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New results on the theoretical precision of the LEP/SLC luminosity [1]

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Abstract

We consider the error budget for the calculation of the LEP/SLC luminosity in the Monte Carlo event generator BHLUMI4.04 from the standpoint of new calculations of the exact results for the $\mathscr{O}(\alpha^2)$ contributions to the processes $e^+e^- \to e^+e^- + (n)\gamma$, $n = 0,1,2$, in the low angle luminosity regime at SLC/LEP energies, in context of Yennie-Frautschi-Suura exponentiation. We find that the error on the $\mathscr{O}(\alpha^2)$ photonic correction can be reduced from the currently published value 0.1% to the value 0.027%. This leads to an over-all precision tag for the currently available program BHLUMI4.04 of 0.061%. This reduction of the precision of the calculation is important for the final LEP1 EW precision Z physics tests of the Standard Model.

Currently, new luminometers at LEP [1] have made measurements of the luminosity process $e^+e^- \to e^+e^- + n(\gamma)$ at the experimental precision tags below 0.1%. This should be compared with the prediction by the Knoxville-Krakow (KK) Collaboration in the program BHLUMI4.04 [2] wherein the theoretical precision tag of 0.11% is realized for this process in the ALEPH SICAL-type [3] acceptance. If one combines the experimental results, one arrives at an experimental precision of $\lesssim 0.05\%$. Evidently, for the final EW precision tests data analysis for LEP1, it would be desirable to reduce the theoretical precision tag on the luminosity cross section prediction at least to the comparable 0.05%-regime in order not to obscure unnecessarily the comparison between experiment and the respective Standard Model of the electroweak interaction. With this as our primary motivation, we have examined the error budget arrived at in Refs. [2,4,5] in view of recent exact results impacting both the technical and physical precision of the errors quoted in that budget.

More precisely, if one looks into the error budget shown in Table 1, Refs. [2,4], one sees that the largest contribution is associated with the $\mathscr{O}(\alpha^2)$ photonic corrections, which contribute 0.1% in quadrature to the total 0.11% quoted for the total precision of the BHLUMI4.04 prediction in these references for the ALEPH SICAL-type acceptance. Accordingly, we have used the exact results in Refs.

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Table 1
Summary of the total (physical + technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within $1^\circ - 3^\circ$, and for LEP2 it covers energies up to 176 GeV, and angular range within $1^\circ - 3^\circ$ and $3^\circ - 6^\circ$

Type of correction/error	LEP1		LEP2	
	Past [5,4]	Present	Past [5,4]	Present
(a) Missing photonic $\mathcal{O}(\alpha^2)$ [12]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$ [13]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [14,15]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [16,17]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [18]	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.122%

[6,8,9] and the exact result in Ref. [7] to make a more realistic estimate of the true size of this dominant error quoted in Refs. [2,4].

In re-examining the photonic corrections used in BHLUMI4.04 at the $\mathcal{O}(\alpha^2)$, which is the relevant order of the corrections, one needs look at the approximations made in the matrix element used in the calculation encoded in the program in comparison to available exact results. This will allow us to re-assess the physical precision of the corresponding part of the BHLUMI4.04 matrix element, which is the exact $\mathcal{O}(\alpha^2)$ LL (leading-log) Yennie-Frautschi-Suura (YFS) exponentiated matrix element. The implementation of the Monte Carlo algorithm in BHLUMI4.04 for two hard photon emission needs also to be checked at this level of precision, since our previous checks on it do not cover sufficiently the two hard photon phase space as we were always working in the leading-log approximation for two hard photons. This check, which we have recently completed, will allow us to give a more realistic estimate of the technical precision of the realization of the corresponding aspect of the matrix element in BHLUMI4.04. The net result is a new estimate of the total precision of the prediction of the luminosity cross section by BHLUMI4.04 at LEP1 energies.

Our discussion is organized as follows. We first discuss the effect of including the exact result in Ref. [6] for the $\mathcal{O}(\alpha)$ correction to the single hard bremsstrahlung process in BHLUMI4.xx in comparison to the LL result for this correction that is used in BHLUMI4.04. We then turn the the technical precision test of the implementation of the two hard bremsstrahlung matrix element in BHLUMI4.04, wherein this matrix element is also computed in the LL approximation. We will carry-out this technical precision test in comparison to the analogous test when the exact two hard bremsstrahlung matrix element [8,9] is implemented in BHLUMI4.xx. This will verify that indeed the physical precision of the LL approximation for the two hard bremsstrahlung matrix element is indeed small in comparison to the other errors in the error budget in Table 1 of Refs. [2,6]. Finally, we turn to the effect of including the exact two-loop virtual correction in BHLUMI4.xx in comparison to the LL approximation of the $\mathcal{O}(\alpha^2)$ virtual correction that is used in BHLUMI4.04. By combining the results of these analyses, we arrive at a more realistic estimate of the error on the theoretical prediction for the luminosity process at LEP1/LEP2 energies as it is calculated by BHLUMI4.04.

Considering now the exact $\mathcal{O}(\alpha)$ correction to the single hard bremsstrahlung in the luminosity process, we have implemented the results in Ref. [6] into BHLUMI4.xx and made a systematic study of the net change in the prediction for the luminosity relative to the prediction of BHLUMI4.04 in which this correction is treated to the LL level. What we find is illustrated in Figs. 1 and 2 for the ALEPH SICAL-type acceptance at the Z^0 peak. In the language of the YFS theory, this correction enters the hard photon residuals as $\bar{\beta}_1^{(2)}$, the $\mathcal{O}(\alpha^2)$ contribution to the one-hard photon residual $\bar{\beta}_1$. In the Fig. 1, we show this part of the SICAL-type accepted cross section as it is given by our exact result in Ref [6] and as it is given by several different approximations to our exact result: the LL approximation in BHLUMI4.04, the approximate ansatz in Eq. (3.25) of Ref. [6], and the result (NLLB) of Ref. [10] which is calculated

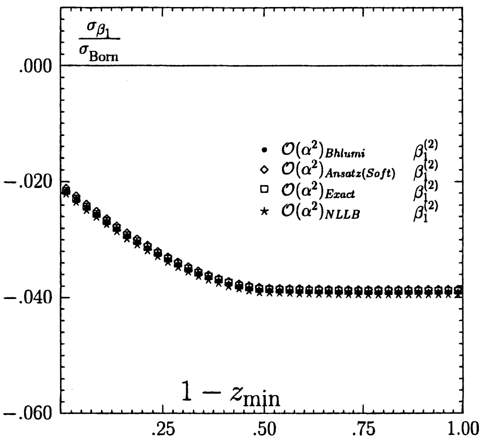

Fig. 1. Monte Carlo result (10^6 events) for the entire cross section associated with $\overline{\beta}_1^{(2)}$ for the SICAL Wide-Narrow trigger.The first and second order results are divided by the Narrow-Narrow Born cross section. z_{min} is as it is defined in Fig. 2 of Phys. Lett. B353 (1995) 362.

using a semi-collinear approximation that the respective authors of Ref. [10] argue includes the LL and NLL effects.

In Fig. 2, we show the difference between the corresponding LL result in BHLUMI4.04 and the other three results in Fig. 1 in ratio to the respective Born cross section. What we see is that the BHLUMI4.04 results are within 0.02% of the exact result in units of the respective Born cross section throughout the experimentally interesting regime 0.2 $\leq 1 - z_{\text{min}} \leq 1.0$. This is the main reason we will be able to reduce the estimated precision of the BHLUMI4.04 prediction in comparison to Refs. [2,4].

Turning next to the technical precision of the 2-γ bremsstrahlung calculation in BHLUMI4.04, we have constructed a completely independent realization of the two photon phase space integration compared to what is used in BHLUMI4.04 by way of an independent Monte Carlo algorithm. We have implemented this new Monte Carlo realization of the two photon phase space and compared its result with that of BHLUMI4.04's for the hard photon residual $\overline{\beta}_2$ contribution to the luminosity cross section, both for the LL matrix element in BHLUMI4.04 and for the exact matrix element for the two-photon bremsstrahlung in Ref. [8,9]. We stress that the two photon phase space in BHLUMI4.04 is exact. What we find is shown in Fig. 3 for the ALEPH SICAL-type acceptance at the Z^0-peak. We find that the difference between the two realizations of the 2-γ bremsstrahlung is below 0.003% of the Born cross section. Moreover, we get an estimate of the physical precision of the LL approximation for this part of the cross section from comparing the LL and exact results as 0.012%, in agreement with our estimate in Ref. [2].

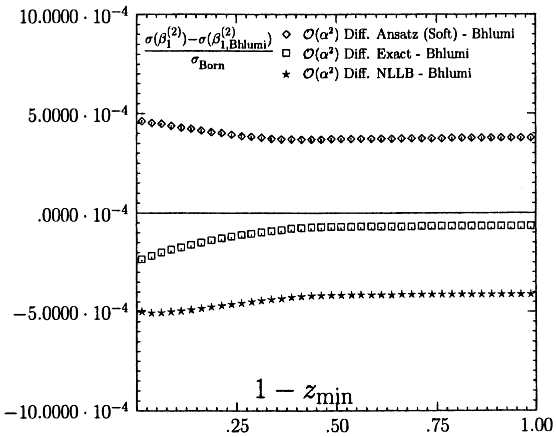

Fig. 2. Pure second order Monte Carlo result for $\overline{\beta}_1^{(2)} - \overline{\beta}_{1,\text{Bhlumi}}^{(2)}$ differences for the SICAL Wide-Narrow trigger, divided by the Narrow-Narrow Born cross section, with z_{min} as given in Fig. 1.

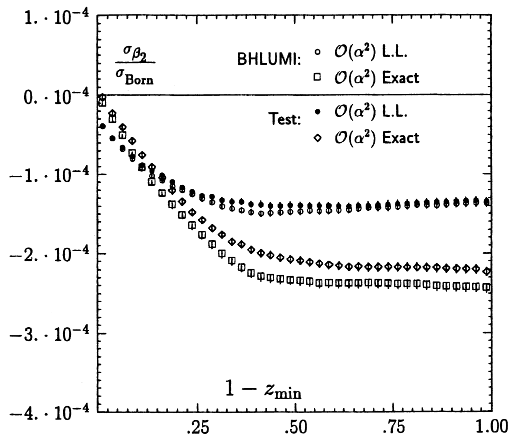

Fig. 3. Comparison of Monte Carlo results for $\overline{\beta}_2^{(2)}$ for the LL and exact matrix elements for 10^6 events. Results are shown for the BHLUMI generator and for an alternative 'Test' generator for a technical precision test. The results are for the SICAL Wide-Narrow trigger, and are divided by the Narrow-Narrow Born cross section; z_{min} is as given in Fig. 1.

Finally, we turn to the exact result for the two-loop contribution of the hard photon residual $\overline{\beta}_0$ to the cross section in comparison to the LL result used for it in BHLUMI4.04. We have analytically continued the result of Ref [7] from the s-channel to the t-channel for the required two-loop contribution to the respective charge form factor in QED. In this way, using the YFS theory, we have found that the difference between the LL result in BHLUMI4.04 and the exact result corresponds to the shift of the function υ in Eq. (2) in Ref. [11] by

$$\Delta\upsilon^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 L\left(6 + 6\zeta(3) - \tfrac{45}{8} - \frac{\pi^2}{2}\right) + \left(\frac{\alpha}{\pi}\right)^2 \Big[6 - 9\zeta(3) + \left(\tfrac{17}{8} - 2\ln 2\right)\pi^2 - \tfrac{8}{45}\pi^4\Big], \qquad (1)$$

where the big logarithm is defined as $L = \ln|t|/m_e^2$ and $\zeta(3)$ is the Riemann *zeta* function of argument 3. For the ALEPH SICAL type acceptance at the Z^0 peak, this corresponds to 0.014% in the cross section. Collecting the results above in quadrature, we obtain the result that the current calculation of the $\mathcal{O}(\alpha^2)$ photonic corrections in BHLUMI4.04 are accurate to

$$\left.\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}\right|_{\mathcal{O}(\alpha^2)\text{-photonic}} = 0.027\%. \qquad (2)$$

Using this result in Table 1 for Ref. [2] we arrive at the precision tag 0.061% for the currently available calculation in BHLUMI4.04 at the Z^0 peak. At the LEP2 energy of 176GeV, if we repeat the analysis just described, we find that the corresponding precision of BHLUMI4.04, for both the SICAL and LCAL type acceptances, is now reduced to 0.122% compared to the estimate in Ref. [2] of 0.25%. The current situation is now summarized in our Table 1.

A more detailed exposition of the results in this paper will appear elsewhere [12].

Our result that the size of the error associated with the missing sub-leading bremsstrahlung correction at $\mathcal{O}(\alpha^2)$ in BHLUMI4.04 is.027% agrees with the estimate of 0.03% made by Montagna et al. [19] using a structure function convolution of a hard collinear external photon with an acollinear internal photon. As these authors have argued, while such a pairing of convolutions does not represent a complete set of photonic $\mathcal{O}(\alpha^2 L)$ corrections, one expects it to contain the bulk of such corrections. Indeed, our exact result of 0.027% shows that indeed the approximation made in Ref. [19] does give the bulk of the respective $\mathcal{O}(\alpha^2 L)$ correction. Evidently, that we now have two independent results, one exact, that presented by us in this paper, and one approximate, that in Ref. [19], which agree on the size of the error associated with the missing photonic $\mathcal{O}(\alpha^2 L)$ correction in BHLUMI4.04 enhances the results in this paper.

Moreover, concerning the light pairs effect contribution to the error budgets in Table 1, the authors in Ref. [20] have recently made an independent cross check of the results obtained with the program BHPAIR [17] in Table 4 of Ref. [5] for the size of this effect in the relevant kinematical variable cut range for the LEP1 and LEP2 luminometer acceptances used in our Table 1 here. When we compare the results of BHPAIR with those of Ref. [20], the both of which have been also cross checked with the results of Ref. [21], we find that the total error, physical + technical precision error, on the prediction of BHPAIR or of the calculation in Ref. [20] for the size of the light pairs effect is 0.01% (0.015%) for the LEP1 and LEP2 acceptances in Table 1. It follows that, if this pairs effect would be implemented by the respective LEP Collaborations in the analysis of their experimental luminometer data, using either BHPAIR, the calculation in Ref. [20], or the more realistic Monte Carlo event generator BHLUMI2.30 [17], then, the entry for the pairs effect in Table 1 would be correspondingly reduced to 0.01% (0.015%) for LEP1 (LEP2), in agreement with the conclusions of Ref. [20]. This would further reduce the error on the theoretical prediction for the luminosity to 0.054% (0.113%) for LEP1 (LEP2), as noted by Ref. [20]. We understand that such implementation is not entirely excluded [22].

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