

SAMPLE LABORATORY REPORT

VECTOR ANALYSIS

Introduction

Vectors are a mathematical construction used to represent objects that have a magnitude and direction. In this experiment, we will construct vector sums using displacement vectors drawn on the laboratory table, and investigate the relationship between the components of a vector and its magnitude and direction.

Procedure

A chalk mark was made on our lab table. We constructed a vector labeled \vec{A} from the corner of the table to the mark. We then measured the vector using a meter stick marked in mm and a protractor marked in 1-degree increments. My partner and I both measured the length and angle of the vector, and found that we could agree on these to within ± 1 mm for the distance and $\pm 2^\circ$ for the angle, so we will take these as an estimate of our experimental accuracy. After measuring the length and angle, we drew perpendicular lines from the mark to the edges of the table, taking care that the distance of each end of the perpendicular lines to an edge of the table were the same. We used these perpendicular lines to measure the x and y components A_x and A_y of the vector \vec{A} . We will use these measurements to test the equations relating the Cartesian and polar coordinates for the location of the mark.

Then, we were asked to construct a second vector \vec{B} of given magnitude 30.0 cm and direction 20.0° , starting from the original mark. We did this by using the protractor to mark an angle of 20.0° relative to our previously-constructed line parallel to the x axis through the mark. Then we used the meter stick to find a point 30 cm away at the correct angle. We used this vector together with \vec{A} to construct the vector sum, or *resultant*, $\vec{R} = \vec{A} + \vec{B}$. The purpose of this exercise is to see whether the length and angle of the resultant obtained through our geometrical construction agrees with the calculated resultant found by adding the components of \vec{A} and \vec{B} and using trigonometry. The precision of our construction and measurements should be comparable to the estimates in the previous paragraph, about ± 1 mm for each distance and $\pm 2^\circ$ for each angle.

The vector \vec{R} was constructed by drawing a line from the corner of the desk to the end of the vector \vec{B} . We measured the length and angle of vector \vec{R} using exactly the same procedure as we used to measure \vec{A} . We measured the x and y components of \vec{B} as well, and used them to compare the measured length and angle of \vec{R} to the vector sum obtained by adding the components of \vec{A} and \vec{B} numerically.

Analysis

The results of the measurements of the vector \vec{A} are shown in Table 1, together with calculated values obtained as follows: The calculated values of the components A_x and A_y were obtained from the measured length A and angle θ_A using the trigonometric equations

$$A_x = A \cos \theta_A, \quad A_y = A \sin \theta_A$$

The calculated values of the length A and angle θ_A are obtained using the measured components A_x and A_y using the Pythagorean Theorem and the definition of the tangent,

$$A = \sqrt{A_x^2 + A_y^2}, \quad \theta_A = \text{atan}\left(\frac{A_y}{A_x}\right).$$

TABLE 1

coordinate	measured value	calculated value
A_x	22.0 cm	20.9 cm
A_y	38.5 cm	39.3 cm
A	44.5 cm	44.3 cm
θ	62°	60.2°

TABLE 2

measured values

A	θ_A	B	θ_B	R	θ_R
44.5 cm	62°	30.0 cm	20°	70.0 cm	44°

calculated values

$A_x = A \cos \theta$	$A_y = A \sin \theta$	$B_x = B \cos \theta$	$B_y = B \sin \theta$
22.0 cm	38.5 cm	28.2 cm	10.3 cm

calculated resultant

$R_x = A_x + B_x$	$R_y = A_y + B_y$	$R = \sqrt{R_x^2 + R_y^2}$	$\theta_R = \text{atan}(R_y/R_x)$
50.2 cm	48.8 cm	70.0 cm	44.2°

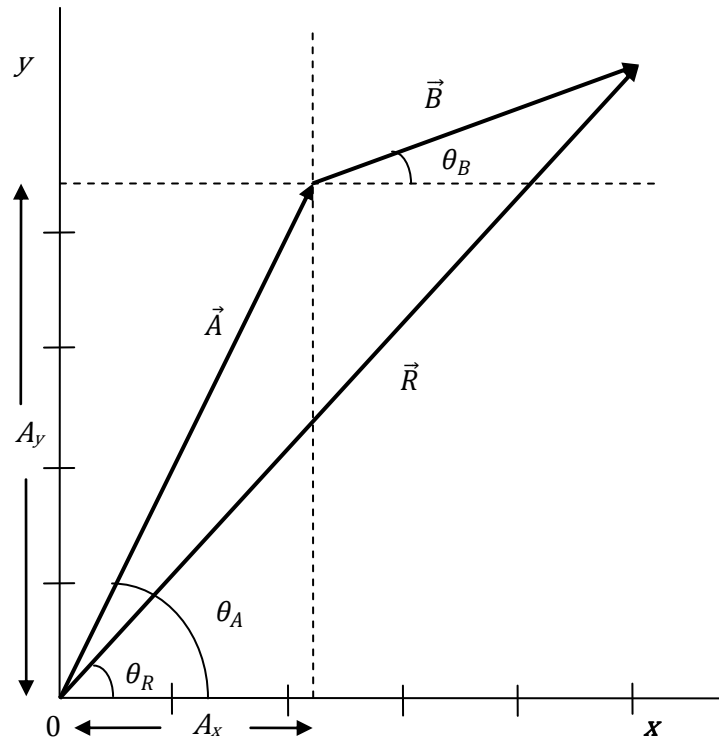


Figure 1 - Diagram showing the constructed vectors drawn to scale: each mark on the axis is 10 cm on the table.

The measured length R and angle θ_R of the resultant $\vec{R} = \vec{A} + \vec{B}$ are shown in the first line of Table 2, and the calculated values appear in the bottom line. The equations used in each step are reproduced in the table. A scale drawing of the three vectors is shown in Figure 1.

Discussion

We first compared two ways of describing a vector: by using the magnitude and direction, or by using the Cartesian (rectangular) coordinates with respect to axes along the edges of the desk, with the corner as origin. We measured both representations, and compared the values calculated from the other representation. Table 1 shows that the calculated and measured coordinates differ by 1.0 cm to 1.2 cm. This difference seems larger than expected, given that we estimate our errors in distance measurements to be approximately 1 mm. However, this comparison also depends on the angle. If the measured angle had been 60° instead of 62° , which is within the error we estimated for protractor readings, then the results for A_x and A_y

would have been 22.3 cm and 38.5 cm, respectively. The maximum error is then 3 mm. Considering that there is some error associated with locating the exact location of the chalk dot, and in drawing the perpendicular axis lines, together with the error of about 1 mm in distance readings, we conclude that the calculated and measured components agree to within our experimental errors.

The calculated and measured magnitudes are within 2 mm, consistent with our above analysis, and the angle is within 2° , consistent with our $\pm 2^\circ$ error estimate for protractor readings. Therefore, these results also agree to within our experimental errors.

In the second part of the experiment, our calculated and measured values of the resultant vector were remarkably close: they agree to within 1 mm, confirming that the vector sum calculated by adding components of the individual vectors agrees with the vector sum constructed geometrically by following the vectors tail-to-head. The 0.2° difference between the calculated and measured angles of the resultant vectors may also be considered insignificant, given that the protractor can be read to only 2° , by our estimate. Thus, we have obtained compelling evidence that the geometrical and algebraic approaches to vector addition agree, within the limits of our experimental technique.

In the process of conducting this experiment, we noted several possible improvements in making such measurements. Some error was due to difficulty in locating the exact corner of the desk. More precise coordinate axes would have been helpful. The chalk dot should have been smaller, but we had no control over that. Also, even a small error in reading the protractor could, in principle, lead to a large error in constructing the second vector \vec{B} . A larger protractor with finer angle markings would help with this, since the angle could then be measured more clearly. Although we didn't think of it at the time, using two meter sticks simultaneously would have made it easier to draw the perpendicular lines to the axis which we needed to measure component. Considering our estimate of the errors inherent in the experimental set-up, we found good agreement with the equations for vector analysis, especially in the case of vector addition.