

Charged black holes in two-dimensional string theory

Michael D. McGuigan * and Chiara R. Nappi **

Institute for Advanced Study, School of Natural Sciences, Princeton, NJ 08540, USA

Scott A. Yost ***

University of Florida, Department of Physics, Gainesville, FL 32611, USA

Received 25 November 1991

Accepted for publication 23 December 1991

We discuss two-dimensional string theories containing gauge fields, introduced either via coupling to open strings, in which case we get a Born–Infeld type action, or via heterotic compactification. The solutions of the modified background field equations are charged black holes, and we compute their masses and charges. We also discuss the possibility of novel space-time geometries.

1. Introduction

Recently, black hole solutions to two-dimensional string theory have attracted much interest [1–6]. The black hole discussed so far is a solution of the bosonic closed string theory on the sphere, and it is neutral. It is interesting to ask what happens to the black hole in string theories which include a gauge field, permitting it to have a charge. It is also interesting to ask how loop corrections to the string theory modify the conformally invariant tree-level solutions. Both of these modifications appear naturally in the open string case, since the gauge field couples to gravity through loop effects [7,8]. Therefore we compute the corrections to the black hole solution which come from adding holes and crosscaps to the closed string worldsheet and coupling a gauge field to the boundary. As in refs. [7,8], we work in the approximation of a slowly-varying electromagnetic field, and the effective action for the gauge field is then of the Born–Infeld type. We solve the beta functions of the two-dimensional string theory that are valid at the leading

* Research supported in part by the Department of Energy, contract DE-FG02-90ER-40542.
Present address: University of Florida, Department of Physics, Gainesville FL 32611, USA.

** Research supported in part by the Ambrose Monell Foundation.

*** Research supported in part by the Department of Energy, contract DE-FG05-86ER-40274.
Present address: University of Tennessee, Department of Physics, Knoxville TN 37996, USA.

order in α' , and find a charged black hole solution. For completeness and comparison we also discuss a two-dimensional heterotic string solution, which also gives rise to a charged black hole. Of course in this case the gauge field comes in at the tree level. However the black hole solution is of the same general type as those obtained from loop corrections.

This paper is organized as follows. In sect. 2 we review the black hole solution of two-dimensional string theory, and show how it is modified in the presence of generic loop corrections. In sect. 3 we derive the specific modification to the string tree-level equation of motion that arise from coupling gauge fields to open strings. As in the critical string, these modifications can be derived by standard background field calculations, but they can really be understood only if one interprets them as loop corrections. Therefore, in sect. 4, as in ref. [9], we actually perform a loop calculation by building the boundary operator for holes and crosscaps, and requiring the BRST anomaly of the tree-level state to cancel against that of the boundary operator. This calculation is similar to that of ref. [9], but somehow more subtle since we are in two dimensions. The results are the same as those from background field calculations. In sect. 5 we find an exact solution for the Born–Infeld black hole. In sect. 6 we discuss and solve the heterotic theory. In sect. 7 we discuss the geometric interpretation of these black holes. In some cases, the solutions look very similar to Reissner–Nordstrom black holes, although the parameters in the solution often have a different interpretation. For example, the role of the charge in the Reissner–Nordstrom solution may be played by the cosmological constant instead. In the presence of generic loop corrections, one could expect new space-time geometries, with Penrose diagrams that tile the plane with black holes. Finally, in sect. 8, we derive the masses and charges in the various cases.

2. The black hole solution

In ref. [1] it was proved that the black hole in a solution of an exact conformal field theory, namely the WZW model with gauge group $SL(2, \mathbb{R})/U(1)$. However, a black hole solution can be derived also by solving the two-dimensional beta function equations for the graviton–dilaton system at the leading order in α' , as in ref. [2]. To leading order in α' , the beta functions of the closed bosonic string in D dimensions are [10]

$$\beta_{\mu\nu}^G = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi = 0, \quad (2.1)$$

$$\beta^\Phi = -R - 4\nabla^2\Phi + 4(\nabla\Phi)^2 + c = 0, \quad (2.2)$$

where $c = 2(D - 26)/3\alpha'$. We are assuming here that the tachyon field is zero, and drop it from the equations. Our curvature conventions follow ref. [11], as in ref. [10].

We look for a metric of the form

$$G_{tt} = -g(r), \quad G_{rr} = \frac{1}{g(r)}, \tag{2.3}$$

where t is a timelike coordinate and r is spacelike, and assume that $\Phi(r)$ is static. Then the graviton beta functions become

$$\beta_{tt}^G = \frac{g}{2}(g'' - 2g'\Phi'), \tag{2.4}$$

$$\beta_{rr}^G = 2\Phi'' - \frac{1}{2g}(g'' - 2g'\Phi'), \tag{2.5}$$

and the dilaton beta function becomes

$$\beta^\Phi = g'' + 4g(\Phi')^2 - 4g\Phi'' - 4g'\Phi' + c, \tag{2.6}$$

where primes denote derivatives with respect to r . Also, $\beta_{tr}^G = 0$ identically.

Together, eqs. (2.4) and (2.5) imply that $\Phi'' = 0$, so the dilaton background takes the form

$$\Phi = \Phi_0 - \frac{1}{2}Qr. \tag{2.7}$$

Then a linear combination of eqs. (2.4) and (2.6) gives

$$\beta^\Phi - \frac{2}{g}\beta_{tt}^G = Qg' + Q^2g + c = 0, \tag{2.8}$$

with the family of solutions $g(r) = -(cQ^{-2} + 2m e^{-Qr})$. Requiring the metric to be asymptotically flat for $r \rightarrow \infty$ fixes $Q = \sqrt{-c}$. It was shown in ref. [1] that this solution is the two-dimensional version of the four-dimensional Schwarzschild solution. (Notice that this solution would be trivial if $c = 0$.) The free parameter m is proportional to the mass, as described in sect. 8.

String loops can modify the beta functions [12,13]. To zeroth order in derivatives of the fields, the effect is simply to add a cosmological term

$$S_{\text{loop}} = \int d^Dx \sqrt{-G} \Lambda e^{2(n-1)\Phi} \tag{2.9}$$

to the effective action, weighted by the dilaton factor appropriate for n handles [8,13]. From these corrections to the effective action, we can infer corrections to the beta functions. Specifically, eqs. (2.1) and (2.2) take the form

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi = \frac{1}{2}n\Lambda G_{\mu\nu} e^{2n\Phi},$$

$$-R - 4(\nabla^2\Phi) + 4(\nabla\Phi)^2 + c = (1-n)\Lambda e^{2n\Phi}, \quad (2.10)$$

where n is chosen to represent the leading non-vanishing loop correction.

Then the r.h.s. of (2.8) becomes $\Lambda e^{2n\Phi_0} e^{-nQr}$, and the generic solution is

$$g(r) = \begin{cases} 1 - 2m e^{-Qr} + b_n e^{-nQr} & \text{if } n \neq 1 \\ 1 - 2m e^{-Qr} + b_1 Qr e^{-Qr} & \text{if } n = 1, \end{cases} \quad (2.11)$$

where

$$b_n = \frac{\Lambda}{c(n-1)} e^{2n\Phi_0} \quad \text{for } n \neq 1, \quad b_1 = -\frac{\Lambda}{c} e^{2\Phi_0}. \quad (2.12)$$

The most general loop-generated dilaton potential is a linear combination of the terms on the r.h.s. of eq. (2.10), and the black hole metric includes a sum over the b_n terms in eqs. (2.11) and (2.12). It is interesting to point out that (2.11) implies that in the presence of loop corrections flat space is no longer allowed as a solution.

We will show that open strings obey non-linear Born-Infeld electrodynamics. For hole and crosscap-type corrections [9,14] in the absence of a gauge field, it will turn out that $n = 1/2$ in the above formulas. Open strings with a weak electric charge but no cosmological constant will have $n = 3/2$.

Heterotic strings have charged solutions with an electromagnetic field which will mimic loop corrections with $n = 2$, so they also fit into this general class of corrections to the black hole. The first closed-string loop corrections (torus-level) will have $n = 1$. Although this case is very interesting, since closed-string loop corrections appear in every string theory, we will not discuss it here any further, except in sect. 8 when we compute masses.

3. Open strings in two dimensions

The original means of introducing a gauge field into string theory was to couple it to charges on the ends of an open string [15]. In this section, we will derive the open-string beta functions for the $D = 2$ string, and use them to find the equations which must be satisfied by a black hole coupled to an abelian gauge field.

We first consider the simplest case, that of a U(1) electromagnetic field. The combined open and closed string worldsheet action is [8]

$$I = \frac{1}{4\pi\alpha'} \int d^2z \sqrt{-\gamma} \left\{ \gamma^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)} \Phi(X) + \frac{\alpha'}{\epsilon^2} T(X) \right\} + \frac{1}{2\pi\alpha'} \oint ds \left\{ A_\mu(X) \frac{\partial}{\partial s} X^\mu + \alpha' k \Phi(X) + \frac{\alpha'}{\epsilon} \Theta(X) \right\}, \quad (3.1)$$

where T and Θ are respectively the closed and open string ‘‘tachyons’’. The bare tachyon couplings include factors of a worldsheet cutoff ϵ with dimension of length [16]. The gauge field A_μ has been rescaled by a factor of $2\pi\alpha'$, so that $F_{\mu\nu}$ is dimensionless. The term containing the extrinsic worldsheet curvature k is needed to complete the coupling of the dilaton to the Euler density.

Demanding conformal invariance of the worldsheet action (3.1) requires two new beta functions, β_μ^A and β^Θ , to vanish. This implies new equations of motion, which to lowest order in derivatives of the gauge fields are given by

$$\beta_\mu^A = (G - F^2)^{-1}_{\lambda\nu} \nabla^\nu F_\mu^\lambda + \nabla^\nu \Phi F_{\nu\mu} = 0, \quad (3.2)$$

$$\beta^\Theta = (G - F^2)^{-1}_{\mu\nu} \nabla^\mu \nabla^\nu \Theta - \nabla^\mu \Phi \nabla_\mu \Theta + (1/\alpha') \Theta = 0. \quad (3.3)$$

Up to Θ -dependent terms in eq. (3.2), these are the variational equations of an open-string contribution

$$S_{\text{open}} = -\kappa \int d^2x e^{-\Phi} \sqrt{-\det(G + F)} \left\{ 1 + \alpha' (G - F^2)^{-1}_{\mu\nu} \nabla^\mu \Theta \nabla^\nu \Theta - \Theta^2 \right\} \quad (3.4)$$

to the effective action, where κ is a positive open-string coupling constant with dimensions of inverse-length-squared*. The gauge-field part of eq. (3.4) is the Born–Infeld action for non-linear electrodynamics [7,17]. The presence of boundary fields in (3.1) does not affect [7,8] the beta functions (2.1) and (2.2), so that the closed-string tree-level effective action [10,18] remains

$$S_{\text{closed}} = \int d^2x \sqrt{-G} e^{-2\Phi} \left\{ R + 4(\nabla\Phi)^2 - (\nabla T)^2 + (4/\alpha') T^2 - c \right\}. \quad (3.5)$$

Like the tachyon contributions to the graviton and dilaton beta functions [16], the Θ -terms in β^A require a sum over all loop-orders in the sigma model. We will not pursue this here, but simply note that eqs. (3.2)–(3.4) can be expected to receive higher-order corrections in Θ , just as the closed-string action contains

* If κ were negative, the gauge field would couple to gravity with the wrong sign. This is a consequence of our curvature conventions, which follow refs. [10,11].

non-trivial higher-order tachyon interactions. The tachyons in the effective action are renormalized with respect to the bare tachyon backgrounds in the worldsheet action [16].

As in the closed string case, we will consider a linear dilaton background of the form (2.7). Then the field redefinitions $T = e^\Phi \hat{T}$, $\Theta = e^{\Phi/2} \hat{\Theta}$ remove the negative-mass terms from the tachyon equations of motion, showing that both ‘‘tachyons’’ are actually massless when $D = 2$, in agreement with the expected spectrum [19]. The massless open and closed string tachyons are the physical degrees of freedom of the $D = 2$ open string. The metric, dilaton, and gauge field are non-dynamical backgrounds. We will look for black hole solutions without tachyon backgrounds, and drop T and Θ in the following.

The total space-time effective action associated with (3.1) is obtained [8] by adding (3.4) to (3.5). Then the equation of motion derived from total action are

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi + \frac{1}{4}\kappa e^\Phi \sqrt{\det(1 + G^{-1}F)} \left(\frac{G + F^2}{G - F^2} \right)_{\mu\nu} = 0,$$

$$-R + 4(\nabla\Phi)^2 - 4\nabla^2\Phi + c + \frac{1}{2}\kappa e^\Phi \sqrt{\det(1 + G^{-1}F)} = 0. \quad (3.6)$$

The above results are based on background field calculations. They rely on an expansion in derivatives of the fields, whose momenta are therefore supposed to be small. However, due to the linear dilaton background, and the fact that in two dimensions, the backgrounds are massive in the world-sheet sense (except the tachyon), such an expansion might be unjustified. In sect. 4 we will show that the corrections to the beta functions inferred from putting together the effective actions (3.4) and (3.5) can be interpreted as loop corrections due to the effect of small holes and crosscaps in the world-sheet. This loop calculation is of interest of its own merits, since it involves massive modes of the string, and is a nice check of the results of background field calculations.

4. Loop corrections from the boundary operator

The basic ideas and methods of our derivation in this section are the same as in the critical case [8,9,12–14], but with modifications due mostly to the linear dilaton background. Namely, we impose the BRST invariance of the closed-string field state and get the string tree-level equations of motion (2.1) and (2.2), as in ref. [20]. Then we represent the insertion of a hole in the worldsheet via a boundary operator [9,14], and require the BRST invariance of the sum of both the closed string tree-level state and the boundary operator. This will give the loop-corrected equations as in refs. [8,9,21].

In this calculation, we will use a weak-field expansion for the closed-string backgrounds. We first derive the tree-level beta function starting with the string state

$$|\Psi\rangle = \left\{ \hat{T}(x) + \hat{h}_{\mu\nu}(x)\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu + \hat{\phi}(x)(c_{-1}\tilde{b}_{-1} - b_{-1}\tilde{c}_{-1}) \right\} c_1 \tilde{c}_1 |iQ/2\rangle, \quad (4.1)$$

where $|iQ/2\rangle = e^{-Q \cdot x/2} |0\rangle$ is the proper Liouville gravitational dressing factor for $D=2$ strings [22]. The fields $\hat{h}_{\mu\nu}$ and $\hat{\phi}$ are perturbations around the background graviton and dilaton, and the hats are a reminder that they are related to those in the sigma model lagrangian by a field redefinition, yet to be determined. The $c_1 \tilde{c}_1 e^{-Q \cdot x/2}$ factor in (4.1) is what converts the $SL_2(\mathbb{R})$ vacuum $|0\rangle$ to a puncture vacuum, appropriate for attaching a fixed vertex [9,14,22].

Before discussing the BRST invariance conditions, it is useful to write the mode expansion of the coordinates X^μ . The closed-string coordinates may be expanded as [23]

$$X^\mu(\sigma, \tau) = x^\mu + 2\alpha' P^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \left\{ \alpha_n^\mu e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)} \right\}. \quad (4.2)$$

The linear dilaton background makes the definition of P^μ somewhat subtle. The operator X^μ is hermitian, so P^μ must be hermitian as well. If $|0\rangle$ is the ordinary $SL_2(\mathbb{R})$ vacuum and $|p\rangle \equiv e^{ip \cdot X(0)} |0\rangle$, then the linear dilaton background implies [23,24] that there is a background momentum: $\langle p + iQ | p \rangle = 1$. Consequently the momentum in eq. (4.2) must be shifted by half the background charge, so that $P^\mu |p\rangle = (p - iQ/2)^\mu |p\rangle$.

The Virasoro generators are now

$$L_n^X = \frac{1}{2} \sum_m \eta_{\mu\nu} : \alpha_m^\mu \alpha_{n-m}^\nu : - \frac{in}{2} \sqrt{\frac{\alpha'}{2}} Q_\mu \alpha_n^\mu - \frac{D}{24} \delta_{n0}, \quad (4.3)$$

$$L_n^{\text{ghost}} = \sum_m (m-n) : c_m b_{n-m} : + \frac{1}{12} \delta_{n0}, \quad (4.4)$$

and the BRST charge is

$$Q_{\text{BRST}} = \sum_n c_{-n} \left(L_n^X + \frac{1}{2} L_n^{\text{ghost}} \right). \quad (4.5)$$

Here, $\alpha_0^\mu = \sqrt{\alpha'/2} P^\mu$. The additional Q -dependent terms are from the contribution of the linear dilaton background to the stress tensor. Such terms were originally found for Liouville theory in ref. [25]. These terms are analogous to ones appearing for bosonized ghosts, for the same reason [23,26]. The ghost part is unchanged from the critical case, and the right-movers are defined analogously.

Applying the technique of ref. [9] with the modified Virasoro operators, we find that $(Q + \tilde{Q})_{\text{BRST}}|\Psi\rangle = 0$ implies

$$\begin{aligned}\frac{1}{4}\alpha'\nabla^2\hat{h}_{\mu\nu} - \hat{h}_{\mu\nu} &= 0, \\ \frac{1}{4}\alpha'\nabla^2\hat{\phi} - \hat{\phi} &= 0.\end{aligned}\tag{4.6}$$

The BRST gauge conditions coming from $L_{\pm 1}$ terms in Q_{BRST} are

$$\begin{aligned}(\partial + \frac{1}{2}Q)^\nu\hat{h}_{(\mu\nu)} - (\partial - \frac{1}{2}Q)_\mu\hat{\phi} &= 0, \\ (\partial + \frac{1}{2}Q)^\nu\hat{h}_{[\mu\nu]} &= 0,\end{aligned}\tag{4.7}$$

where the parentheses and brackets around the indices denote symmetrization and antisymmetrization, respectively. These equations need to be compared with the linearized version of (2.1) and (2.2). To this purpose, as in ref. [9], we will use a weak-field expansion

$$\begin{aligned}G_{\mu\nu}(X) &= \eta_{\mu\nu} + h_{\mu\nu}(X), \\ \Phi(X) &= \Phi_0 - \frac{1}{2}Q_\mu X^\mu + \phi(X)\end{aligned}\tag{4.8}$$

with $Q_\mu = Q\delta_{\mu r}$ as in (2.7). Substituting (4.8) into the beta functions (2.1) and (2.2), we find

$$\beta_{\mu\nu}^G = -\frac{1}{2}(\partial + \frac{1}{2}Q)^2 h_{\mu\nu} + \frac{1}{8}Q^2 h_{\mu\nu} + \frac{1}{2}(\partial_\mu J_\nu + \partial_\nu J_\mu),\tag{4.9}$$

$$\beta^\Phi + \eta^{\mu\nu}\beta_{\mu\nu}^G = (\partial + \frac{1}{2}Q)^2\phi - \frac{1}{4}Q^2\phi - J_\mu(\partial^\mu\phi - \frac{1}{2}Q^\mu),\tag{4.10}$$

where

$$J_\mu = (\partial + Q)^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h^\nu{}_\nu + 2\partial_\mu\phi.\tag{4.11}$$

Fixing the gauge $J_\mu = 0$ and making the field redefinitions

$$\begin{aligned}\hat{h}_{\mu\nu} &= e^{\Phi_0 - \Phi} h_{\mu\nu}, \\ \hat{\phi} &= e^{\Phi_0 - \Phi} (\frac{1}{2}h^\nu{}_\nu - 2\phi)\end{aligned}\tag{4.12}$$

reproduces eq. (4.6), while the gauge condition $J_\mu = 0$ turns into (4.7). Note that $\hat{\phi}$ is a linear combination of the dilaton vertex and trace of the graviton [9,20], and the $e^{\Phi_0 - \Phi}$ scaling is the same as for the tachyon. This scaling is associated with the Euler characteristic of a puncture.

Although we have not discussed the antisymmetric tensor before, we may note that the gauge condition (4.7) on the antisymmetric part of $\hat{h}_{\mu\nu}$ implies that the antisymmetric part of eq. (4.6) is identically zero (remember $Q^2/4 = 4/\alpha'$), so that at the tree-level the antisymmetric tensor beta function vanishes identically. This has important consequences, which we will come back to.

We now compute the loop corrections. The boundary state that describes the insertion of a hole on the world-sheet with abelian gauge field attached is [8,9,21]

$$|B\rangle_F = \lambda e^{\Phi_0} \sqrt{-\det[\eta(x) + F(x)]} \times \exp\left\{-\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\eta - F}{\eta + F}\right)_{\mu\nu} \alpha_{-n}^{\mu} \tilde{\alpha}_{-n}^{\nu}\right\} (c_0 + \tilde{c}_0) |iQ/2\rangle_X |B\rangle_{\text{ghost}} \quad (4.13)$$

since $|iQ/2\rangle$ satisfies the zero-mode condition $P^\mu = 0$. The presence of a background momentum makes this reminiscent of the boundary states for bosonized ghosts [9] and superghosts [27]. The ghost boundary state is the same as in the critical case [9]. The open-string coupling constant λ will be proportional to κ in (3.4). The state $|B\rangle$ is attached to a world-sheet via a cylinder. We need to integrate over the modulus (length) τ of the attached cylinder, which has the effect of multiplying $|B\rangle$ by a closed-string propagator, and also need to insert the required ghost zero-modes (which eventually will combine with those in (4.13) to reproduce the appropriate vacuum [9]). Therefore the complete state for inserting a boundary in the world-sheet is [9]

$$|F\rangle = -(b_0 + \tilde{b}_0) \int_0^\infty d\tau e^{-\tau(L_0 + \tilde{L}_0)} |B\rangle_F = -(b_0 + \tilde{b}_0) (L_0 + \tilde{L}_0)^{-1} |B\rangle_F. \quad (4.14)$$

We need to remember [9] that the state $|B\rangle$ itself is BRST-invariant by virtue of its boundary conditions, and that

$$\{(Q + \tilde{Q})_{\text{BRST}}, b_0 + \tilde{b}_0\} = L_0 + \tilde{L}_0. \quad (4.15)$$

Therefore commuting $(Q + \tilde{Q})_{\text{BRST}}$ past the ghost insertion in (4.14), produces an inverse propagator, which cancels the propagator in (4.14). This cancellation presents no problem for the massive modes, and for them the correction to the tree-level equations (4.6) can be read directly from the boundary operator, and no divergence calculation is involved. (This is not too striking. In the BRST approach we derived directly eqs. (4.6) without computing any divergence, while in the sigma model approach the same equations emerged from a beta function calculation.) However the cancellation is more problematic for the massless modes, since in this

case one gets a zero over zero ambiguity which can be dealt with by some regularization procedure, for instance giving the boundary state an off-shell momentum as in ref. [21].

Therefore, requiring as in critical case [9] that the *sum* of (4.1) and (4.14) be BRST invariant one gets here for the massless tachyon

$$\frac{1}{4}\alpha'\nabla^2\hat{T} = \lambda e^{\Phi_0}\sqrt{-\det(\eta + F)}, \tag{4.16}$$

including the *F*-dependence. This is completely analogous to the derivation of the loop corrections for massless modes in the critical string.

Similarly one derives for the massive modes the loop corrected equations

$$\begin{aligned} \frac{1}{4}\alpha'\nabla^2\hat{h}_{\mu\nu} - \hat{h}_{\mu\nu} &= -\lambda e^{\Phi_0}\sqrt{-\det(\eta + F)} \left(\frac{\eta + F^2}{\eta - F^2} \right)_{\mu\nu}, \\ \frac{1}{4}\alpha'\nabla^2\hat{\phi} - \hat{\phi} &= \lambda e^{\Phi_0}\sqrt{-\det(\eta + F)} \end{aligned} \tag{4.17}$$

while the gauge condition (4.7) receives no loop corrections.

The loop corrections (4.17) are exactly what are needed to explain the extra terms in the graviton and dilaton beta functions which can be inferred from the open-string effective action (3.4) (here we restrict ourselves to the case $\Theta = 0$). Indeed, eqs. (4.17) agree with the linearized equations of motion (3.6) if $\kappa = 8\lambda/\alpha'$. This agreement confirms the validity of our procedure to obtain loop correction to massive, as well as massless states. In eq. (4.16) \hat{T} is the massless tachyon, which is related to the sigma model tachyon by $\hat{T} = T e^{\Phi_0 - \Phi}$. This relation is consistent with the tachyon field redefinition discussed earlier. (Subtracting Φ_0 in the field redefinition is optional. It is done to maintain the connection between e^Φ and the string-loop coupling.) The tachyon correction (4.16) may be accounted for by adding a term

$$-\kappa e^\Phi T(X)\sqrt{-\det(G + F)} \tag{4.18}$$

to the effective lagrangian, whose variational equation cancels the r.h.s. of (4.16).

A remark is in order here. On the r.h.s. of (4.17), we have symmetrized the matrix $((\eta - F)/(\eta + F))_{\mu\nu}$ which comes from the boundary state. Its antisymmetric part may be considered to be the correction to the equation for the antisymmetric tensor field $B_{\mu\nu}$. However, as noted earlier, since its field strength $H_{\lambda\mu\nu}$ vanishes identically in $D = 2$, the closed-string tree-level equations are independent of $B_{\mu\nu}$. Therefore the antisymmetric part of the boundary corrections must vanish too, which can happen only if $F = 0$.

The same conclusion can be reached also in the approach of sect. 3. Indeed in the presence of a boundary, the antisymmetric tensor enters the beta functions and effective action via the substitution $F_{\mu\nu} \rightarrow F_{\mu\nu} + B_{\mu\nu}$ everywhere [8]. Varying the action with respect to $B_{\mu\nu}$ gives the antisymmetric tensor beta function, which turns out to imply $F + B = 0$. Therefore, the gauge field can be gauge away.

This problem can be solved by considering a non-abelian gauge group instead, and choosing a background in an abelian subgroup. Orientable open strings can couple to $U(N)$, while unorientable ones can couple to $SO(N)$ or $Sp(N)$, where in the latter case, N is even [28]. In the orientable case, the antisymmetric tensor is simply absent. In the unorientable case, it will eat a $U(1)$ factor by the process described above, leaving an $SU(N)$ group and no antisymmetric tensor.

In the case of non-orientable strings, one needs to take in account also the crosscap contribution which appears at the same order in the loop expansion as the boundary state. The insertion of a cross-cap on the world-sheet can be represented by the operator [9,14]

$$|C\rangle = 2^{D/2} \eta \lambda e^{\Phi_0} \exp \left\{ \sum_{n=1}^{\infty} (-)^{n+1} \frac{1}{n} \eta_{\mu\nu} \alpha_{-n}^{\mu} \tilde{\alpha}_{-n}^{\nu} \right\} |iQ/2\rangle_X |C\rangle_{\text{ghost}}, \quad (4.19)$$

which will also contribute to eqs. (4.17).

If one takes the background field in an abelian subgroup, the final form of the effective action is indeed (omitting the tachyon)

$$S_{\text{open}} = -\kappa \int d^D x e^{-\Phi} \left\{ \text{Tr} \sqrt{-\det(G + F)} + \eta 2^{D/2} \sqrt{-G} \right\}, \quad (4.20)$$

where $D = 2$ for our purposes, and $\eta = 0, -1, \text{ or } 1$ when the group is $SU(N)$, $SO(N)$ or $Sp(N)$, respectively. The trace is over the group indices, which are suppressed, and the term proportional to $2^{D/2}$ comes from including the crosscap contribution.

It is useful to choose a particular abelian subgroup, so that we can evaluate the trace in (4.20). We will take the background to be in abelian subgroup with a single generator λ^a . The group elements are represented by $N \times N$ matrices, and we normalize the generators by $\text{Tr} \lambda^a \lambda^b = 2\delta^{ab}$. The generator λ^a of the abelian subgroup can be chosen so that

$$\text{Tr} 1 = N, \quad \text{Tr} F^{2n} = 2F^{2n}, \quad \text{Tr} F^{2n+1} = 0. \quad (4.21)$$

Using (4.21) and the fact that $\det(G + F)$ is even in F (since F is antisymmetric) we rewrite the open-string part of the action as

$$S_{\text{open}} = -\kappa \int d^2 x e^{-\Phi} \left\{ 2\sqrt{-\det(G + F)} + (N - 2 + 2\eta)\sqrt{-G} \right\}. \quad (4.22)$$

The equations of motion derived by adding (4.22) to (3.5) are now

$$\begin{aligned}
 R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi + \frac{1}{2}\kappa e^\Phi\sqrt{\det(1+G^{-1}F)}\left(\frac{G+F^2}{G-F^2}\right)_{\mu\nu} + \frac{1}{4}\kappa(N+2\eta-2)e^\Phi G_{\mu\nu} &= 0, \\
 -R + 4(\nabla\Phi)^2 - 4\nabla^2\Phi + c + \kappa e^\Phi\sqrt{\det(1+G^{-1}F)} + \frac{1}{2}\kappa(N+2\eta-2)e^\Phi &= 0.
 \end{aligned}
 \tag{4.23}$$

Comparing eqs. (4.22) and (4.23) for $F=0$ with the general forms (2.9) and (2.10) shows that the open-string contributes a cosmological constant $\Lambda = -\kappa(N+2\eta)$, which vanishes when the gauge group is $\text{SO}(2)$. We will solve eqs. (4.23) in sect. 5.

5. Open-string Born–Infeld black holes

We will now search for a charged black hole solution to the background field equations (4.23) for the open $\text{SU}(N)$, $\text{SO}(N)$, or $\text{Sp}(N)$ string, with an abelian background $F_{\mu\nu}$. In two dimensions, the field strength has just one independent component $F_{r\tau} = -F_{\tau r} = f$. The gauge-field beta function (3.2) is

$$\beta_r^A = (1-f^2)^{-1}f' - \Phi'f. \tag{5.1}$$

Note that β_t^A vanishes identically.

Rewriting the beta functions (4.23) using the metric (2.3) gives the equations

$$\begin{aligned}
 -\frac{1}{2}g'' + 2g'\Phi' &= -\frac{1}{2}g'' + 2g'\Phi' + 2g\Phi'' \\
 &= -\frac{1}{2}\kappa e^\Phi\left\{\frac{1+f^2}{\sqrt{1-f^2}} + \frac{1}{2}N + \eta - 1\right\}, \\
 g'' + 4g(\Phi')^2 - 4g\Phi'' - 4g'\Phi' + c &= -\kappa e^\Phi\left\{\sqrt{1-f^2} + \frac{1}{2}N + \eta - 1\right\},
 \end{aligned}
 \tag{5.2}$$

for the metric and dilaton background. The graviton beta functions again imply that that $\Phi'' = 0$, so the ansatz (2.7) is still appropriate. The equations to solve are then

$$(f^2)' = -Qf^2(1-f^2), \tag{5.3}$$

$$g' + Qg = -\frac{c}{Q} - \frac{2\hat{\kappa}}{Q}e^\Phi\left\{(1-f^2)^{-1/2} + \frac{1}{2}N + \eta - 1\right\}. \tag{5.4}$$

The solution to (5.3) must be positive and vanish at infinity. A family of solutions is

$$f^2(r) = q^2(e^{2r} + q^2)^{-1}. \tag{5.5}$$

Substituting this into (5.4) gives

$$(g(r) e^{Qr})' = -\frac{c}{Q} e^{Qr} + \frac{\hat{\kappa}}{Q} (2 - 2\eta - N) e^{Qr/2} - \frac{\hat{\kappa}}{Q} \sqrt{e^{Qr} + q^2}, \tag{5.6}$$

where $\hat{\kappa} = \kappa e^{\Phi_0}$. Integrating eq. (5.6) and using $Q^2 = -c$ gives

$$g(r) = 1 - 2m e^{-Qr} + \frac{2\hat{\kappa}}{c} (N + 2\eta - 2) e^{-Qr/2} + \frac{4\hat{\kappa}q}{c} e^{-Qr} \left\{ \sqrt{q^{-2} e^{Qr} + 1} - \sinh^{-1}(q e^{-Qr/2}) \right\}. \tag{5.7}$$

The curvature is then

$$R = -2mc e^{-Qr} + \left(\frac{1}{2}N + \eta - 1\right)\hat{\kappa} e^{-Qr/2} + \hat{\kappa}(e^{Qr} + q^2)^{-1/2} - 4\hat{\kappa}q e^{-Qr} \sinh^{-1}(q e^{-Qr/2}). \tag{5.8}$$

This has a singularity at $r = -\infty$, which is where the usual curvature singularity occurs. Event horizons occur when $g(r) = 0$. In general there will be more than one, as for the usual charged black hole.

To first order in q^2 , the black hole (3.4) corresponds to a solution coupled to ordinary linear electrodynamics with a cosmological constant,

$$g(r) = 1 - 2m e^{-Qr} + \frac{2\hat{\kappa}}{c} (N + 2\eta) e^{-Qr/2} - \frac{2\hat{\kappa}q^2}{c} e^{-3Qr/2}. \tag{5.9}$$

When $q = 0$, the pure cosmological constant correction is present, and has the form (2.11) with $n = 1/2$. The cosmological constant $\Lambda = -\kappa(N + 2\eta)$ may be cancelled by coupling non-orientable strings to SO(2), as noted in sect. 4.

The U(1) string has no gauge field, due to the fact that the antisymmetric tensor $B_{\mu\nu}$ acts as a Lagrange multiplier. However, it still has a cosmological constant, which the action (3.4) shows to be $\Lambda = -\kappa$. In that case, the modified black hole metric is of the form (2.11) for $n = 1/2$, specifically,

$$g(r) = 1 - 2m e^{-Qr} + (\hat{\kappa}/c) e^{-Qr/2}. \tag{5.10}$$

6. Black holes in two-dimensional heterotic string theory

Heterotic strings provide an alternative way to couple a gauge field to string theory. In this section, we find solutions of the form of a two-dimensional charged black hole to a string theory heterotically compactified to two dimensions. In the two non-compact space-time dimensions the left-moving sector will be taken to be bosonic, and the right-moving sector supersymmetric, so that the heterotic sigma model coupling the string to the bosonic backgrounds of interest is [10]

$$I = \frac{1}{2\pi\alpha'} \int d^2z \, d\theta \left\{ G_{\mu\nu}(X) DX^\mu \bar{\partial} X^\nu - 4\alpha' (D\bar{\partial}\sigma) \Phi(X) + A_{\mu a}(X) DX^\mu \bar{\partial} J^a \right\}, \quad (6.1)$$

where J^a is the current of the internal gauge group, whose nature will be discussed shortly. Above we have used superfield notations for the fields in the right sector and the world-sheet conformal gauge $\gamma_{ab} = e^{2\sigma} \delta_{ab}$. We will assume that the background A_μ takes values in an abelian subgroup of the full gauge group, as in the open-string case.

The metric, dilaton and gauge field beta functions at string tree-level and to leading order in α' are given by [10]

$$\begin{aligned} \beta_{\mu\nu}^G &= R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - \frac{1}{2} F_\mu^\lambda F_{\nu\lambda}, \\ \beta^\Phi &= \frac{1}{4} F^2 - R + c + 4(\nabla\Phi)^2 - 4\nabla^2\Phi, \\ \beta_\nu^A &= \nabla_\mu F_\nu^\mu - 2\nabla_\mu \Phi F_\nu^\mu. \end{aligned} \quad (6.2)$$

These beta functions can all be derived from the effective action

$$S = \int d^2x \sqrt{-G} \, e^{-2\Phi} \left\{ R + 4(\nabla\Phi)^2 - c - \frac{1}{4} F^2 \right\}. \quad (6.3)$$

Unlike the heterotic string in critical dimensions, the central charge c will be non-zero here, which is crucial for the existence of two-dimensional black hole solutions, as in the bosonic case. The actual value of c may differ from the bosonic case, and will be given later. It is the only way in which (6.3) depends on the specific details of the heterotic string construction. The antisymmetric tensor background contributes only through its field strength $H_{\lambda\mu\nu}$, which vanishes in two dimensions, so it has been omitted.

We look for a metric of the form (2.3) and a dilaton background (2.7). The requirement that the beta functions vanish then reduces to

$$-g'' - Qg' + f^2 = 0, \quad (6.4)$$

$$(f e^{-2\Phi})' = 0, \tag{6.5}$$

where we have written $F_{tr} = -F_{rt} = f$. The second equation can be easily solved by $f(r) = \sqrt{2} Qq e^{-Qr}$. Then the first equation becomes

$$-g'' - Qg' + 2Q^2q^2 e^{-2Qr} = 0. \tag{6.6}$$

This is a special case of the equations considered in sect. 2 with solution

$$g = 1 - 2m e^{-Qr} + q^2 e^{-2Qr}. \tag{6.7}$$

As usual, asymptotic flatness requires $c = -Q^2$. Notice that (6.5) tells that a combination of the gauge field and the dilaton field is a constant. This seems to be a general feature of these two dimensional models, and a similar relation holds also in the Born–Infeld case as shown in (5.5). The solution (6.7) can have two horizons and is qualitatively different from the charged black hole of ref. [4], which was a solution of a $D = 2 + 1$ string theory compactified to two dimensions. The global structure of this solution as well as that of the half-loop open-string solution is discussed in sect. 7.

We need to describe the gauge group and internal space which gives rise to a heterotic theory of the type described above. Heterotic string compactifications with a linear dilaton background and tree-level cosmological constant have been constructed in ref. [29], but in *four* non-compact dimensions. Non-critical superstrings have also been discussed in refs. [30,31]. Our construction differs in that we choose a GSO projection which allows the tachyon to be present. For two non-compact dimensions this mode turns out to be massless, as in the bosonic string, and poses no problem.

Requiring that the total central charge of the left (right) ghost, non-compact space-time, dilaton background and internal field theories vanish yields the conditions

$$\begin{aligned} -26 + 2 + \frac{3}{2}\alpha'Q^2 + \tilde{c}_1 &= 0, \\ -15 + 3 + \frac{3}{2}\alpha'Q^2 + c_1 &= 0, \end{aligned} \tag{6.8}$$

where (\tilde{c}_1, c_1) are the (left, right) central charges of the internal theory. Subtracting the two equations in (6.8), we get $\tilde{c}_1 - c_1 = 12$. We take the internal charges to be $\tilde{c}_1 = 12, c_1 = 0$. Comparison with eqs. (6.8) then shows that $c = -Q^2 = -8/\alpha'$. Our construction could be interpreted as a heterotic construction with a two-dimensional superstring on the right and a 14-dimensional bosonic string on the left compactified down to two dimensions and giving rise to a rank-12 group. Here we take the internal theory on the left to consist of an $E_8 \times SO(8)$ or $SO(24)$ current algebra.

To be consistent at loop level, the appropriately GSO projected theory must be modular invariant. The approach we follow is to start with a left–right symmetric superstring theory with worldsheet fermions on the left and right, that is modular invariant. Then, by using Gepner’s construction [32], we will then obtain a modular invariant heterotic theory with the left worldsheet fermions replaced by the fermionic representation of a group current-algebra.

We imitate the construction of refs. [33,34], where new types of ten-dimensional superstring theories were found by imposing a new GSO projection which projects out the spin- $\frac{3}{2}$ gravitino. Our theory will not be supersymmetric and will contain a tachyon, but in $D = 2$ that state will turn out to be massless. We begin as in refs. [33,34], by choosing a modular invariant set of boundary conditions in σ and τ which consists of summing over the same boundary conditions for left and right movers. Namely, we project the (NS, NS) sector onto states with $(-)^{F+\tilde{F}} = 1$ and add a (R, R) sector also projected onto states with $(-)^{F+\tilde{F}} = 1$. In $d = D - 2$ transverse dimensions one obtains three sectors (NS, NS), $(\overline{\text{NS}}, \overline{\text{NS}})$ and (R, R), where $\overline{\text{NS}}$ refers to NS states of odd fermion-number. The partition function is the left–right symmetric theory with worldsheet fermions on the left and right is then

$$\begin{aligned} & \text{Tr}\left[\frac{1}{2}(1 + (-)^F) e^{2\pi i\tau M^2}\right]_{\text{NS}} \text{Tr}\left[\frac{1}{2}(1 + (-)^{\tilde{F}}) e^{2\pi i\tau \tilde{M}^2}\right]_{\text{NS}}^* \\ & + \text{Tr}\left[\frac{1}{2}(1 - (-)^F) e^{2\pi i\tau M^2}\right]_{\overline{\text{NS}}} \text{Tr}\left[\frac{1}{2}(1 - (-)^{\tilde{F}}) e^{2\pi i\tau \tilde{M}^2}\right]_{\overline{\text{NS}}}^* \\ & + \text{Tr}\left[\frac{1}{2}(1 + (-)^F) e^{2\pi i\tau M^2}\right]_{\text{R}} \text{Tr}\left[\frac{1}{2}(1 + (-)^{\tilde{F}}) e^{2\pi i\tau \tilde{M}^2}\right]_{\text{R}}^*, \end{aligned} \tag{6.9}$$

where M^2, \tilde{M}^2 are the right and left (mass)² operators.

Performing the traces in $d = D - 2$ transverse dimensions, we obtain the modular invariant partition function

$$\begin{aligned} & 2^{d-2} \tau_2^{-d/2} \frac{1}{|\theta'_1|^d} \{ (\theta_3^{d/2} + \theta_4^{d/2})(\theta_3^{d/2} + \theta_4^{d/2})^* \\ & + (\theta_3^{d/2} - \theta_4^{d/2})(\theta_3^{d/2} - \theta_4^{d/2})^* + 2(\theta_2^{d/2})(\theta_2^{d/2})^* \} \\ & = 2^{d-1} \tau_2^{-d/2} \frac{1}{|\theta'_1|^d} (|\theta_3|^d + |\theta_4|^d + |\theta_2|^d). \end{aligned} \tag{6.10}$$

Now one can obtain a heterotic construction from the above theory following ref. [32]. The point is that the above expressions $\theta_3^{d/2} + \theta_4^{d/2}$, $\theta_3^{d/2} - \theta_4^{d/2}$ and $\theta_2^{d/2}$ are proportional to the characters of the $\text{SO}(d)$ helicity group current-algebra in the scalar, vector and spinor representation (denoted respectively by b_0 , b_v and $b_s + b_{\bar{s}}$). It turns out that the characters of an $\text{E}_8 \times \text{SO}(8 + d)$ or $\text{SO}(24 + d)$

current algebra in the scalar, vector and spinor representation (denoted respectively by B_0 , B_v and $B_s + B_{\bar{s}}$) transform in exactly the same way as the helicity characters provided one replaces b_0 by B_v , b_v by B_0 and $b_s + b_{\bar{s}}$ by $-B_s - B_{\bar{s}}$. This amounts to projecting onto states with $(-)^{F+\bar{F}} = -1$ in the NS sector of the fermionic formulation of $E_8 \times SO(8 + d)$ or $SO(24 + d)$. The partition function in the $SO(24 + d)$ heterotic string theory case is then given by

$$\begin{aligned} & \text{Tr}\left[\frac{1}{2}(1 + (-)^F) e^{2\pi i\tau M^2}\right]_{\text{NS}} \text{Tr}\left[\frac{1}{2}(1 - (-)^{\bar{F}}) e^{2\pi i\tau \bar{M}^2}\right]_{\text{NS},SO(24+d)}^* \\ & + \text{Tr}\left[\frac{1}{2}(1 - (-)^F) e^{2\pi i\tau M^2}\right]_{\text{NS}} \text{Tr}\left[\frac{1}{2}(1 + (-)^{\bar{F}}) e^{2\pi i\tau \bar{M}^2}\right]_{\text{NS},SO(24+d)}^* \\ & - \text{Tr}\left[\frac{1}{2}(1 + (-)^F) e^{2\pi i\tau M^2}\right]_{\text{R}} \text{Tr}\left[\frac{1}{2}(1 + (-)^{\bar{F}}) e^{2\pi i\tau \bar{M}^2}\right]_{\text{R},SO(24+d)}^*. \end{aligned} \quad (6.11)$$

Note that the overall minus sign for the (R, R) sector indicates that this is the fermionic portion of the partition function. Performing the traces in eq. (6.11), the partition function becomes

$$\begin{aligned} & 2^{2+2d/3} \tau_2^{-d/2} \frac{1}{|\theta'_1|^d} \frac{1}{(\theta'_1)^4} \left((\theta_3^{d/2} + \theta_4^{d/2})(\theta_3^{12+d/2} - \theta_4^{12+d/2})^* \right. \\ & \left. + (\theta_3^{d/2} - \theta_4^{d/2})(\theta_3^{12+d/2} + \theta_4^{12+d/2})^* + 2(\theta_2^{d/2})(-\theta_2^{12+d/2})^* \right), \end{aligned} \quad (6.12)$$

and is modular invariant. Again the GSO projection amounts taking an extra minus sign for $(-1)^{\bar{F}}$ in the NS sector of the fermionic formulation of $SO(24 + d)$. For the special case of $D = 2$, with no transverse dimensions, the partition function is given by

$$\frac{8}{(\theta'_1)^4} (\theta_3^{12} - \theta_4^{12} - \theta_2^{12})^*. \quad (6.13)$$

The first two theta functions give the bosonic partition function, and the last one is the fermionic partition function.

It can be shown quite generally using the transformation laws of ref. [32] that $\alpha B_v + \beta B_s + \gamma B_{\bar{s}}$ is a modular invariant combination of the characters of $SO(24)$ whenever $\alpha + \beta + \gamma = 0$. Our construction picks out precisely the combination $2B_v - B_s - B_{\bar{s}}$. The same combination of characters of $SO(8)$ times the singlet of E_8 yields a modular invariant partition function associated with the gauge group $E_8 \times SO(8)$.

We will now describe the low-lying states of the $SO(24)$ heterotic theory just constructed. The only physical particle is a massless ‘‘tachyon’’ transforming under the fundamental representation of $SO(24)$. The vertex operator for this state is

$e^{-\phi(z)}\psi^I(\bar{z}) e^{-Q \cdot X/2}$, where ϕ is the bosonized superghost and $i\psi^I\psi^J$ yields the fermionic realization of the current algebra of SO(24), with $I, J = 1, \dots, 24$. The notation $Q \cdot X$ is as in the open-string section. Only the r -component of Q is non-zero.

In addition, although they do not exist as particles, it is possible to associate vertex operators with the graviton, dilaton, antisymmetric tensor and gauge field, whose backgrounds can appear in the sigma model. The vertex for the gravity sector is $e^{-\phi(z)}\psi^\mu(z)\bar{\partial}X^\nu e^{-Q \cdot X/2}$. An SO(24) gauge field is associated with the operator $e^{-\phi(z)}\psi^\mu(z)(i\psi^I\psi^J)(\bar{z}) e^{-Q \cdot X/2}$.

In the (R, R) sector we have fermionic backgrounds associated with the vertex operators $e^{-\phi(z)/2}S^\alpha(z)S^A(\bar{z}) e^{-Q \cdot X/2}$ and $e^{-\phi(z)/2}S^{\dot{\alpha}}(z)S^{\dot{A}}(\bar{z}) e^{-Q \cdot X/2}$ where S^α ($S^{\dot{\alpha}}$) is the positive (negative) chirality spin operator of SO(2) and S^A ($S^{\dot{A}}$) is the positive (negative) chirality spin operator of SO(24).

7. Black hole geometry

The usual charged black hole metric is

$$ds^2 = - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2} \right) dt^2 + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2. \tag{7.1}$$

The asymptotic flat region is at $r = \infty$, and the curvature singularity at $r = 0$. The Reissner-Nordstrom (RN) geometry corresponds to the case where the two roots $r_\pm = m \pm \sqrt{m^2 - q^2}$ are both positive. The cases with $m < 0$ (both roots are negative), or the case $m^2 - q^2 < 0$ correspond to a naked singularity, since the singularity at $r = 0$ is not protected by an horizon. The case $r_- < 0 < r_+ < \infty$ is not usually considered since it would correspond to imaginary q . This would however be a Schwarzschild type of black hole, since one of the horizons is behind the singularity and does not matter anymore.

We first discuss the space-time interpretation of the metric obtained in the heterotic case,

$$ds^2 = - (1 - 2m e^{-Qr} + q^2 e^{-2Qr}) dt^2 + (1 - 2m e^{-Qr} + q^2 e^{-2Qr})^{-1} dr^2. \tag{7.2}$$

The asymptotic flat region is $r = +\infty$ (since Q is positive), and the curvature singularity is at $r = -\infty$ (since $R = -g''$). If we define $y = e^{-Qr}$, the curvature singularity is at $y = \infty$, the asymptotic flat region is at $y = 0$, and there are two

apparent singularities at $y_{1,2} = q^{-2}(m \pm \sqrt{m^2 - q^2})$. We study the case where the two roots are distinct $y_2 > y_1 > 0$ (We are assuming $m > 0$, and q real. In the case $m < 0$, one gets two negative roots, and a naked singularity as in (7.1). Finally, the case when one root is positive and the other is negative is of the Schwarzschild type, since one of the apparent singularities is behind the asymptotic flat region.) In the y -variable the metric reads

$$ds^2 = -q^2(y - y_1)(y - y_2) dt^2 + \frac{1}{q^2 Q^2} \frac{dy^2}{y^2(y - y_1)(y - y_2)}. \tag{7.3}$$

Along the lines of ref. [35], this metric can be put in conformal form

$$ds^2 = -q^2(y - y_1)(y - y_2)(dt^2 - dr^{*2}) \tag{7.4}$$

by the transformation

$$r^* = \frac{1}{q^2 Q} \log y^{a_0}(y - y_1)^{a_1}(y - y_2)^{a_2}. \tag{7.5}$$

The a_i are fixed by the partial fraction decomposition

$$\frac{1}{y(y - y_1)(y - y_2)} = \frac{a_0}{y} + \frac{a_1}{y - y_1} + \frac{a_2}{y - y_2}, \tag{7.6}$$

with $a_0^{-1} = y_1 y_2$, $a_1^{-1} = y_1(y_1 - y_2)$, $a_2^{-1} = y_2(y_2 - y_1)$, and $a_0 + a_1 + a_2 = 0$. Notice that $a_0 > 0$, $a_1 < 0$, $a_2 > 0$. The metric (7.3) can be put in the null-form of the Kruskal–Szekeres variables by introducing

$$u_{1,2} = e^{\gamma_{1,2}(r^* + t)}, \quad v_{1,2} = e^{\gamma_{1,2}(r^* - t)}, \tag{7.7}$$

where $\gamma_{1,2} = Qq^2(2a_{1,2})^{-1}$. This choice of $\gamma_{1,2}$ eliminates the apparent singularity at $y = y_{1,2}$. The null-coordinates satisfy the relations

$$\begin{aligned} u_1 v_1 &= y^{a_0/a_1}(y - y_1)(y - y_2)^{a_2/a_1}, \\ u_2 v_2 &= y^{a_0/a_1}(y - y_1)^{a_1/a_2}(y - y_2). \end{aligned} \tag{7.8}$$

The Penrose diagram can be built by introducing the coordinates ψ and ξ defined by $u = \tan \frac{1}{2}(\psi + \xi)$ and $v = \tan \frac{1}{2}(\psi - \xi)$. Patching together these two patches give rise to the usual RN geometry, as shown in fig. 1.

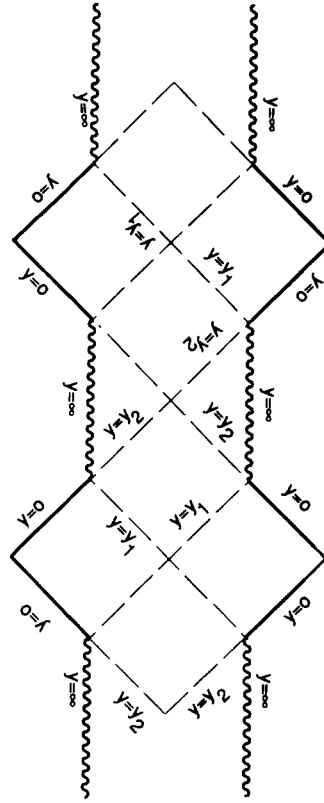


Fig. 1. Penrose diagram for the Reissner-Nordstrom geometry.

The cosmological constant contribution from open string loops gives rise to a very similar metric

$$ds^2 = -(1 - 2m e^{-Qr} + 2b e^{-Qr/2}) dt^2 + (1 - 2m e^{-Qr} + 2b e^{-Qr/2})^{-1} dr^2. \tag{7.9}$$

The U(1) string has $b = \hat{\kappa}/c$, while the $q = 0$ limit of the non-abelian open string has $b = (\hat{\kappa}/c)(N + 2\eta)$. In terms of the variable $y = e^{-Qr/2}$ the two apparent singularities are at $y_{\pm} = (1/m)(b \pm \sqrt{b^2 + m})$. For $m > 0$, one always gets a positive and a negative root, therefore a Schwarzschild-type solution. For $m < 0$ and $b < 0$, the roots are both negative, and one gets a naked singularity. If $m < 0$ and $b > 0$, both roots are positive and one gets a RN black hole in this case. Notice that we can get a RN black hole even though in this case the black hole is not charged. (However, the case $b < 0$ appears to be more relevant to the string solution, since $\hat{\kappa} > 0$ and $c < 0$.)

For the general charged open-string black hole, certain ranges of parameters can lead to more interesting solutions. In this case, using the order q^2 solution (5.9), the metric is

$$ds^2 = -\left(1 - 2m e^{-Qr} + 2b e^{-Qr/2} + \hat{q}^2 e^{-3Qr/2}\right) dt^2 + \left(1 - 2m e^{-Qr} + 2b e^{-Qr/2} + \hat{q}^2 e^{-3Qr/2}\right)^{-1} dr^2, \quad (7.10)$$

with $\hat{q} = 2\hat{\kappa}^{1/2}q/Q$. In terms of the variable $y = e^{-Qr/2}$ the coefficient of the metric is cubic, and therefore the metric has three apparent singularities at the three roots y_1, y_2 and y_3 , which satisfy

$$y_k = \frac{2m}{3\hat{q}^2} \left\{ 1 - 2\sqrt{1 - \frac{3b\hat{q}^2}{2m^2} \cos \theta_k} \right\},$$

$$\cos(3\theta_k) = -\left(1 - \frac{3b\hat{q}^2}{2m^2}\right)^{-3/2} \left(1 - \frac{9b\hat{q}^2}{4m^2} - \frac{27\hat{q}^4}{16m^3}\right). \quad (7.11)$$

The geometry is simplest when $b = 0$, as for the SO(2) case. When $0 \leq \hat{q}^4/m^3 \leq \frac{8}{27}$, there are three real roots. Since the θ_k are each separated by an angle $2\pi/3$ when $b = 0$, it follows that for $m > 0$, two roots are positive, and one negative, giving a RN geometry. If $m < 0$, then one root is positive, giving a Schwarzschild geometry. Outside this range of parameters, there is one real root, whose sign is always negative, giving a naked singularity. If $b \neq 0$, then the condition on the parameters ($m < 0, b < 0$, and $\hat{q}^2 < 0$) could allow for all three roots to be real and positive. However, since $N + 2\eta$ is always positive, b and \hat{q}^2 always have opposite sign, and this condition can never be satisfied. One can show that this is true also for the complete Born–Infeld black hole metric in (5.9). Namely, in principle the function in (5.9) could have up to three zeroes at positive y_i , but this does not actually happen for the same reason mentioned above. Nevertheless, geometry of the case of three positive roots is sufficiently interesting to merit closer attention. In fact, higher-loop corrections will lead to higher-order polynomials in y , as discussed in sect. 2. Therefore multiple positive roots may be expected to occur in the generic loop-corrected string theory.

When the three real roots are all distinct and positive, $0 < y_1 < y_2 < y_3 < \infty$, the equivalent of (7.5) is

$$r^* = a_0 \log y + a_1 \log(y - y_1) + a_2 \log(y - y_2) + a_3 \log(y - y_3), \quad (7.12)$$

where the a_i coefficients are defined by the partial fraction decomposition

$$\frac{1}{y(y - y_1)(y - y_2)(y - y_3)} = \frac{a_0}{y} + \frac{a_1}{y - y_1} + \frac{a_2}{y - y_2} + \frac{a_3}{y - y_3}, \quad (7.13)$$

with $a_0 = -(y_1 y_2 y_3)^{-1}$ etc. and $a_0 + a_1 + a_2 + a_3 = 0$.

The transformation (7.12) allows to write the metric in the conformal form, and the equivalent of (7.7) puts it in a null-form. This time we need to choose three γ_i , each to eliminate an apparent singularity,

$$u_i = e^{\gamma_i(r^*+t)}, \quad v_i = e^{\gamma_i(r^*-t)}, \tag{7.14}$$

where $\gamma_i = (2a_i)^{-1}$, $i = 1, 2, 3$. To understand the geometry of each patch, the relations

$$\begin{aligned} u_1 v_1 &= y^{a_0/a_1} (y - y_1) (y - y_2)^{a_2/a_1} (y - y_3)^{a_3/a_1}, \\ u_2 v_2 &= y^{a_0/a_2} (y - y_1)^{a_1/a_2} (y - y_2) (y - y_3)^{a_3/a_2}, \\ u_3 v_3 &= y^{a_1/a_3} (y - y_1)^{a_1/a_3} (y - y_2)^{a_2/a_3} (y - y_3) \end{aligned} \tag{7.15}$$

are helpful. Since the a_i have alternate signs, the ratios of a_2/a_3 and a_2/a_3 are negative. Therefore, $u_1 v_1 = 0$ at $y = y_1$ and $u_1 v_1 = \infty$ at $y = y_2$, and similarly in the other patches. Every time we cross a line $y = y_i$, the space and time coordinate interchange role. So in this case the singularity at $y = \infty$ is spacelike again, as in the Schwarzschild case. The final Penrose diagram is the two-dimensional lattice illustrated in fig. 2. When higher-order loop corrections are included, one can obtain even more positive roots in principle, leading to more intricate geometries.

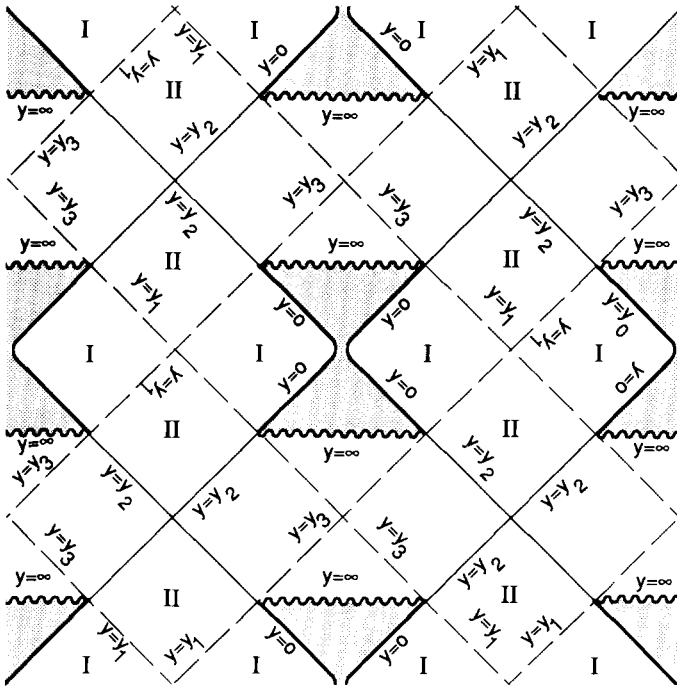


Fig. 2. Penrose diagram in the case of three positive roots. The dotted areas are not part of space-time.

8. General charge and mass formulas

The computation of the charges is a straightforward application of Gauss’s law.

First, consider the heterotic string effective action (6.3). Adding a source $-\int d^2x \sqrt{-G} J_\mu A^\mu$ to the action (6.3), shows that the conserved current density is

$$J_\mu = \nabla^\nu (e^{-2\Phi} F_{\nu\mu}). \tag{8.1}$$

Charge is the integral of J_t over r , which is just the value at $r \rightarrow \infty$ of $e^{-2\Phi} F_{tr}$. This quantity is independent of r by the equations of motion. Then the charge of the black hole is

$$\mathcal{Q} = \int dr J^t(r) = (e^{-2\Phi} f(r))_{r \rightarrow \infty} = \sqrt{2} Q q e^{-2\Phi_0}, \tag{8.2}$$

using the solution of (6.5) together with $Q = \sqrt{8/\alpha'}$.

The Born–Infeld case proceeds similarly. In this case, A^μ was rescaled by a power of α' , giving it dimensions of length, so the appropriate current coupling is $-(2\pi\alpha')^{-1/2} \int d^2x \sqrt{-G} \text{Tr}(J_\mu A^\mu)$. Then the current derived from (4.20) is

$$J_\mu = \sqrt{2\pi\alpha'} \kappa \nabla^\nu \left[e^{-\Phi} \sqrt{-\det(G+F)} \left(\frac{F}{G-F^2} \right)_{\nu\mu} \right], \tag{8.3}$$

where the r.h.s. of (8.3) is connected to (3.2) via Bianchi identities as in ref. [7]. The quantity in brackets is constant for a solution to the equations of motion, and gives the physical charge

$$\mathcal{Q} = \sqrt{2\pi\alpha'} \kappa q e^{-\Phi_0} = 8\lambda q \sqrt{(2\pi/\alpha')} e^{-\Phi_0} \tag{8.4}$$

of the black hole, where λ is the dimensionless parameter defining the normalization of the one-loop partition function in (A.1) of appendix A.

We now discuss the masses. In general relativity the mass is determined from the asymptotic behaviour of the gravitational field. Any gauge field or loop induced cosmological constant that might be present could change the mass via its contribution to the behaviour of the gravitational field at infinity. In ref. [1] the mass of the black hole was computed using the standard ADM prescription, namely by perturbing the black hole solution around asymptotically flat space with a linear dilaton. We could follow the ADM procedure as well. We would then find that in all cases discussed in this paper the mass stays unchanged, and it is the same as in ref. [1]. The reason is that the changes in the dilaton stress-energy tensor

$$T_{\mu\nu}^\Phi = e^{-2\Phi} \left(\beta_{\mu\nu}^G + \frac{1}{2} g_{\mu\nu} \beta^\Phi \right) \tag{8.5}$$

coming from loop corrections, as in (2.10), are cancelled against the change in the metric, as in (2.11). However we will not follow the ADM derivation here, but choose to start from a quasi-local mass formula, an approach that might be of interest in its own merits. We will find of course the same results, and even the details of the calculation are practically the same. In any number of dimensions, it is actually possible to give a local definition of the energy density, provided one considers only spherically symmetric solutions [11]. The physical reason is that there are no gravitational S-waves, so one can unambiguously define the mass enclosed in a sphere, and construct a conserved matter stress-energy tensor. In $D = 2$, there are never any gravity waves. We can think of this as a degenerate spherical case, and look for a mass density function whose integral from $-\infty$ to r gives the mass to the left of r . We will find a conserved vector, whose time component is the mass density, and show that its integral reproduces the ADM results. The main point is that in two dimensions one does not need to perturb about flat space-time in order to define a conserved current. Indeed, taking inspiration from ref. [36], one can define

$$S_\mu = \frac{1}{2}\sqrt{\alpha'}\epsilon^{\lambda\nu}T_{\lambda\mu}^m\partial_\nu\Phi. \quad (8.6)$$

Since $R_{\mu\nu} - \frac{1}{2}RG_{\mu\nu}$ vanishes identically in two dimensions, the equations of motion imply $T_{\mu\nu}^\Phi + T_{\mu\nu}^m = 0$, (we omit the tachyon or include it in $T_{\mu\nu}^m$). Therefore (8.6) can be also written as

$$S_\mu = -\frac{1}{2}\sqrt{\alpha'}\epsilon^{\lambda\nu}T_{\lambda\mu}^\Phi\partial_\nu\Phi. \quad (8.7)$$

Starting from the definition (8.7) we show that in all the cases we examine in this paper S^μ is a total derivative, $S^\mu = \epsilon^{\mu\nu}\partial_\nu\omega$ with an appropriate ω , so that S_μ is a conserved current.

We will first compute ω for the standard blackhole [1]. We find convenient to work in the target space conformal gauge $G_{\mu\nu} = g\eta_{\mu\nu}$ so that our variables are t and r^* , as in sect. 7. Then the components of $T_{\mu\nu}^\Phi$ (with no loop correction included so far) are

$$\begin{aligned} T_{tt}^\Phi &= e^{-2\phi}\left(2\frac{\dot{g}}{g}\dot{\Phi} + 2\frac{g'}{g}\Phi' - 4\dot{\Phi}^2 + 4\Phi'^2 - 4\Phi'' + cg\right), \\ T_{r^*r^*}^\Phi &= e^{-2\phi}\left(2\frac{\dot{g}}{g}\dot{\Phi} + 2\frac{g'}{g}\Phi' + 4\dot{\Phi}^2 - 4\Phi'^2 - 4\ddot{\Phi} - cg\right), \\ T_{tr^*}^\Phi &= e^{-2\phi}\left(2\frac{\dot{g}}{g}\Phi' + 2\frac{g'}{g}\dot{\Phi} - 4\dot{\Phi}'\right), \end{aligned} \quad (8.8)$$

where the primes denote derivatives with respect to r^* , i.e. $\Phi' = g \, d\Phi/dr$. In this case

$$\omega = \frac{\sqrt{\alpha'}}{g} e^{-2\Phi} (\dot{\Phi}^2 - \Phi'^2) + \frac{4}{\sqrt{\alpha'}} e^{-2\Phi}. \tag{8.9}$$

This implies that $\partial_\mu S^\mu = 0$, so that S^μ is a conserved current. The component S^t represents the conserved mass density, and S^{r^*} measures its flow in non-static situations. In an asymptotically flat case with a linear dilaton background, S^μ reduces to the linearized S^μ defined in ref. [1]. Therefore it will reproduce the ADM mass result in that case, as we will check.

The idea of using the scalar field ω to define a local mass has been discussed extensively by Poisson and Israel [37]. For a static solution, S^t defines a conserved mass $M(r^*) = \int^{r^*} dr^* S^t(r^*)$. Therefore, up to an integration constant, $M(r^*) = \omega(r^*)$ defines the mass contained inside a distance r^* . For a non-singular solution, the integration constant would be unambiguously [11], and would correspond to taking the lower limit to $-\infty$ in this integral. However, we are interested in a solution which is singular at that point, so we instead define the black hole mass to be $M = \omega(\infty) - \omega_0(\infty)$, where ω_0 is the mass function for “empty space”, i.e. for parameters $m = q = 0$ in the black hole metric. In the absence of a cosmological constant $\omega_0 = 0$.

For the static solutions we have considered, M can be expressed as an integral over the matter stress tensor component T_{tt}^m , giving further evidence that this definition of the mass is physically what one would expect. Note that evaluating (8.6) for $\mu = t$ gives

$$\omega' = \frac{\sqrt{\alpha'}}{2g} (-T_{tt}^m \Phi' + T_{t^*t^*}^m \dot{\Phi}). \tag{8.10}$$

Using the dilaton ansatz $\Phi = \Phi_0 - (2/\sqrt{\alpha'})r$, we obtain

$$M = \int dr^* T_{tt}^m(r^*) = \int dr g^{-1} T_{tt}^m. \tag{8.11}$$

This shows that the mass is determined by the energy density of the matter source. (The expression (8.11) is an alternative way to verify the normalization of S^μ .)

Evaluating the mass formula (8.9) on the classical solution one can relate the parameter m of the black hole solution to the physical mass. The result is

$$M = \frac{8}{\sqrt{\alpha'}} m e^{-2\Phi_0}. \tag{8.12}$$

This expression is in agreement with Witten's [1] definition of the mass if we take $2m = 1$. This agreement with the ADM result is a sign that our definition of the mass is physically reasonable.

Given the black hole mass one can now determine the black hole entropy, which is given by

$$S = \beta_H M + \ln Z = (\pi\sqrt{\alpha'}) \left(\frac{8}{\sqrt{\alpha'}} m e^{-2\Phi_0} \right) = 8\pi m e^{-2\Phi_0} = \pi\sqrt{\alpha'} M. \quad (8.13)$$

Here β_H is the inverse Hawking temperature of the two-dimensional black hole and $\ln Z$ is the euclidean action which can be shown to vanish for the ordinary closed-string case. (This expression has also been obtained by ref. [38].)

As derived, the mass formula (8.9) is valid only for the ordinary closed-string action (3.5). However, the same principles will apply in general. Other theories will have new contributions to the stress-energy tensor, which must be included in eq. (8.7), leading to a modified mass function ω . For example, the loop-corrected effective action to the two-dimensional closed string, including the cosmological constant term (2.9) for $n = 1$, yields a mass formula

$$\begin{aligned} \omega &= \frac{e^{-2\Phi}}{g} (\dot{\Phi}^2 - \Phi'^2) \sqrt{\alpha'} + \frac{4}{\sqrt{\alpha'}} e^{-2\Phi} - \frac{1}{2} \sqrt{\alpha'} \Lambda \Phi \\ &= \frac{8}{\sqrt{\alpha'}} m e^{-2\Phi_0} + \frac{\sqrt{\alpha'}}{2} \Lambda \Phi_0. \end{aligned} \quad (8.14)$$

Subtracting $\omega_0 = \frac{1}{2} \sqrt{\alpha'} \Lambda \Phi_0$ gives the same value (8.12) as the standard black hole. In the open-string case, the loop-induced cosmological constant modifies the mass formula as follows:

$$\omega = \frac{e^{-2\Phi}}{g} (\dot{\Phi}^2 - \Phi'^2) \sqrt{\alpha'} + \frac{4}{\sqrt{\alpha'}} e^{-2\Phi} + \frac{1}{2} \sqrt{\alpha'} \Lambda e^{-\Phi}. \quad (8.15)$$

However, the actual value of M is independent of Λ , and is identical with the standard result (8.12). (In this case, as in all of the following open-string results, $\omega_0 = 0$, and no subtraction is needed to obtain the mass.)

Finally, we turn to the mass formulas of the charged black holes. In the heterotic case we obtain

$$\omega = \frac{4}{Qg} e^{-2\Phi} (\dot{\Phi}^2 - \Phi'^2) + Q e^{-2\Phi} + \frac{1}{2Q} \mathcal{Q}^2 e^{2\Phi} \quad (8.16)$$

while the open-string Born–Infeld action yields

$$\omega = \frac{\sqrt{\alpha'}}{g} e^{-2\Phi} (\dot{\Phi}^2 - \Phi'^2) + \frac{4}{\sqrt{\alpha'}} e^{-2\Phi} + \kappa\sqrt{\alpha'} \left(1 - \eta - \frac{N}{2} \right) e^{\Phi} - \kappa\sqrt{\alpha'} e^{\Phi} \left(1 + \frac{Q^2 e^{2\Phi}}{2\pi\alpha'\kappa^2} \right)^{1/2} + \frac{Q}{\sqrt{2\pi}} \sinh^{-1} \left(e^{\Phi} \frac{Q}{\sqrt{2\pi\alpha'\kappa}} \right). \quad (8.17)$$

In either case, evaluating these formulas gives the mass $2Qm e^{-\Phi_0}$ but for the heterotic case $Q = \sqrt{8/\alpha'}$ whereas in the bosonic case $Q = 4/\sqrt{\alpha'}$.

9. Conclusions

In this paper we have investigated more general black hole solutions to two-dimensional string theory. The modifications have been obtained introducing gauge fields by coupling them to the boundary of open strings. We have shown how these corrections can be derived in the context of two-dimensional strings by applying the techniques of boundary operators [9] and BRST invariance. Our results hold only in the leading order in α' . Since the completion of this work, some further analysis of the two-dimensional open string has appeared [39], which possibly leads to an exact treatment.

Another way to couple a gauge field to strings is to construct a heterotic string. We found such a theory with an $E_8 \times SO(8)$ or $SO(24)$ gauge group. Applying background field methods allowed us to find a second form of charged black hole solutions. These are pure closed-string tree-level solutions, although the space-time metric has the same form as the loop-corrected ones. Our solution appears to be different from those recently discussed in refs. [4,6].

The solutions have geometries which can be identified with Schwarzschild or Reissner–Nordstrom black hole solutions. Both the open and heterotic string theory can give rise to either class of solutions, depending on the range of charge and mass parameters. Models with a cosmological constant can lead to a Reissner–Nordstrom geometry even in the absence of charge.

Multiple event horizons could be the generic case when higher-order loop corrections are included, leading to very intricate geometries in general. We examined in detail the case of three event horizons. In this case the Penrose diagram tiles the plane.

We wish to thank D. Kutasov, O. Lechtenfeld, S. Trivedi, J. Polchinski and especially Z. Qiu for useful discussions. S. Yost thanks the Institute for Advanced Study for its hospitality during the completion of this work.

Appendix A. Open-string partition function and factorization

In this appendix, we check that the boundary state plays the usual role [9] in factorizing the open-string partition function. This fixes its normalization, once the one-loop open-string vacuum amplitude is given.

The one-loop open string vacuum amplitude may be expressed as a product of partition functions for the string modes,

$$Z = \lambda^2 \int_0^\infty \frac{d\tau}{\tau} Z_X(\tau) Z_{\text{ghost}}(\tau), \quad (\text{A.1})$$

where we are considering a strip of width π , whose ends are identified after a propagation time $2\pi i\tau$. The open-string one-loop coupling constant is λ^2 , the square of the tree-level coupling. Choosing a convenient normalization,

$$Z_X(\tau) = 8\pi\alpha' \int \frac{d^2P}{(2\pi)^2} \text{Tr} e^{2\pi i\tau H} = (-\pi i\tau)^{-1} \eta^{-2}(\tau), \quad (\text{A.2})$$

where the open-string hamiltonian is [23]

$$H = \alpha' P^2 + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \tilde{\alpha}_{-n} - \frac{1}{12} \quad (\text{A.3})$$

including the normal-ordering constant $-1/24$ per boson [23], and

$$\eta(\tau) = (-i\tau)^{-1/2} \eta(-1/\tau) = e^{\pi i\tau/12} \prod_{n=1}^{\infty} (1 - e^{2\pi in\tau})$$

is the Dedekind eta function. The momentum P_r includes the shift by $-iQ/2$ explained in sect. 3, but this does not affect Z_X since P is integrated from $-\infty$ to ∞ . The ghost partition function is simply [23]

$$Z_{\text{ghost}}(\tau) = \eta^2(\tau). \quad (\text{A.4})$$

A Jacobi transformation $\tau \rightarrow -1/\tau$ transforms the partition function to the closed-string channel, where it may be interpreted as an overlap of boundary states. The result of the Jacobi transformation is

$$\begin{aligned} Z_X(\tau) &= \frac{1}{\pi} a^{-1/6} \prod_{n=1}^{\infty} (1 - a^{2n})^{-2}, \\ Z_{\text{ghost}}(\tau) &= -\frac{\ln a}{\pi} a^{1/6} \prod_{n=1}^{\infty} (1 - a^{2n})^2, \end{aligned} \quad (\text{A.5})$$

where $a = e^{-\pi i/\tau}$ is the modulus of an annulus conformally equivalent to the tube of length $-\ln a$. The total partition function is then

$$Z = \lambda^2 \int_0^1 \frac{da}{a}.$$

This has a logarithmic divergence at $a \rightarrow 1$ from the ‘‘tachyon’’ tadpole. In the presence of background fields, Z is multiplied by [8]

$$\det[G(x) + F(x)]. \quad (\text{A.6})$$

Each of the partition function factors in (A.1) may be individually expressed as an overlap of the appropriate boundary states in the closed string channel. The ghost partition function is factorized in ref. [9], and it is easy to check that

$$Z_X(\tau) = \langle B | a^{L_0 + \tilde{L}_0} | B \rangle_X. \quad (\text{A.7})$$

Therefore, the boundary states play the same role in factorizing the loop amplitude as they do in the critical case. The normalization of the boundary state is fixed by the normalization of the one-loop partition function via (A.7), which in turn fixes the normalization of the loop corrections to the beta functions.

References

- [1] E. Witten, Phys. Rev. D44 (1991) 314
- [2] G. Mandal, A. Sengupta and S. Wadia, Classical solutions of two dimensional string theory, IAS preprint IASSNS-HEP-91/10 (1991)
- [3] R. Dijkgraaf, H. Verlinde and E. Verlinde, String propagation in a black hole geometry, Princeton preprint PUPT-1252 (1991)
- [4] N. Ishibashi, M. Li and A. Steif, UCSB preprint UCSBTH-91-28-REV (1991)
- [5] S.P. De Alwis and J. Lykken, 2d Gravity and the black hole solution in 2d critical string theory, Fermilab-pub-91/198-T
- [6] S. Pratik Khastgir and Alok Kumar, Bhubaneswar preprint (1991)
- [7] A. Abouelsaood, C. Callan, C. Nappi and S. Yost, Nucl. Phys. B280 (1987) 599
- [8] C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl. Phys. B288 (1987) 525
- [9] C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl. Phys. B293 (1987) 83
- [10] C. Callan, D. Friedan, E. Martinec and M. Perry, Nucl. Phys. B262 (1985) 593
- [11] C. Misner, K. Thorne and J.A. Wheeler, Gravitation (Freeman, New York, 1973)
- [12] C. Lovelace, Nucl. Phys. B273 (1986) 413
- [13] W. Fischler and L. Susskind, Phys. Lett. B173 (1986) 262
- [14] J. Polchinsky and Y. Cai, Nucl. Phys. B296 (1988) 91
- [15] J. Paton and H. Chan, Nucl. Phys. B10 (1969) 516
- [16] S. Das and B. Sathiapalan, Phys. Rev. Lett. 56 (1986) 2664;
C. Itoi and Y. Watabiki, Phys. Lett. B198 (1987) 486
- [17] M. Born and L. Infeld, Proc. R. Soc. 144 (1934) 425;
E. Fradkin and A. Tseytlin, Phys. Lett. B163 (1985) 123
- [18] C. Callan and Z. Gan, Nucl. Phys. B272 (1986) 647

- [19] S. Das and A. Jevicky, *Mod. Phys. Lett. A5* (1990) 1639;
J. Polchinsky, *Nucl. Phys. B346* (1990) 253
- [20] W. Siegel and B. Zweibach, *Nucl. Phys. B263* (1986) 105;
T. Banks and M. Peskin, *Nucl. Phys. B264* (1986) 513
- [21] C. Callan, C. Lovelace, C. Nappi and S. Yost, *Phys. Lett. B206* (1988) 41; *Nucl. Phys. B308* (1988) 221
- [22] J. Distler and H. Kawai, *Nucl. Phys. B321* (1989) 509
- [23] M. Green, J. Schwarz and E. Witten, *Superstring Theory, Vol. 1* (Cambridge University Press, Cambridge, 1987)
- [24] D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys. B271* (1986) 93
- [25] D. Fairlie, unpublished;
A. Chodos and C. Thorn, *Nucl. Phys. B72* (1974) 509
- [26] C. Thorn, *Phys. Rep. 175* (1989) 1
- [27] S. Yost, *Nucl. Phys. B321* (1989) 629
- [28] N. Marcus and A. Sagnotti, *Phys. Lett. B119* (1982) 97
M. Green, J. Schwarz and E. Witten, *Superstring Theory, Vol. 2* (Cambridge University Press, Cambridge, 1987) ch. 8.1.2
- [29] S. deAlwis, J. Polchinski and R. Schimmrigk, *Phys. Lett. B218* (1989) 449
- [30] D. Kutasov and N. Seiberg, Princeton preprint PUPT-1193, RU-90-41
- [31] Kounnas, B. Rostand and E. Tomboulis, UCLA preprint UCLA/91/TEP 1
- [32] D. Gepner, *Nucl. Phys. B296* (1988) 757
- [33] L. Dixon and J. Harvey, *Nucl. Phys. B274* (1986) 93
- [34] N. Seiberg and E. Witten, *Nucl. Phys. B276* (1986) 272
- [35] J.C. Graves and D.R. Brill, *Phys. Rev. 120* (1960) 1507
- [36] W. Fischler, D. Morgan and J. Polchinski, *Phys. Rev. D41* (1990) 2638; *D42* (1990) 4042
- [37] E. Poisson and W. Israel, *Phys. Rev. D41* (1991) 1796
- [38] S. Trevidi, private communication
- [39] M. Bershadsky and D. Kutasov, Princeton preprint PUPT-1283