

QED \otimes QCD THRESHOLD CORRECTIONS AT THE LHC

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We use the theory of YFS resummation to compute the size of the expected resummed soft radiative threshold effects in precision studies of heavy particle production at the LHC, where accuracies of 1% are desired in some processes. We find that the soft QED threshold effects are at the level of 0.3% whereas the soft QCD threshold effects enter at the level of 20% and hence both must be controlled to be on the conservative side to achieve such goals.

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The physics objectives of the LHC entail a precise knowledge of the Standard Model processes that are either background to the would-be discovery processes, such as Higgs and SUSY processes, or important for the normalization of the observed signal and background processes so that the cross-sections of both types of processes can be determined in a way which maximizes the physics output of this pioneering high energy colliding beam device. One of the many sources of uncertainty that must be controlled is that associated with higher order radiative corrections to all aspects of the 15 TeV pp collisions. The dominant source of these effects is the higher order QCD corrections and many authors¹ have worked and are still working on these corrections. But at the level of precision required for some of the LHC objectives, one must also check that the higher order EW corrections are under the appropriate control. It is well known that initial state QED corrections can be large, particularly for resonate heavy particle production, and hence, already in Refs. 2 and 3 estimates were made of the size of the QED corrections to the structure function evolution for the LHC energies. An effect at the level of a few per mille was found in most of the

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relevant range of the Bjorken variable x . More recently, in Ref. 4, a similar result was found with however a qualitatively different character — whereas the results in Refs. 2 and 3 all show that the QED correction decreases the structure functions in the relevant regime of x , those in Ref. 4 actually show a QED correction which changes sign in the relevant regime of x , but which is still similar in magnitude, a few per mille in general in the relevant regime. This could just be the difference in the bases used to present the respective results, as a recent more complete treatment of the these same effects in Ref. 5 confirms both behaviors found in Refs. 3 and 4. In the current discussion, we do not address these effects further. Rather, we focus on the potentially larger issue of the size of the non-universal QED corrections associated with the threshold behavior of the heavy particle production at the LHC. To illustrate these effects, we treat as prototypical processes the processes $pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X$, where $V = W^\pm, Z$, and $\ell = e, \mu$, $\ell' = \nu_e, \nu_\mu(e, \mu)$ respectively for $V = W^+(Z)$, and $\ell = \nu_e, \nu_\mu$, $\ell' = e, \mu$ respectively for $V = W^-$. These processes are potential luminosity processes for the LHC.^{6,a} What we want to do is to estimate the expected size of the threshold contributions from these corrections in the realistic acceptance for the process in the LHC detectors. In this way, we show what role if any of these threshold radiative effects may play in precision LHC studies where the luminosity is expected to be needed at the few percent precision level. We recall for reference that the current precision on the FNAL luminosity is $\sim 6\%$.⁷ We stress that, for an overall precision of 2–3%, the theory budget in the error must be held to $\sim 1\%$ so that it does not figure too prominently in the total LHC luminosity budget.

Physically, the source of the effects which we calculate will be the following. In the complete calculation, assuming we have a factorization scale μ_F comparable to the renormalization scale μ_R , both of which are set near the heavy particle production scale M , one has the big logs $L_s \equiv \ln M^2/\mu_s^2$, $L_{th} \equiv L = \ln \hat{s}/M^2$, where here we assume with Refs. 2 and 3 that the structure functions have been evolved from input data at scale μ_s that have QED radiative effects in them. \hat{s} is the invariant mass squared of the hard parton production process.^b Hence, all of the familiar big logs of the type $\ln Q^2/m_f^2$ that one would expect from analogy with the big logs in e^+e^- annihilation at high energies are removed by the factorization of the cross-section's mass singularities as usual. Here, Q is any hard scale and m_f would be the respective quark mass. What do remain are the residual big logs L_s, L_{th} , where the former are presumably all summed up by the structure function evolution. This leaves the latter, which we now study using the YFS methods in Refs. 9 and 10.

^aWe understand⁷ that FNAL experiments are also considering these processes as potential luminosity processes.

^bWe stress that there are apparently data available⁸ from which the QED corrections from quarks have been removed so that the attendant QED correction to the structure function evolution would have a different boundary condition than that used in Refs. 2 and 3. We are not aware of any results in the published literature where this type of calculation has been done.

We start from the basic formula

$$d\sigma_{\text{exp}}(pp \rightarrow V + X \rightarrow \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{\text{exp}}(x_i x_j s), \quad (1)$$

where the squared pp cms energy is s and the structure functions have their usual meaning here so that $\hat{\sigma}_{\text{exp}}(x_i x_j s)$ is the reduced YFS exponentiated V production cross-section, $V = W, Z$. The YFS formula which we use in the latter cross-section was already developed in Ref. 11, where the dominant higher order corrections from the QCD corrections for top production at FNAL were estimated. Here, we can extend that result by including the dominant QED corrections in addition to the ISR QCD corrections treated in Ref. 11.

We point out that the threshold resummation result which we obtained in Ref. 11 is in agreement with the results of Catani *et al.* in Ref. 12 for the size of the higher order threshold resummation effects in QCD for $t\bar{t}$ production at the Tevatron. As we have explained in Ref. 13, this is not an accident. The Serman–Catani–Trentadue (SCT)^{14,15} resummation theory for QCD used in Ref. 12 is fully consistent with the QCD extension of the YFS theory that we discuss in this paper. As we explain in Ref. 13, one has to use the corresponding values of the $\bar{\beta}_n$ residuals in the extension of the YFS theory to QCD and the corresponding higher order corrections to the infrared exponent $\text{SUM}_{\text{IR}}(\text{QCD})$ in Eqs. (15)–(18) in Ref. 13 to get an exact correspondence between the two resummation approaches. As we will see below, the result which we obtain for the QCD resummed threshold effects in Z production using our extension of the YFS methods to QCD will account for most of the actual QCD correction as given by the exact result through $\mathcal{O}(\alpha_s^2)$ in Refs. 16 and 17. This again shows the applicability of our methods to this problem and, by our correspondence in Ref. 13, the applicability of the SCT method as well. We do not pursue the latter method further here.

Specifically, we recall the need to use Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) synthesized¹³ YFS formulas so that we avoid double counting of effects. In practice, this amounts to applying the factorization theorem to the YFS real and virtual infrared functions, removing those parts of these functions which generate the big logs L_s . The factorization scheme used should be that used in the isolation of these big logs to the respective structure functions. This means that the functions B^{nls} , \tilde{B}^{nls} , \tilde{S}^{nls} defined in Ref. 13 should be used in the YFS formulas applied to the factorized reduced hard parton–parton production cross-section. Here, we stress that we are doing this for both the QCD and the EW QED corrections. This means that, in the basic YFS algebra illustrated in Ref. 11, we need to make the replacements, using an obvious notation,

$$\begin{aligned} B_{\text{QCD}}^{nls} &\rightarrow B_{\text{QCD}}^{nls} + B_{\text{QED}}^{nls} \equiv B_{\text{QCED}}^{nls}, \\ \tilde{B}_{\text{QCD}}^{nls} &\rightarrow \tilde{B}_{\text{QCD}}^{nls} + \tilde{B}_{\text{QED}}^{nls} \equiv \tilde{B}_{\text{QCED}}^{nls}, \\ \tilde{S}_{\text{QCD}}^{nls} &\rightarrow \tilde{S}_{\text{QCD}}^{nls} + \tilde{S}_{\text{QED}}^{nls} \equiv \tilde{S}_{\text{QCED}}^{nls} \end{aligned} \quad (2)$$

in the YFS exponentiation algebra in Ref. 11, with the attendant replacement of the reduced parton cross-sections and structure functions to reflect the incoming pp state as it is compared to the incoming $p\bar{p}$ state in Ref. 11 and to reflect the V production and decay here versus the $t\bar{t}$ production there.

In this way, we start from the basic result

$$\begin{aligned}
 d\hat{\sigma}_{\text{exp}} &= e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^n \frac{d^3 k_{j_1}}{k_{j_1}} \\
 &\times \prod_{j_2=1}^m \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\text{QCED}}} \\
 &\times \tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \tag{3}
 \end{aligned}$$

where the new YFS residuals $\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)$, with n hard gluons and m hard photons, represent the successive application of the YFS expansion first for QCD as in Refs. 11, 18 and 19 followed by that for QED as some of us have used in many applications as in Ref. 10. The infrared functions are now given by

$$\begin{aligned}
 \text{SUM}_{\text{IR}}(\text{QCED}) &= 2\alpha_s \mathfrak{R}B_{\text{QCED}}^{nls} + 2\alpha_s \tilde{B}_{\text{QCED}}^{nls}, \tag{4} \\
 D_{\text{QCED}} &= \int \frac{dk}{k^0} (e^{-iky} - \theta(K_{\text{max}} - k^0)) \tilde{S}_{\text{QCED}}^{nls},
 \end{aligned}$$

where the dummy parameter K_{max} is just that — nothing depends on it. We have taken the liberty, not to be viewed as permanent, to set the dummies in QCD and QED to be the same value. The infrared algebra realized here for QCD and QED we will sometimes denote as Quantum ChromoElectroDynamics (QCED).

The result (3) may be realized in the context of (1) using Monte Carlo methods as illustrated in Ref. 19. We shall present those methods in detail elsewhere. Here, to illustrate the size of the QED effects in the presence of the QCD resummation as well, we use a semi-analytical evaluation of (1).

Specifically, in order to isolate explicitly ISR effects, we take the case of single Z production and refer the case of single W production to Ref. 20. We also call attention to the complete EW $\mathcal{O}(\alpha)$ corrections to single heavy gauge boson production in hadron colliders in Refs. 21–25. We work in the approximation that, when we retain the dominant infrared effects at the $\tilde{\beta}_{0,0}^{(0,0)}$ -level, these two sets of corrections commute with one another: physically, gluons are EW singlets and photons are color singlets. Thus, the only issue is whether particles such as quarks with both color and EW charge have significant higher order corrections in which the two sets of corrections do not commute. Here, we appeal to the Heisenberg uncertainty principle and note that the average soft photon energy fraction in the YFS formalism is $x_{\text{avg}}(\text{QED}) \cong \gamma(\text{QED})/(1 + \gamma(\text{QED}))$ and the average YFS soft gluon

energy fraction is correspondingly $x_{\text{avg}}(\text{QCD}) \cong \gamma(\text{QCD})/(1 + \gamma(\text{QCD}))$, where the YFS radiation probability strengths are given by $\gamma(A) = \frac{2\alpha_A C_A}{\pi}(L_s - 1)$, $A = \text{QED}, \text{QCD}$. For quarks, $C_A = Q_f^2$, C_F , respectively, for $A = \text{QED}, \text{QCD}$, where Q_f is the quark electric charge and C_F is the quadratic Casimir invariant for the quark color representation. Here, we variously use $\alpha_{\text{QED}} \equiv \alpha$, $\alpha_{\text{QCD}} \equiv \alpha_s$. As the ratio of the two fine structure constants is ~ 10 in the LHC/Tevatron environment in which we work, we expect that the typical time for the dominant QCD corrections we exponentiate to occur is more than an order of magnitude shorter than the corresponding time for the analogous QED corrections which we exponentiate. Thus, we do not expect that significant effects in our leading exponentiated analysis are missing due to sets of QCD and EW QED corrections that do not commute. We then compute the ratio $r_{\text{exp}} = \sigma_{\text{exp}}/\sigma_{\text{Born}}$ with and without the QED contributions, where, for definiteness, we note that the partonic Born cross-section we use here is well known and we generate it by the standard methods from Eq. (43) in Ref. 26, with the understanding that we retain only the Z contribution and that we substitute the respective incoming quark chiral couplings for the incoming e chiral couplings accordingly. (We stress that we *do not* use the narrow resonance approximation here.) We get the results

$$r_{\text{exp}} = \begin{cases} 1.1901, & \text{QCED} \equiv \text{QCD} + \text{QED}, \text{LHC} \\ 1.1872, & \text{QCD}, \text{LHC} \\ 1.1911, & \text{QCED} \equiv \text{QCD} + \text{QED}, \text{Tevatron} \\ 1.1879, & \text{QCD}, \text{Tevatron}. \end{cases} \quad (5)$$

We see that indeed the QED threshold correction is at the level of 0.3% at both the Tevatron and at the LHC and is in principle significant for studies at the percent level such as those that are envisioned for the precision luminosity determination for the LHC.^{6,c} We also corroborate here the well-known^{16,17} result that the QCD threshold correction is needed through $\mathcal{O}(\alpha_s^2)$ for such studies. The results in (5) are entirely consistent with the exact $\mathcal{O}(\alpha_s^2)$ results in Refs. 16 and 17, where comparison with the pure QCD results in Eq. (5) shows that our threshold effects reproduce most of the complete exact results at both the LHC and the Tevatron. The size of our exponentiated QED effects in (5) is fully consistent with the exact $\mathcal{O}(\alpha)$ results in Refs. 21–23, where the ISR QED correction was found to be 0.4%. In arriving at our results in (5), we have used the parton distributions from Ref. 27.

Our result for the size of the QED threshold effect is similar to the size found in Refs. 2–5 for the size of the QED effects on the structure function evolution itself. From these two results, we can conclude that the (higher order) QED correction to heavy particle production at the LHC will be at the several per mille level and it cannot really be ignored for percent level precision studies.

^cWe checked that the variation of the renormalization scale, which we set equal to the factorization scale, above and below its nominal value M_Z by a few widths of the Z does not change our result that the QED ISR threshold correction enters at the 0.3% level.

In conclusion, we have introduced a new theoretical construct in this paper, QCED, the regime of the large IR effects in the large momentum transfer regime, where we expect the QED and QCD exponentiation algebras to commute to a large extent and to the extent that they do, we identify the resulting theory as Quantum ChromoElectroDynamics. Corrections to it are then to be treated perturbatively in the respective combined YFS residuals $\tilde{\beta}_{n,m}^{\text{QCED}}$. We have used this paradigm to estimate the size of the QED threshold effects in single gauge boson production at the LHC type energies. We find that such effects are large enough that they must be taken into account for precision studies, such as those envisioned for precision luminosity determinations.

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References

1. See for example T. Gehrmann, in *Proc. LP2003*, ed. C. Sazama (FNAL, 2004), in press, and references therein.
2. S. Haywood, P. R. Hobson, W. Hollik and Z. Kunszt, in *Proc. 1999 CERN Workshop on Standard Model Physics (and more) at the LHC*, CERN-2000-004, eds. G. Altarelli and M. L. Mangano (CERN, Geneva, 2000), p. 122.
3. H. Spiesberger, *Phys. Rev.* **D52**, 4936 (1995).
4. W. J. Stirling, Electroweak effects in parton distribution functions, talk presented at ESF Exploratory Workshop, *Electroweak Radiative Corrections to Hadronic Observables at TeV Energies*, Durham, September 2003.
5. M. Roth and S. Weinzierl, *Phys. Lett.* **B590**, 190 (2004).
6. M. Dittmar, F. Pauss and D. Zurcher, *Phys. Rev.* **D56**, 7284 (1997); M. Rijssenbeek, in *Proc. HCP2002*, ed. M. Erdmann (Karlsruhe, 2002), p. 424; M. Dittmar, *ibid.*, p. 431, and references therein.
7. S. Klimenko, in *Proc. HCP2002*, ed. M. Erdmann (Karlsruhe, 2002), p. 413, and references therein.
8. R. Thorne, private communication (2003).
9. D. R. Yennie, S. C. Frautschi and H. Suura, *Ann. Phys.* **13**, 379 (1961); see also K. T. Mahanthappa, *Phys. Rev.* **126**, 329 (1962), for a related analysis.
10. See also S. Jadach *et al.*, *Comput. Phys. Commun.* **102**, 229 (1997); S. Jadach, M. Skrzypek and B. F. L. Ward, *Phys. Rev.* **D55**, 1206 (1997); S. Jadach, B. F. L. Ward and Z. Was, *ibid.* **D63**, 113009 (2001); S. Jadach, B. F. L. Ward and Z. Was, *Comput. Phys. Commun.* **130**, 260 (2000); S. Jadach *et al.*, *ibid.* **140**, 432; 475 (2001), and references therein.
11. S. Jadach *et al.*, *Mod. Phys. Lett.* **A12**, 2425 (1997).
12. S. Catani, M. L. Mangano, P. Nason and L. Trentadue, *Phys. Lett.* **B378**, 329 (1996).
13. B. F. L. Ward and S. Jadach, *Mod. Phys. Lett.* **A14**, 491 (1999).
14. G. Sterman, *Nucl. Phys.* **B281**, 310 (1987).

15. S. Catani and L. Trentadue, *Nucl. Phys.* **B327**, 323 (1989); *ibid.* **B353**, 183 (1991).
16. R. Hamberg, W. L. van Neerven and T Matsuura, *Nucl. Phys.* **B359**, 343 (1991).
17. W. L. van Neerven and E. B. Zijlstra, *Nucl. Phys.* **B382**, 11 (1992); *ibid.* **B680**, 513 (2004), and references therein.
18. B. F. L. Ward and S. Jadach, *Acta Phys. Pol.* **B33**, 1543 (2002), and references therein.
19. B. F. L. Ward and S. Jadach, QCD soft gluon exponentiation: YFS MC approach, in *Proc. ICHEP2002*, eds. S. Bentvelsen *et al.* (North-Holland, 2003), p. 275 and references therein.
20. S. Jadach *et al.*, to appear.
21. U. Baur, S. Keller and W. K. Sakumoto, *Phys. Rev.* **D57**, 199 (1998).
22. U. Baur, S. Keller and D. Wackerth, *Phys. Rev.* **D59**, 013002 (1998), and references therein.
23. U. Baur *et al.*, *Phys. Rev.* **D65**, 033007 (2002), and references therein.
24. S. Dittmaier and M. Kramer, *Phys. Rev.* **D65**, 073007 (2002), and references therein.
25. Z. A. Zykunov, *Eur. Phys. J.* **C3**, 9 (2001), and references therein.
26. S. Jadach, B. F. L. Ward and Z. Was, *Phys. Rev.* **D63**, 113009 (2001).
27. A. D. Martin *et al.*, *Phys. Rev.* **D51**, 4756 (1995).