Physics 221

Department of Physics
The Citadel

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September 25, 2009

Circular Motion and Newton's Laws – Part 1

Announcements

- Due next Wednesday: problem set 6.
- Assigned Problems: Ch. 6:

  5, 11, 13, 18, 57

  (18, 57 are modified from the book versions: 18 has more parts, 57 has less.)
- Reading: Chapter 6, sec. 1 - 2.
- Start looking at Ch. 7: Energy, sec. 1 – 3.
- Monday: finish Ch. 6, start Ch. 7.

Uniform Circular Motion

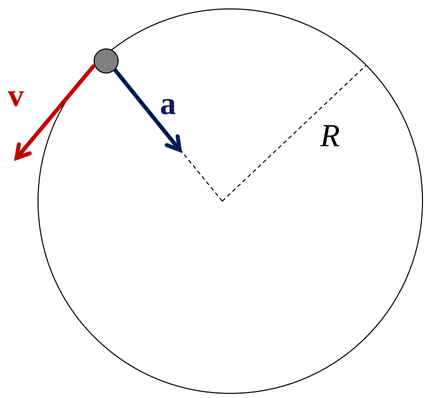
An object in uniform circular motion with period T has speed given by

$$v = 2\pi R/T$$

and acceleration given by

$$a = 2\pi v/T = v^2/R$$

directed toward the center.

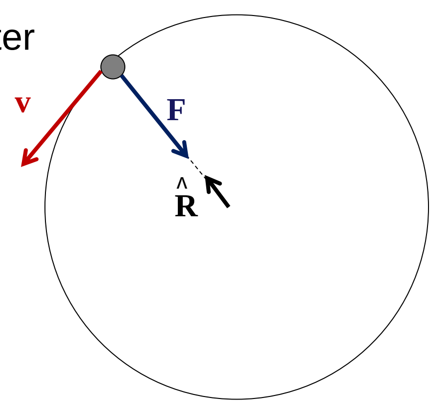


Centripetal Force

From the fact that there is centripetal acceleration directed toward the center of a circle in uniform circular motion, we can infer that the **net force** on the object is

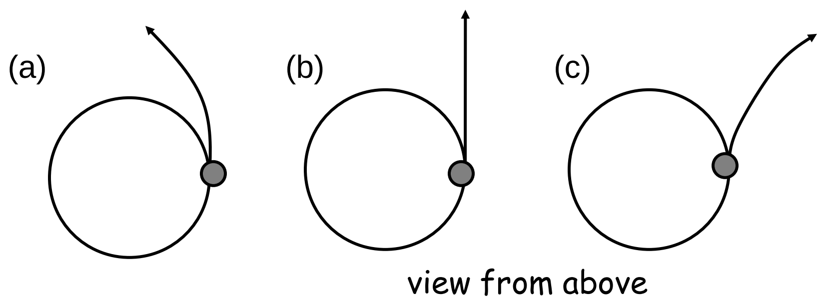
$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = -\frac{mv^2}{R}\hat{\mathbf{R}}$$

also directed toward the center of the circle ($-\hat{\mathbf{R}}$).



Circular Motion

If you twirl an object around your head on a string and then let go, which way does it travel?



Circular Motion

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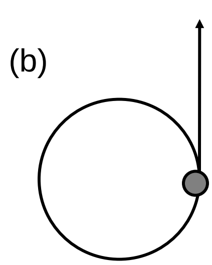


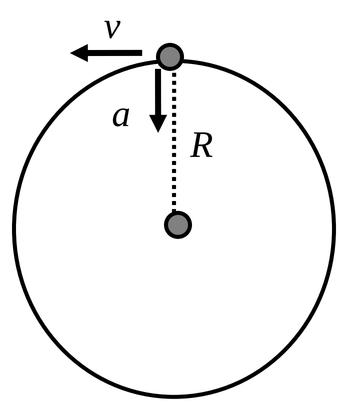
As soon as you let go, the centripetal force no longer acts, so the object moves in a straight line at a constant speed (horizontally): the same speed it was going in a circle, and in the same initial direction (tangent to the circle).

The Moon

Newton realized that gravitation is also the cause of the centripetal acceleration of the moon toward the earth:

$$a = v^2/R$$

What is the moon's acceleration?



The Moon

The velocity is related to the Moon's period.

$$v = 2\pi\, R\, /\, T$$

Therefore,

$$a = 4\pi^2 R/T^2$$

For the moon,

R = 3.84 x 10^8 m, T =2.36 x 10^6 s

Therefore,

$$a = 4\pi^2 R/T^2 = 2.72 \times 10^{-3}\ \text{m/s}^2$$

The Moon

This is much smaller than the acceleration of gravity on earth. In fact, $a = g/3600$

Newton compared this to the fact that the moon is 60 earth radii from the center of the Earth and noted that

$$ma/mg = a/g = (R_e/R)^2$$

The force of gravity is inversely proportional to the distance from the center of the earth.

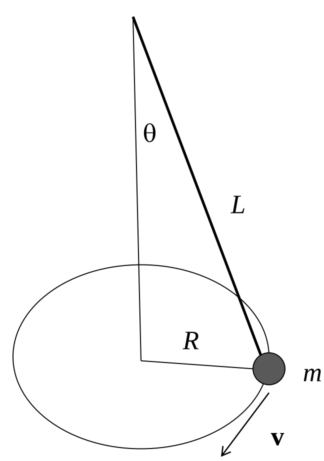
From this, Newton inferred the inverse square law of gravity: F_g is proportional to $1/R^2$.

Conical Pendulum

An 80 kg bob hangs on a 10 m string making an angle of 5º to the vertical.

- What is the period of the pendulum?

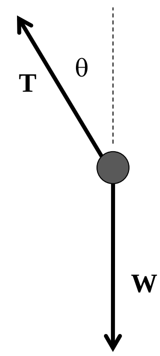
Apply $\mathbf{F} = m\mathbf{a}$ to the pendulum bob.



Conical Pendulum

First, isolate the bob and draw a free body diagram, showing all forces on it.

There are only two forces: tension and weight.



Newton's law: we know

$a = v^2/R$ to the left, so

y: $W = mg = T\cos\theta$

x: $T\sin\theta = ma = mv^2/R$.

Conical Pendulum

y: $mg = T\cos\theta$

x: $T\sin\theta = mv^2/R$.

$T\sin\theta = mg\sin\theta/\cos\theta = mv^2/R$.

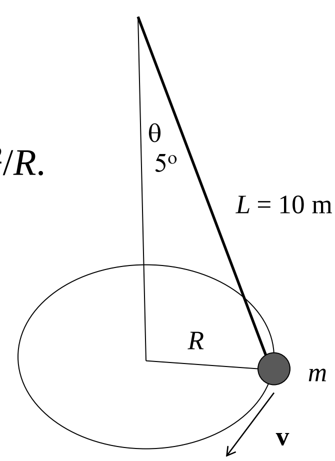
$g\tan\theta = v^2/R$.

$R = L\sin\theta = 0.872$ m

$v^2 = Rg\tan\theta = 0.0747\ \mathrm{m^2/s^2}$

$v = 0.273$ m/s

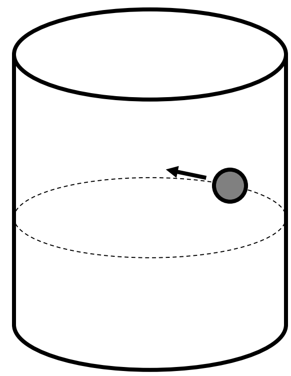
Period = $2\pi R/v = 20.0$ s.



The Rotor

The Rotor is a ride with a rotating cylinder in which people stand along the wall, as it spins, and then the floor drops out when it reaches a fast enough speed for them to stick to the walls.

If $\mu_s = 0.75$ and the radius of the cylinder is 4.0 m, what is the minimum rpm to avoid slipping down?



The Rotor

y: forces balance: $F_f = mg$

$F_f = \mu F_N$ if just about to slip.

x: $F_N = mv^2/R$

$R = 4.0$ m
$g = 9.8$ m/s^2
$\mu = 0.75$

Combine:

$mg = \mu F_N = \mu m v^2/R$

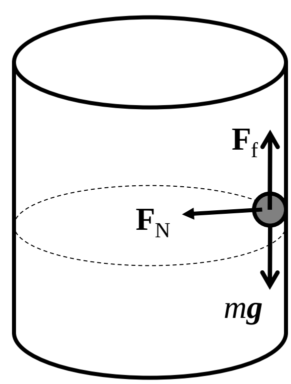
$v^2 = Rg/\mu = 52.27$ m/s

$v = 7.23$m/s.

$f = 1/T$

$= v/2\pi R = 0.288$ rev / sec

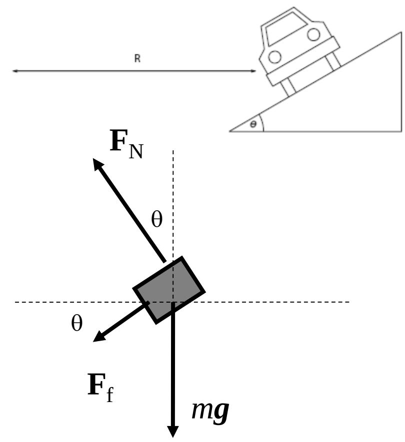
$= 17.3$ rpm.



Banked Curve

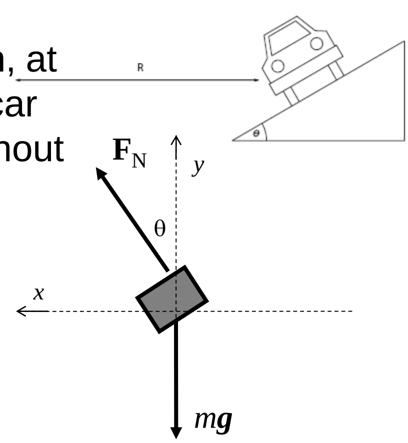
An architect is designing a circular track for race cars. It has a 200 m radius and 30° slope.

a) Draw the free body diagram for a car going around the track. Include friction.



Banked Curve

b) If there were **no** friction, at what speed v_0 could a car go around the track without sliding up or down?



x: $F_N \sin\theta = ma = mv_0^2/R$

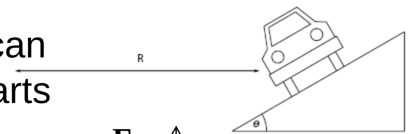
y: $F_N \cos\theta = mg$

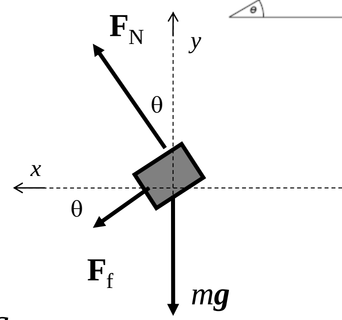
$\tan\theta = v_0^2/Rg.$

$v_0 = \sqrt{Rg\tan\theta} = 34\ \text{m/s}$

Banked Curve

c) If μ_s = 0.40, how fast can the car go before it starts to slide up?





Call this speed v_1.

Newton's Laws:

x: $F_N \sin\theta + F_f \cos\theta$

$= ma = mv_1^2/R$

y: $F_N \cos\theta - F_f \sin\theta = mg$

$F_f = \mu_s F_N$

Banked Curve

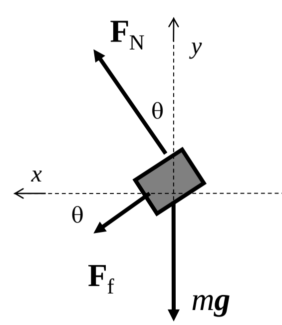
$F_N \sin\theta + F_N \mu \cos\theta = mv_1^2/R$

$F_N \cos\theta - F_N \mu \sin\theta = mg$

Divide the two sides:

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v_1^2}{Rg}$$

= 1.28



$v_1^2 = 1.28\, Rg$

$= 1.28 \times 200 \text{ m} \times 9.8 \text{ m/s}^2$

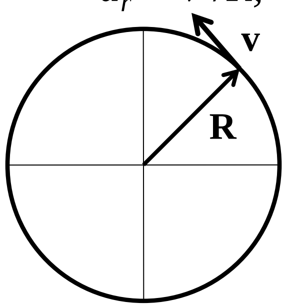
$= 2509 \text{ m}^2/\text{s}^2$

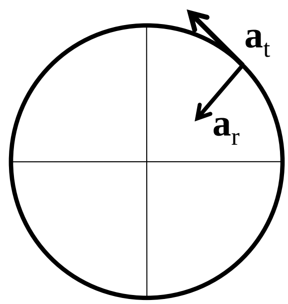
$v_1 = 50.1$ m/s.

Non-Uniform Circular Motion

Recall: An object in non-uniform circular motion has both centripetal and tangential acceleration:

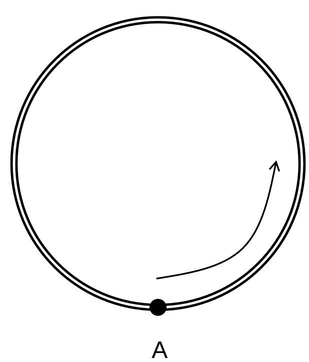
$$a_r = v^2/R, \qquad a_t = dv/dt.$$





Exam Problem

In the exam problem, a car started from rest at point A, and accelerated with constant tangential acceleration around the 1200 m circular track, arriving at the starting point in 52 seconds.



(a) What is the tangential acceleration?

Exam Problem

This is a one-dimensional problem along the road traveled.

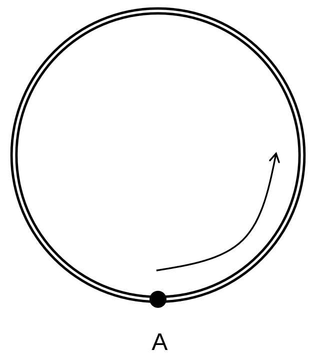
D = 1200 m complete circle

t = 52 s.

$D = ½ a_t t^2$ (starting from rest)

1200 m = $½ a_t (53\ s)^2$

$a_t = 0.888\ m/s^2$.



Exam Problem

(b) What is the velocity vector at point B?

The magnitude is the speed, and the direction is forward, the way the car is headed.

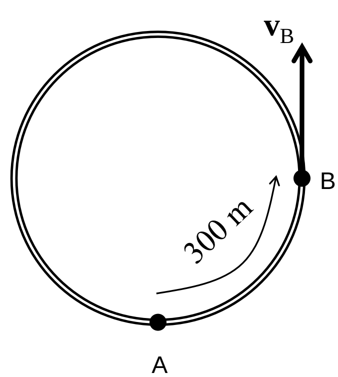
One-dimensional constant acceleration equation:

$v_B^2 = v_A^2 + 2a\,x$

$= 0 + 2(0.888\ m/s^2)(300m)$

$= 532.5\ m^2/s^2$

$v_B = 23.1$ m/s.



Exam Problem

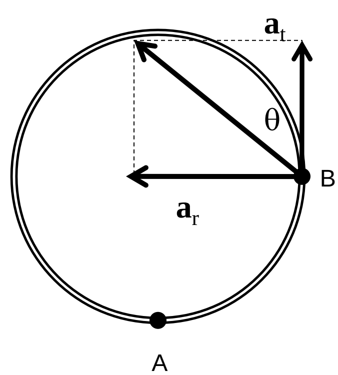
(c) What is the acceleration vector at point B?

The tangential acceleration is

$\mathbf{a}_t = 0.888\ \mathrm{m/s^2}$ forward.

The radial acceleration is

$\mathbf{a}_r = v_B^2/R = 2.74\ \mathrm{m/s^2}$ inward.



$$a = \sqrt{a_t^2 + a_r^2} = 2.88\,\tfrac{\mathrm{m}}{\mathrm{s}^2}$$

$$\theta = \tan^{-1}\left(\frac{2.74}{0.888}\right) = \tan^{-1}(3.09) = 72.0^{\circ}$$