fermion modes are both integer) while in the NS–NS sector [17] (where bosons are integer moded and fermions half-integer moded) the fermion determinant is unity and the boson determinant stands uncancelled. The remaining polynomial dependence of the R–R sector boundary state then comes from the ten R–R sector zero-modes ψ_0^μ.

The sum of hole and crosscap BRST anomalies can now be cancelled against the sphere BRST anomaly (7), giving the loop-corrected field equations

$$g_{\mu\nu,\alpha\alpha}=\kappa\,\mathrm{tr}\{[\det(1+F)]^{1/2}\times[(1-F)/(1+F)]_{\mu\nu}-\delta_{\mu\nu}\}\,, \qquad (20)$$

$$g_{\mu\alpha,\alpha}-\Phi_{,\mu}=0\,, \qquad (21)$$

$$\Phi_{,\alpha\alpha}=-\kappa\,\mathrm{tr}\{[\det(1+F)]^{1/2}-1\}\,, \qquad (22)$$

$$\partial\wedge H=\tfrac{1}{4}\kappa\,\mathrm{tr}(F\wedge F)\,, \qquad (23)$$

$$\partial^\alpha H_{\alpha\mu\nu}=\tfrac{1}{144}\mathrm{i}\kappa\,\mathrm{tr}\{{}^*(F\wedge F\wedge F\wedge F)_{\mu\nu}\}\,. \qquad (24)$$

It was not obvious a priori that tree-level and loop-level anomalies would be of the right form to cancel against each other, but our results show that they are. By adding a BRST transform to (1), we can effect a spacetime coordinate transformation [5,8]

$$h_{\mu\nu}\to h_{\mu\nu}+\epsilon_{\mu,\nu}+\epsilon_{\nu,\mu}\,,$$

$$\Phi\to\Phi+\epsilon_{\mu,\mu}\,. \qquad (25)$$

Since the physical dilaton $\vartheta(x)$ must be a coordinate scalar, we must identify

$$\varphi(x)-\bar\varphi=\Phi(x)-\tfrac{1}{2}h_{\mu\mu}(x)\,, \qquad (26)$$

where

$$\exp(-\bar\varphi/2)=\kappa \qquad (27)$$

is the string loop coupling constant. Then (21) is essentially the harmonic gauge condition. The equations for $H_{\mu\nu\rho}$ show that it is related to an antisymmetric tensor potential $B_{\mu\nu}$ in the familiar way involving the Chern–Simons three-form [18]

$$H=\partial\wedge B+\tfrac{1}{4}\exp(-\bar\varphi/2)\,\mathrm{tr}(A\wedge F-\tfrac{2}{3}A\wedge A\wedge A)\,. \qquad (28)$$

The $\exp(-\bar\varphi/2)$ reflects the fact that the Chern–Simons coupling arises as a loop-order effect.

All of these equations of motion and gauge conditions can be derived from the effective lagrangian

$$\sqrt{g}\mathscr{L}=\sqrt{g}\exp(\varphi)\{R-(\nabla\varphi)^2-2\nabla^2\varphi-\tfrac{1}{12}H^2\}+\exp(\varphi/2)\mathrm{tr}\{\sqrt{g}-[\det(g+F)]^{1/2}\}-\tfrac{1}{144}\mathrm{i}\exp(\varphi/2)\mathrm{tr}\{{}^*(B\wedge F\wedge F\wedge F\wedge F)\}\,. \qquad (29)$$

The $\exp(\varphi)$ term is the standard tree-level result [19]. The $\exp(\varphi/2)$ terms are the string loop corrections. The i will disappear in Minkowski space. Our field equations (20)–(24) can be derived from (29) by expanding the tree term to second order in $h_{\mu\nu}$, Φ, $B_{\mu\nu}$, and the loop terms to first order. It is of course crucial that an effective action exists, since that guarantees that the loop-corrected equations of motion are mutually consistent, something which is also not a priori obvious.

The open string photon is known to interact with itself through a non-linear Born–Infeld lagrangian [14]. We now see that the hole anomaly is its energy–momentum tensor, and the σ-model anomaly which cancels it is the Einstein tensor of gravity. This simple physics already forces us to violate world-sheet conformal invariance. The couplings between $F_{\mu\nu}$ and $B_{\mu\nu}$ in (28) and (29) are just the Chern–Simons three-form and F^4 coupling needed to cancel the hexagon anomaly [18]. Again, essential physics has resulted from the violation of conformal invariance. The non-polynomial Born–Infeld determinant in the NS–NS sector miraculously disappeared from the R–R sector (17), thus maintaining, even for non-zero F, the cancellation of the zero-form between hole and crosscap needed to avert a hexagon anomaly disaster [12].

In string theory remarkable cancellations generating unexpected consistency are evidence that one is on the right track. We are therefore inclined to believe that the interpretation of string loop physics as generating conformal invariance violation in the world-sheet σ-model has deep significance. The loop-corrected σ-model is still required to have an enormous symmetry group, presumably some subtle generalization of naive conformal invariance, in order to decouple all the spurious states of string theory. The challenge is to identify and exploit its structure.

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