Physics 221

Department of Physics
The Citadel

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September 28, 2009

Circular Motion and Newton's Laws – Part 2

Work for Constant Forces

Announcements

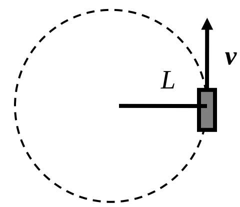
- Wednesday: problem set 6.
- Assigned Problems: Ch. 6:

  5, 11, 12, 13, 18, 57

  (Problem 12 was added to have one on non-uniform circular motion.)
- Today: Sec. 6-2 and
- Ch. 7: Energy, sec. 1 – 4
- Friday: Ch. 7, sec. 5 – 9

Model Rocket

A model rocket rests on round frictionless table, and is joined by a string of length $L = 50$ cm to a fixed point, so that it can move in a circle of radius L when the engine is lit. The engine provides a constant thrust of 40 N in the direction of motion. The rocket's mass of 660 g is approximately constant as the engine burns.



(a) If the string breaks when its tension reaches 800N, how many seconds does the rocket fly before it breaks?

Model Rocket

Is it the total force on the rocket or the radial force that determines when the string breaks?

The **radial** force is due to the string tension.

Breaking condition: $F_r = mv^2/L = T_{max}$.

$v = (T_{max}L/m)^{1/2} = (800 \text{ N} \times 0.50 \text{ m}/0.66 \text{ kg})^{1/2}$

$= 24.6$ m/s.

Tangential acceleration

$a_t = F/m = 40\text{N} / 0.66 \text{ kg} = 60.6 \text{ m/s}^2$.

$t = v/a_t = 0.406$ s.

Model Rocket

(b) What is its velocity when the string breaks?

Speed $v = 24.6$ m/s. Direction?

Distance travelled along circumference:

$d = ½\, a_t t^2$

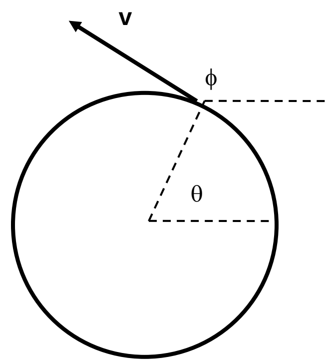
$= ½\,(60.6\ \text{m/s}^2)(\ 0.406\ \text{s})^2$

$= 4.99$ m.

Position Angle θ (radians) = d/L = 9.98 rad = 571.8° = 211.8°

Velocity angle ϕ: add $\pi/2 = 1.57$ rad

Get $\phi = \theta + 90° = 301.8°$

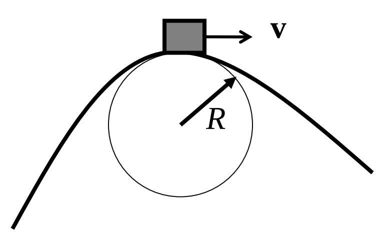


π rad = 180°

Roller Coaster

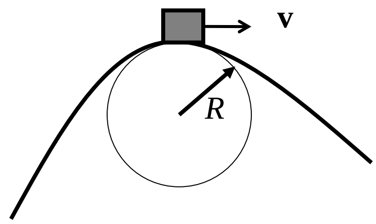
A roller coaster car goes over a hill with radius of curvature $R = 15$ m at speed $v = 10$ m/s.

If a passenger normally weighs 600 N, how heavy would this passenger feel at the top of the hill?



Roller Coaster

Newton's Law:



$W - F_N = ma = mv^2/R.$

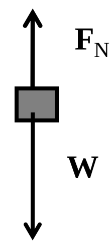
$F_N = W - mv^2/R. \qquad W = mg$

$= W - Wv^2/(\mathrm{Rg})$

$= W\,(1 - v^2/Rg)$

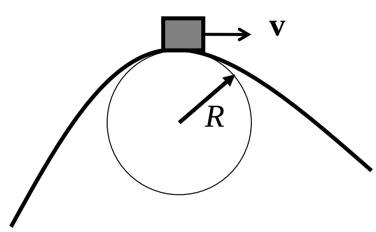
$= (600\mathrm{N})(1 - 100/(15\mathrm{x}9.8))$

$= 192$ N.



Roller Coaster

How fast must it go for the riders to feel weightless at the top?



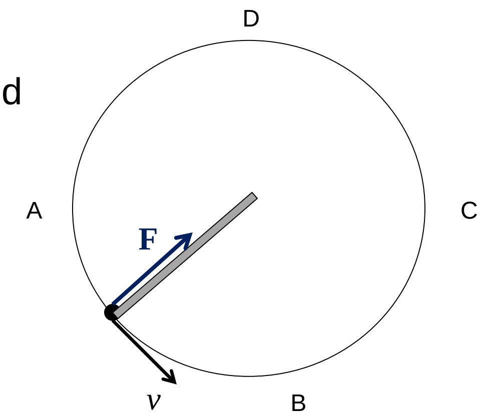
Feeling weightless means that there is no normal force on the passenger, who is therefore in free-fall, acted upon only by gravity. This is true if the normal force is zero: Then $v^2 = gR$. $\quad v = 12.1$ m/s.

Ball on a Rod

A 2 lb ball is carried in a circle at constant speed v by a rotating rod.

- Where is the **net** force on the rod the ball the greatest?

Nowhere: $F = mv^2/R$ everywhere. The net force keeps the ball moving in uniform circular motion.



Ball on a Rod

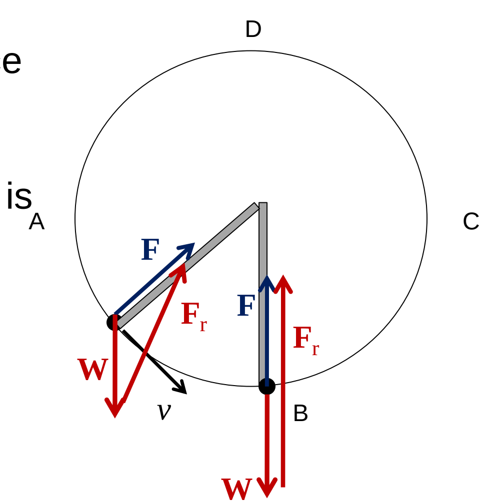
At which point is the force of the rod on the ball greatest?

The force due to the rod is the vector difference

$$\mathbf{F}_r = \mathbf{F} - \mathbf{W}.$$

This vector is longest when **F** and **W** are antiparallel: at point B.

There, $F_r = mv^2/R + W$.



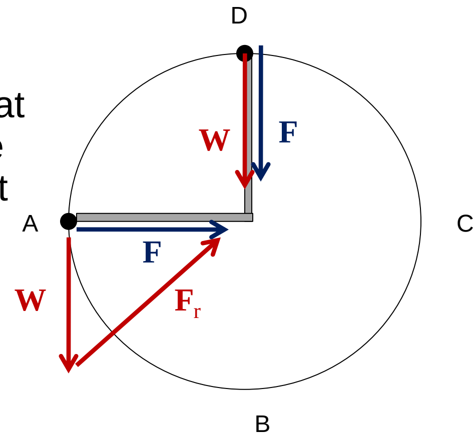
Ball on a Rod

Suppose the force of the rod on the ball is zero at point D. Find the force of the rod on the ball at point A.

At D, $W = F = mv^2/R$. So $F = W$ everywhere.

At A, $\mathbf{F}_r = \mathbf{F} - \mathbf{W}$ gives

$$F_r = W\sqrt{2} = 2.8\ \text{lb}$$



Effect of Force over Distance

- Applying a force to a particle over distance changes its speed in the direction of the force:

$$v_f^2 - v_i^2 = 2ax$$

$$mv_f^2 - mv_i^2 = 2max$$

$$F = ma$$

$$\tfrac{1}{2}\,mv_f^2 - \tfrac{1}{2}\,mv_i^2 = Fx$$

The Work-Energy Theorem

Definitions: Kinetic Energy = $K = \frac{1}{2} mv^2$.

Work = Fx.

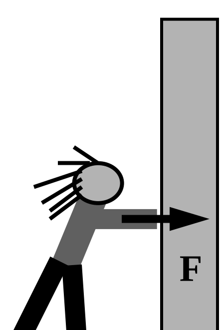
The work done by the net force on a mass force causes a change in kinetic energy:

$$\Delta K = W.$$

This is called the work-energy theorem.

Work Requires Motion

- Work is done only when there is motion.
- $W = Fx$ requires both F and x to be nonzero for W to be nonzero.
- You can push all day on a wall and get very tired, but if it doesn't move, you did no work on it.

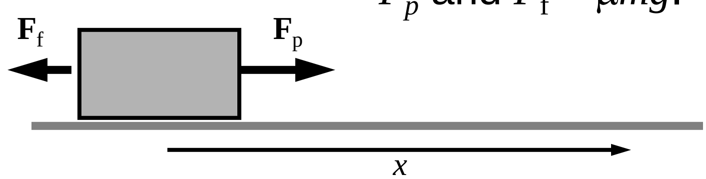


Example

Suppose you apply a force $F_p = 60\ \text{N}$ to a box of mass $m = 15\ \text{kg}$ initially at rest, with coefficient of friction $\mu = 0.3$, causing it to accelerate.

Two forces act on the box:

F_p and $F_f = \mu m g$.



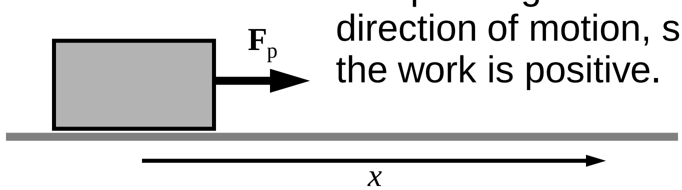
Example

1. How much work do you do on the box when it moves a distance $x = 12\ \text{m}$?

You do an amount of work

$$W = F_p x = (60\ \text{N})(12\ \text{m}) = 720\ \text{Nm}$$
$$= 720\ \text{J}$$

The pushing force is in the direction of motion, so the work is positive.



Example

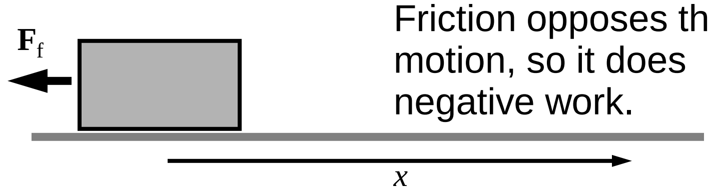
2. How much work does friction do when you move the box 12 m?

The force of friction is

$$F_f = -\mu mg = -(0.3)(15\text{ kg})(9.8\text{ m/s}^2) = -44\text{ N}$$

The work done by friction is

$$W_f = F_f x = (-44\text{ N})(12\text{ m}) = -528\text{ J}$$



Friction opposes the motion, so it does negative work.

Example

3. What is the kinetic energy of the box after being pushed 12 m?

The box was initially at rest, so the kinetic energy is the net work,

$$K = W_p + W_f = 720\text{ J} - 528\text{ J} = 192\text{ J}$$

4. What is the speed of the box after being pushed 12 m?

The kinetic energy is $K = \frac{1}{2} m v^2$.

$$v = \sqrt{2K/m} = \sqrt{2(192\text{ J}) / 15\text{ kg}} = 5.1\text{ m/s}$$