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QED CORRECTIONS TO LUMINOSITY MEASUREMENTS AT LEP[1]

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ABSTRACT

In this short contribution we present certain new results on the QED second-order radiative corrections to low-angle Bhabha cross section. The presented results will be essential to the future reduction of the overall theoretical uncertainty in the measurement of the luminosity at LEP below the present 0.25% level.

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The low-angle Bhabha (LABH) process $e^+e^- \rightarrow e^+e^-$ in the angular range below 100 mrad is used in LEP/SLC accelerators to determine the luminosity and hence the absolute normalization of the cross section of all other processes in the e^+e^- scattering. This LABH cross section is therefore not of physical interest by itself, but, on the contrary, is regarded as completely known from theory, i.e. from Quantum Electrodynamics (QED). The pure experimental precision of the luminosity measurement has improved dramatically from the times of PETRA (2%) and early LEP (0.5%) down to present value close to 0.1%. On the other hand, although the LABH cross section is calculable, in principle, in perturbative QED with arbitrary precision (except a small hadronic correction), it is subject to theoretical uncertainties due to truncation of the perturbative expansion and also due to limitations of the calculational tools (computer programs). All LEP/SLC experiments use a theoretical calculation for LABH based on works published by the actual authors three years ago [1]. This calculation has an overall theoretical/technical precision tag of 0.25% and is embodied in the form of the Monte Carlo event generator BHLUMI version 2.0 [2]. This error was acceptable in 1991 but now, with a factor of more than 2 improvement of the experimental precision, it dominates the present overall luminosity error. It is therefore quite urgent to reduce the theoretical error of the QED calculation down to a precision level at least 0.1%.

The backbone of the 0.25% theoretical precision estimate in Ref. [1] is due to missing second-order $\mathcal{O}(\alpha^2 L^2)$ (0.15%) and $\mathcal{O}(\alpha^2 L)$ (0.09%) contributions in the matrix element encoded in the Monte Carlo calculations. Here $L = \ln(|t|/m_e^2)$ is the so-called big-log in the leading-logarithmic (LL) approximation where t is t-channel transfer (of order of 1 GeV) (see also Fig. 1 for a pictorial definition of the LL approximation). The first of the above contributions (0.15%) includes also the *technical precision* of the Monte Carlo programs due to bugs, rounding errors, quality of random numbers, etc. It is illustrated in Fig. 2 (taken from Ref. [1]) as a difference of three Monte Carlo sub-generators of BHLUMI 2.0: (i) multiphoton $\mathcal{O}(\alpha)_{exp}$ BHLUMI, (ii) $\mathcal{O}(\alpha)$ OLDBIS (without exponentiation), and (iii) $\mathcal{O}(\alpha^3)_{LL}$ leading logarithmic (collinear photon emission) event sub-generator LUMLOG. The difference of the three MC sub-generators provides a solid estimate of the technical precision. In addition, sub-generators (ii) and (iii) have separate estimates of their technical precision at the level below 0.05%; this comes, in the case of (ii), from an independent comparison with a semi-analytical calculation, see Ref. [3], and, in the case of (iii) from another Monte Carlo; see Ref. [4]. The comparison in Fig. 2 provides, therefore, an estimate of the technical precision mainly for the multiphoton BHLUMI sub-generator, which did not have any other independent analytical or Monte Carlo cross-check[2].

As for *physical precision*, which is mainly due to truncation of perturbative calculation, in Ref. [1] the dominant (beyond first-order) correction of $\mathcal{O}(\alpha^2 L^2)$ was under good control because it was calculated using $\mathcal{O}(\alpha^3)_{LL}$ sub-generator LUMLOG; see Ref. [4]. The hybrid Monte Carlo calculation OLDBIS+LUMLOG includes the entire $\mathcal{O}(\alpha^2 L^2)$ correction for the integrated cross section, but due to the zero-angle (collinear) emission of photons, LUMLOG is not very suitable for experimental analysis, where various fine-grain inclusive/multiphoton distributions are checked in the process of reducing the systematic experimental error. In view

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[2] At the time it seemed unthinkable to integrate analytically the total cross section of the $\mathcal{O}(\alpha)_{exp}$ BHLUMI.

of the above, experimentalists have always preferred to use the multiphoton MC generator BHLUMI 2.0, which includes only the part of $\mathcal{O}(\alpha^2 L^2)$ generated by a Yennie-Frautschi-Suura exponentiation (providing excellent realistic differential distributions), and then to employ the OLDBIS+LUMLOG hybrid solution in order to estimate the missing $\mathcal{O}(\alpha^2 L^2)$ correction. For realistic cuts this correction has turned out to be small, typically below 0.2%.

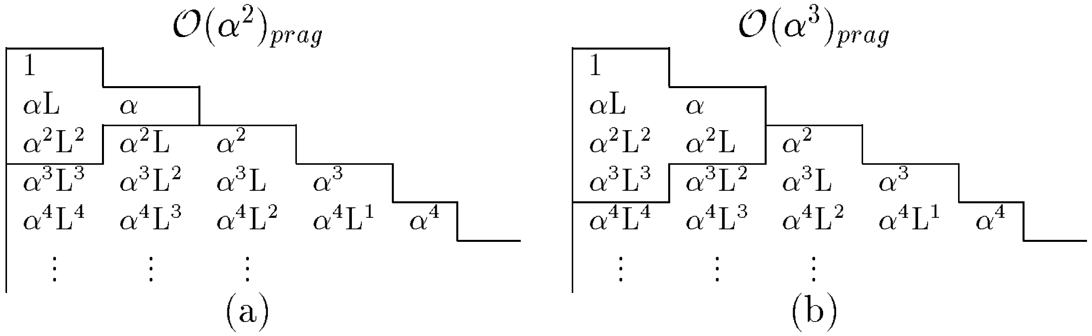
The obvious development path of the above calculation scheme was the following: (i) to implement the $\mathcal{O}(\alpha^2 L^2)$ missing part of the matrix element in the multiphoton exponentiated sub-generator of BHLUMI, (ii) to provide a new independent analytical cross-check of the new matrix element, (iii) to improve the estimate of the next dominant bremsstrahlung-type corrections, i.e. of $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^3)$ corrections, and (iv) to estimate again other higher-order corrections such as light pairs, vacuum polarization, remnant of s-channel Z-exchange, etc. The point (iv) and to some extent also the point (iii) were elaborated in Ref. [5].

Here we shall show the first results concerning points (i) and (ii). In fact these two points are closely related, because it is of little use to implement a new "better matrix element" if there is no immediate cross-check that it was done correctly. If in the process of doing such improvement, one introduces some bugs in the program, then the apparent gain in physical precision will be offset by the loss of technical precision. In the situation when, ultimately, both enter into error of the luminosity measurement, the sum of them counts. In the ideal case we would like to perform such a cross-check of the newly implemented matrix element for realistic cuts, close to the experimental ones, as was done (to some extent) for OLDBIS and LUMLOG [3, 4]. In the example presented in this contribution we had to compromise on the choice of cuts and to integrate over multiphoton phase space for cuts chosen in such a way that the integration was feasible and the result of integration was relatively simple. Such a comparison of the semi-analytical calculation with the Monte Carlo provides an excellent estimate of the technical precision of the MC program, but strictly speaking, it is valid for only one set of cuts. The extension of such an estimate of the technical precision to a wider class of cuts, including experimental ones, has to be worked out separately, and this part of our work will be discussed elsewhere.

In the following we shall (a) characterize the new $\mathcal{O}(\alpha^2)_{prag}$ exponentiated matrix element implemented in the new version of Monte Carlo BHLUMI 4.0, (b) define a set of "academic" cuts used for semi-analytical integration of the above matrix element over multiphoton phase space, (c) characterize briefly methods used in the analytical integration and class of corrections kept in the analytical phase-space integration (it is not the same as in matrix element!),
(d) show the numerical agreement of the Monte Carlo (with the new matrix element) with the semi-analytical formula down to the 0.03% level (technical precision). Due to lack of space we shall not write the explicitly differential distribution implemented in the Monte Carlo and used as an input in the analytical integration of the phase space. We shall include, however, the explicitly analytical result of the integration.

Ad (a):

At the 1% precision level, it is enough to use the complete $\mathcal{O}(\alpha)$ matrix element. All "photonic" corrections in which the additional photon line connects the upper electron line with the lower positron line (so called up-down interference) are strongly suppressed below 100 mrad. This phenomenon was conjectured and proved numerically, using an $\mathcal{O}(\alpha)$ calculation, in Ref. [3].



$\mathcal{O}(\alpha^2)_{prag}$

1				
αL	α			
α^2L^2	α^2L	α^2		
α^3L^3	α^3L^2	α^3L	α^3	
α^4L^4	α^4L^3	α^4L^2	α^4L^1	α^4
⋮	⋮	⋮		

(a)

$\mathcal{O}(\alpha^3)_{prag}$

1				
αL	α			
α^2L^2	α^2L	α^2		
α^3L^3	α^3L^2	α^3L	α^3	
α^4L^4	α^4L^3	α^4L^2	α^4L^1	α^4
⋮	⋮	⋮		

(b)

Figure 1: *QED perturbative leading and subleading corrections. Rows represent corrections in consecutive perturbative orders – the first row is the Born contribution. The first column represents the leading logarithmic (LL) approximation and the second column depicts the next-to-leading (NLL) approximation. In the figure, terms selected for (a) second- and (b) third-order pragmatic expansion are limited with the help of an additional line.*

Since then we are exploiting this approximation routinely. In the $\mathcal{O}(\alpha^2)$, in order to reach a 0.1% level of physical precision, it is probably enough to add in the Monte Carlo matrix element, beyond the regular $\mathcal{O}(\alpha)$, the dominant second-order contribution of $\mathcal{O}(\alpha^2 L^2)$ (with or without exponentiation). This type of calculation we denote as $\mathcal{O}(\alpha^2)_{prag}$, and it is depicted in Fig. 1a. While $\mathcal{O}(\alpha)$ distributions come directly from the Feynman diagrams the additional $\mathcal{O}(\alpha^2 L^2)$ contributions we derive more simply by convoluting twice the Altarelli-Parisi kernel[3]. For this contribution, the soft limit is improved by hand[4] to a well-known behaviour and the finite transverse momenta of photons are introduced in the distributions using the soft limit as a model to follow. The above procedure necessarily creates some contributions of $\mathcal{O}(\alpha^2 L^1)$ and $\mathcal{O}(\alpha^2 L^0)$. The important advantage of the above $\mathcal{O}(\alpha^2 L^2)$ ansatz is that it is simple (also quick in the computer evaluation) and its LL content, which is of main interest, is explicit and very easy to control. The above LL ansatz comes before the Yennie-Frautschi-Suura exponentiation and the exponentiation procedure introduces new contributions of $\mathcal{O}(\alpha^3)$ and higher. Among them, the $\mathcal{O}(\alpha^3 L^3)$ contribution will be numerically dominant. In the future, if $\mathcal{O}(\alpha^2)_{prag}$ will not provide sufficient physical precision, then in the next step we shall have to add valid perturbative contributions of $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L)$ – they are potentially of a similar size. Such a set of perturbative contributions, we denote collectively as $\mathcal{O}(\alpha^3)_{prag}$ and depict it in Fig. 1b.
Ad (b):
The most important criterion used to define our set of kinematical cuts for semi-analytical integration of the $\mathcal{O}(\alpha^2)_{prag}$ new matrix element over the multiphoton phase space (the so-called *academic trigger*) is that, actually, this semi-analytical integration is really feasible. We define cuts of our "academic trigger" as follows: $|t_{min}| < |t| < |t_{max}|$ and $V < V_{max}$, where t is the four-momentum transfer squared transmitted through t-channel photon exchange, and the variable V represents some kind of measure of the total energy carried away by all emitted real photons. We require that $0 < V < V_{max} < 1$. The $V_{max} = 1$ represents the condition of

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[3] The same kind of LL ansatz was used successfully in the YFS2 and YFS3 Monte Carlo programs [6, 7, 8].

[4] In the LL approximation correct soft limit is generally not reproduced.

completeness of the phase space and $V_{max} = \epsilon$ (where ϵ is small but positive number) represent the condition that only configurations with soft photons (or no photons at all) are kept. The V-variable we actually define, in terms of the four-momenta, as follows:

$$V = 1 - \frac{2(p_1 p_2)\ |t|}{(2(p_1 p_2)\ + 2(p_1 K_p))^2}\ \frac{2(q_1 q_2)\ |t|}{(2(q_1 q_2)\ + 2(q_1 K_q))^2}, \tag{1}$$

where $p_i = 1,2$ are the four-momenta of incoming and outgoing electron, $q_i = 1,2$ are four-momenta of incoming and outgoing positron, and K_p and K_q are the total four-momenta of all photons emitted from electron and positron lines respectively.

Ad (c):

With the above definition of the phase-space window, it is rather straightforward to integrate $\mathcal{O}(\alpha^2)_{prag}$ matrix element keeping all terms within the $\mathcal{O}(\alpha^2)_{prag}$ approximation. This we found not sufficient for the purpose of establishing the technical precision at the 0.03% level, because some terms beyond $\mathcal{O}(\alpha^2)_{prag}$ (especially for partial incomplete results!) are of that order. We have therefore decided to follow in the integration the third order $\mathcal{O}(\alpha^3)_{prag}$ approximation (see also Fig. 1b). This means that terms of $\mathcal{O}(\alpha^2 L)$ due to our LL ansatz, and terms of $\mathcal{O}(\alpha^3 L^3)$ due to exponentiation, are integrated analytically over the phase space (with the academic trigger) exactly! To our knowledge it is the only example of an explicit third-order QED analytical calculation (albeit LL-type) with the non-trivial kinematic cuts imposed. The resulting integrated cross section is not very complicated and it reads as follows:

$$\begin{aligned}
&\sigma^{(2)}(t_{min}, t_{max}, V_{max}) = \int_{t_{min}}^{t_{max}} dt \int_0^{V_{max}} dV\ \ \rho^{(2)}_{tot}(t,V),\\
&\rho^{(2)}_{tot}(t,V) = b_0\ F(2\gamma)\ e^{2\Delta_{YFS}(\gamma)}\ 2\gamma V^{2\gamma-1}\Big\{1+\gamma+\gamma^2/2\Big\}\\
&\quad +b_0\ F(2\gamma)\ e^{2\Delta_{YFS}(\gamma)}\ V^{2\gamma}\Big\{\gamma(-2+V) + \frac{\alpha}{\pi}\ln(1-V)(-4+4V-2V^{-1})\\
&\quad +\gamma^2(-2) + \gamma^2\ln(1-V)(3-3V/2-2V^{-1})\\
&\quad +\gamma^3(-7V/4) + \gamma^3\ln(1-V)[5/4+V/2V-2V^{-1}]\\
&\quad +\gamma^3\ln(1-V)^2[-5/8+5V/16+(1/4)V^{-1}] + \gamma^3\mathrm{Li}_2(V)(2-V)\\
&\quad +\gamma\frac{\alpha}{\pi}[1/4+11V-(13/4)(2-V)^{-1}+(1/2)(2-V)^{-2}-6(2-V)^{-3}+2(1-V)^{1/2}]\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(1-V)[39/4-19V/4-2V^{-1}\\
&\qquad\qquad -2(2-V)^{-1}+(2-V)^{-2}-(1/2)(2-V)^{-3}-(3/2)(1-V)^{1/2}]\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(1-V/2)[-9/2+3V/4-4(2-V)^{-1}+2(2-V)^{-2}-4(2-V)^{-3}]\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(1-V)^2[19/8-41V/16+V^{-1}] + \gamma\frac{\alpha}{\pi}\ln(1-V)\ln(2-V)(-1/2+V/4)\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V)(12-10V) + \gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V/2)(-6+5V)\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(1-V)\ln[1-(1-V)^{1/2}](-6+5V)\\
&\quad +\gamma\frac{\alpha}{\pi}\ln(2-V)\ln(1-V/2)(3/2-11V/4)
\end{aligned}$$

$$
\begin{aligned}
&+\gamma\frac{\alpha}{\pi}\ln(1-V/2)^2(3/4-5V/8)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/2)(-3/2+11V/4)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[(1-V)/(2-V)](1/2-V/4)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/(2-V))(1-5V/2)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(-V/2/(1-V))(6-5V)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[1-(1-V)^{-1/2}](6-5V)\\
&-\xi\gamma\chi(V)/(1-V)\Big\}, \qquad (2)
\end{aligned}
$$

where $\gamma = 2(\alpha/\pi)(L-1)$, $b_0 = \chi(\xi)$, $\chi(x) \equiv (1+(1-x)^2)/2$, $\xi = |t|/s$, $F(x) \equiv \exp(-Cx)/\Gamma(1+x)$ and $\Delta_{YFS}(\gamma) = \gamma/4 - (\alpha/\pi)(1/2+\pi^2/6)$. Note that the $\mathcal{O}(\alpha^2)_{prag}$ part of the formula is very compact and its LL content is identical to the non-singlet second-order structure function of the photon in the electron[5]. The terms of $\mathcal{O}(\alpha^3L^3)$ and $\mathcal{O}(\alpha^2L)$, which represent most of the formula, finally turn out to be numerically small, in fact below 0.04%.

Ad (d):

In Fig. 3 we show a comparison of eq. (2) with a Monte Carlo for the angular range $26.125 < \theta < 55.875$ mrad, (translated according to Born-level kinematics into an appropriate $t-$range). Although the integration over $|t|$ and V is feasible analytically we are doing it numerically with the help of the standard Gauss technique, because the integrand is a very smooth function, suitable for this method (peaks are removed either by a change of variables or by a subtraction). As we see the Monte Carlo and semi-analytical result differ by up to 0.03%.

We can, of course, repeat the exercise of comparing the new BHLUMI version 4.0 (instead of 2.0) with the old hybrid solution OLDBIS+LUMLOG. The result of such a comparison, with the scale on the vertical axis inflated by a factor of almost ten, is presented in Fig. 4. Let us stress that the interpretation of this difference is not the same as in Ref. [1]. The multiphoton Monte Carlo BHLUMI 4.0 includes $\mathcal{O}(\alpha^2L^2)$ corrections, hence the plotted difference BHLUMI.4−(OLDBIS+LUMLOG) is potentially dominated by the $\mathcal{O}(\alpha^2L)$ and $\mathcal{O}(\alpha^3L^3)$ contributions, while the $\mathcal{O}(\alpha^2L^2)$ cancels out. The difference also includes, as in the case of Ref. [1], the technical precision of the Monte Carlo programs, mainly of the multiphoton BHLUMI[6]. As we see in Fig. 4 the difference is again well within 0.15%. This result is very encouraging, but it should be treated as very preliminary[7] and it will be soon subject to another round of tests. We can, of course, now look directly at how big the "missing $\mathcal{O}(\alpha^2L^2)$" correction was in the older version 2.0 of the multiphoton/exponentiated version of BHLUMI by comparing it with the new version 4.0. The difference turns out to be below 0.04%, for a typical realistic trigger. This result can be treated as a success of the exponentiation since $\mathcal{O}(\alpha^2L^2)$ can be as big as 0.25% and the above result shows that $\mathcal{O}(\alpha)_{prag}$ exponentiated calculation reproduces almost all of it. (This we could see already to some extent in Ref. [1], where it was found to be, within statistical error, less than 0.15%; see Fig. 2.) The result of Fig. 4 is just one example of the series of tests which are now under preparation. The aim of the ongoing work is to establish the technical precision of BHLUMI 4.0 to within 0.05% or better, and to get a

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[5] This because the variable V in the LL has a very simple meaning.

[6] The estimate of technical precision from Fig. 3 does not apply here automatically, due to the difference in the type of trigger.

[7] In fact this plot has already changed a little bit since its presentation in Meribel.

solid estimate of the physical precision (higher orders) better than 0.1%. Results presented in this talk are encouraging and will certainly be of great help in attaining this goal.

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Figure 2: *This figure is taken from Ref. [1]. We plot the difference of σ_{B2} of BHLUMI 2.0 and σ_{O+L} from OLDBIS and LUMLOG. This represents the missing $\mathcal{O}(\alpha^2 L^2)$ bremsstrahlung correction in BHLUMI version 2.0 of Ref. [2] together with its technical precision. The difference of the cross sections (divided by Born) is calculated for symmetric and asymmetric calorimetric trigger Ξ_{NW} as a function of the energy cut $z_{\rm min}$. Dotted lines mark the 0.15% limit. Vacuum polarization, Z and s channel γ are switched off.*

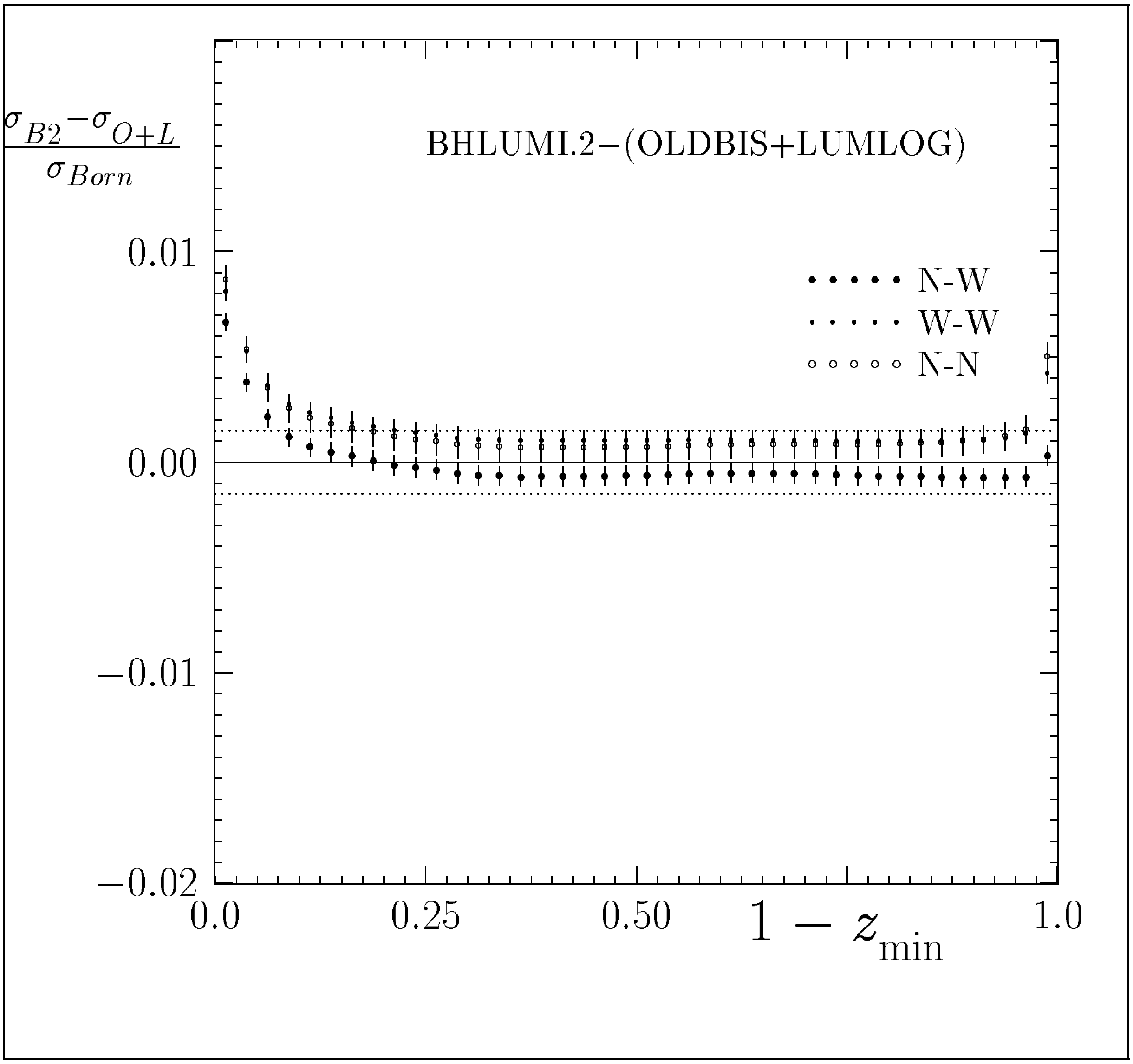

Figure 3: *Comparison of BHLUMI version 4.0 (unpublished) with semi-analytical formula for "academic" trigger defined with* $|t_{min}| < |t| < |t_{max}|$ *and* $V < V_{max}$. *Dotted lines mark the 0.15% limit.*

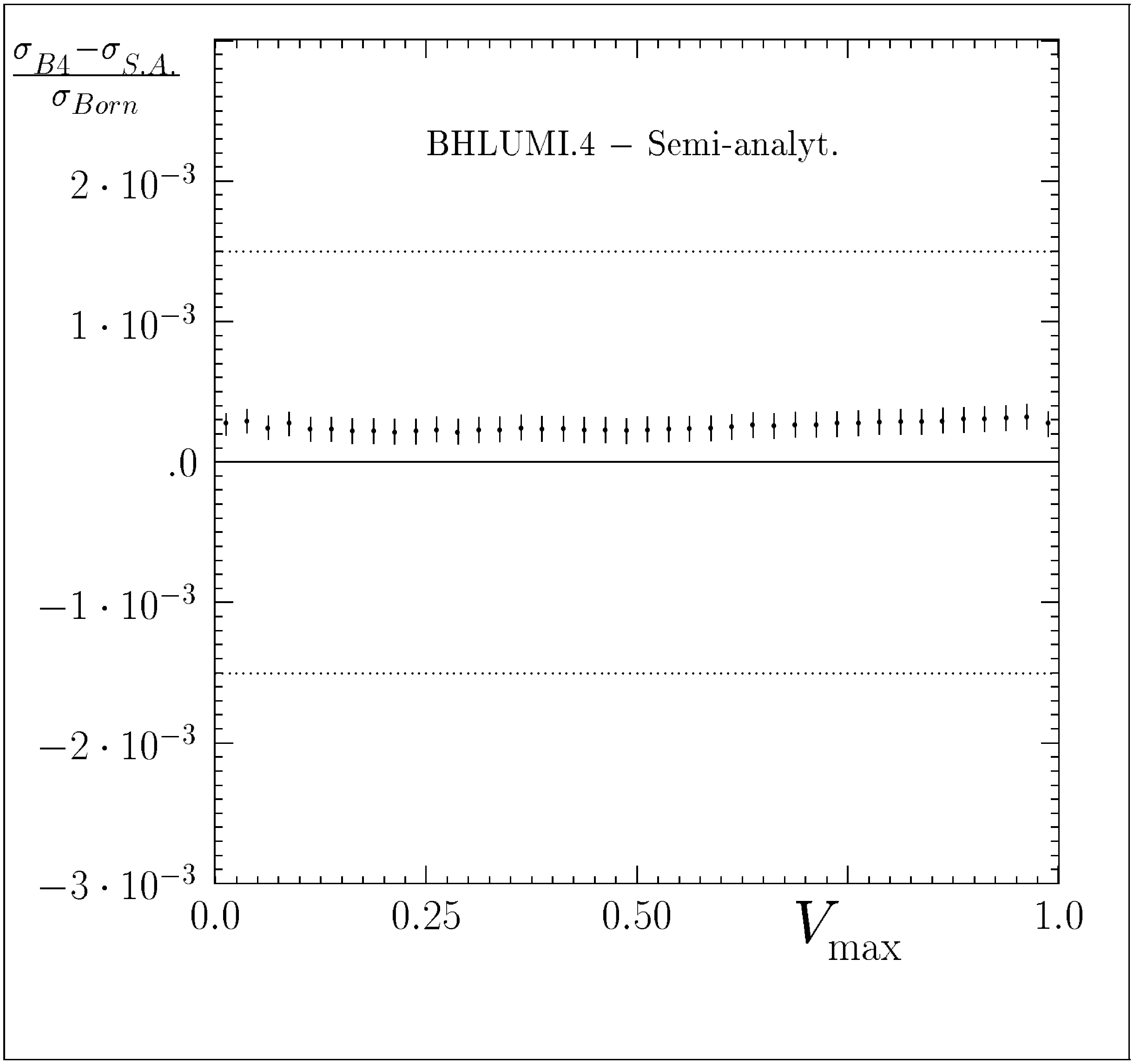

Figure 4: *The same plot as in Fig. 2 for the same kind of trigger and angular range, but using the new BHLUMI version 4.0 (unpublished). Dotted lines mark the same 0.15% limit.*