Physics 221

S. Yost

September 21, 2009

Newton's Laws – Part 2

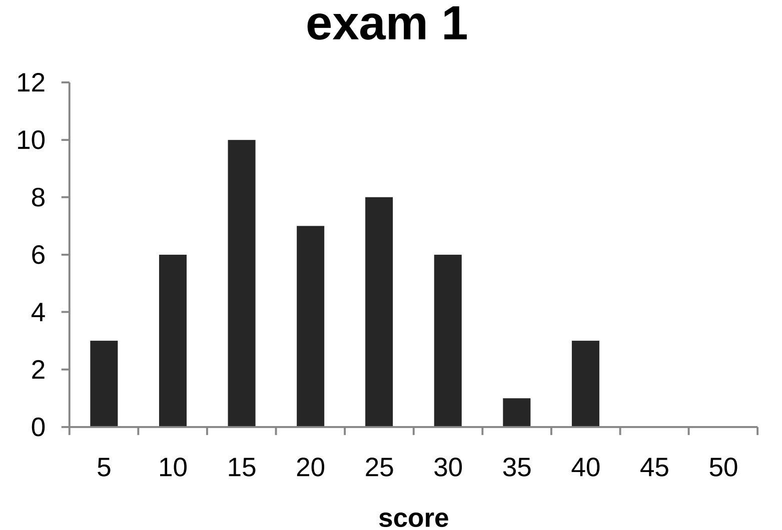
Ropes, pulleys, inclined planes, and friction

Announcements

- Problem set 5 is due Wednesday. The problems are Chapter 5: 3, 17, 20, 28, 29, 40, 43, 57.
- The exams are graded and will be returned Wednesday. The average was 18.3 / 50.
- Today we will consider some applications of Newton's Laws.

Rough grading scale:

C > 15 points,
B > 25 points,
A > 35 points.



Newton's Laws

- Last time, we wrote down Newton's 3 laws of motion:

1. An object experiencing no force moves with constant velocity in an inertial frame of reference.
2. An object of mass m experiencing a net force $\mathbf{F}$ experiences an acceleration $\mathbf{a}$ in an inertial frame of reference given by $\mathbf{F} = m\mathbf{a}$.
3. If object 2 exerts a force $\mathbf{F}_{12}$ on object 2, object 2 exerts a force $\mathbf{F}_{21}$ on object 1 with $\mathbf{F}_{21} = -\mathbf{F}_{12}$.

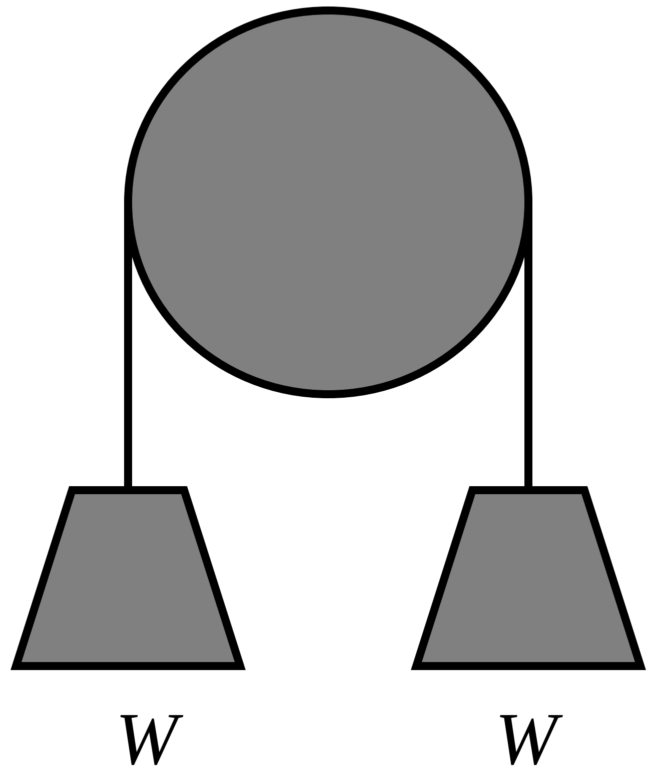
Applying Newton’s Laws

- Newton’s Laws can be used to determine the motion of an object when the forces on it are known, or to determine a forces on it when the motion is known.
- In this chapter, we will be considering primarily applications of $\mathbf{F} = m\mathbf{a}$.
- The forces we will consider in this chapter are weight (gravity), tension, the normal force, and friction.

Tension Question

A rope passes over a frictionless pulley, connecting two masses.

What is the tension in the rope?

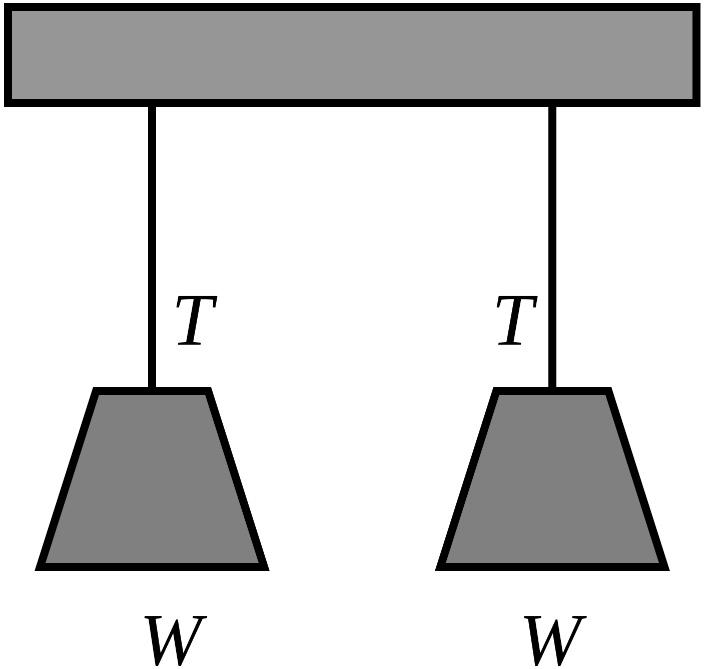


- Is it W or $2W$?

Tension Question

Does it make any difference if I ask for the tensions in this diagram?

- Two separate ropes hang from the ceiling. The tensions are both clearly W.

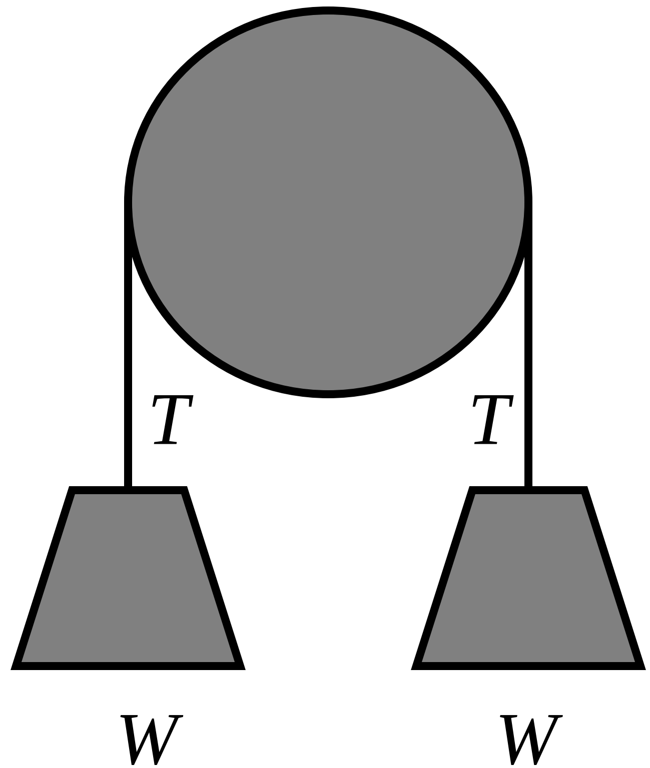


Tension Question

Physically, these are equivalent situations – either way, each half of the rope supports the full weight!

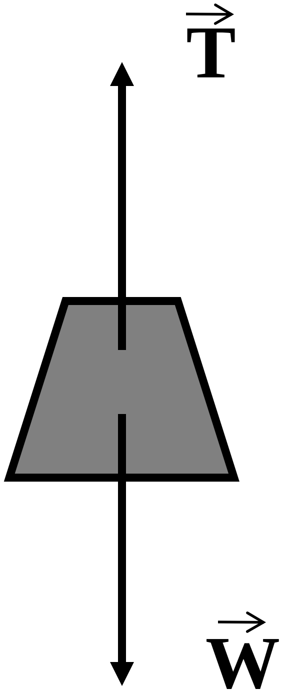
- So $T = W$, not $2W$

Don't add tensions on opposite ends of the rope!!



Free Body Diagram

- Isolate one object in the situation, and show all forces acting on just that object – don't confuse things by putting any forces acting on other objects. They don't determine this object's motion.
- The object is at rest, so $T = W$.



Sign

What is the tension in each rope supporting the sign?

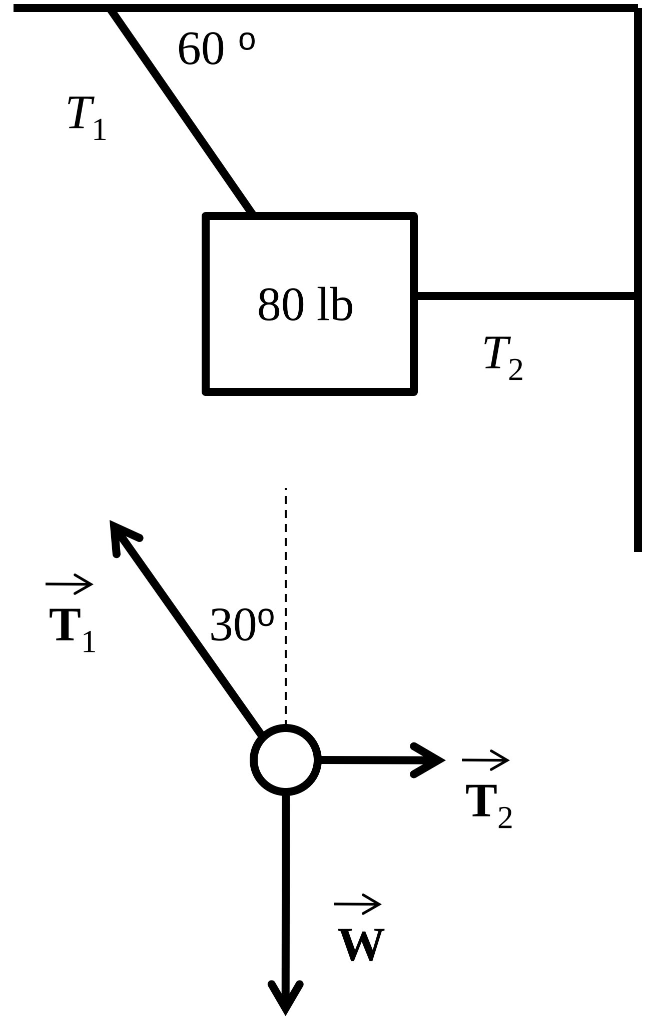
- The sign is in equilibrium.
- The vector sum of the forces on the sign is zero.

x: $T_2 = T_1 \sin 30^o = ½\ T_1$

y: $W = 80\text{ lb} = T_1 \cos 30^o$

$= 0.866\ T_1.$

$T_1 = 92.4\text{ lb}, \quad T_2 = 46.2\text{ lb}$



free body diagram

Pulleys

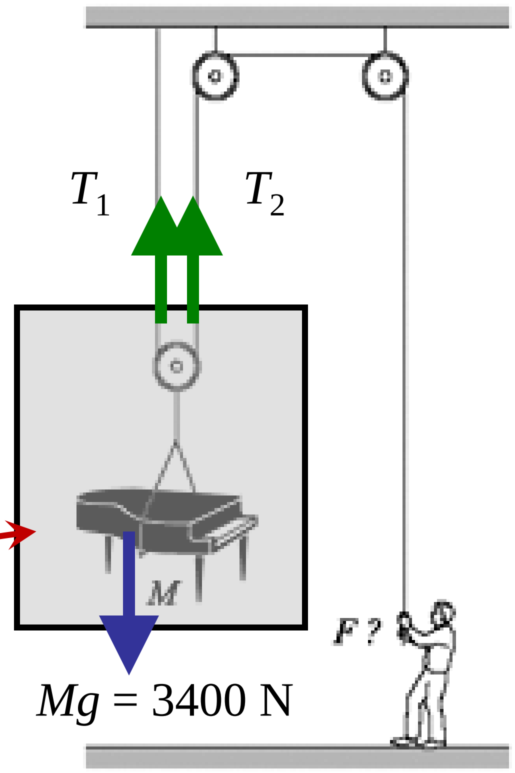
- What force F must be applied to the rope to hold the piano stationary if $M = 340$ kg?
- Take $g = 10$ N/kg. Then the piano weighs

$$Mg = 3400 \text{ N.}$$

Balance forces:

$$T_1 + T_2 = Mg = 3400 \text{ N.}$$

Always isolate the object on which you are balancing the forces!



Show the forces on the isolated object. This is a **free-body diagram** for the piano.

Pulleys

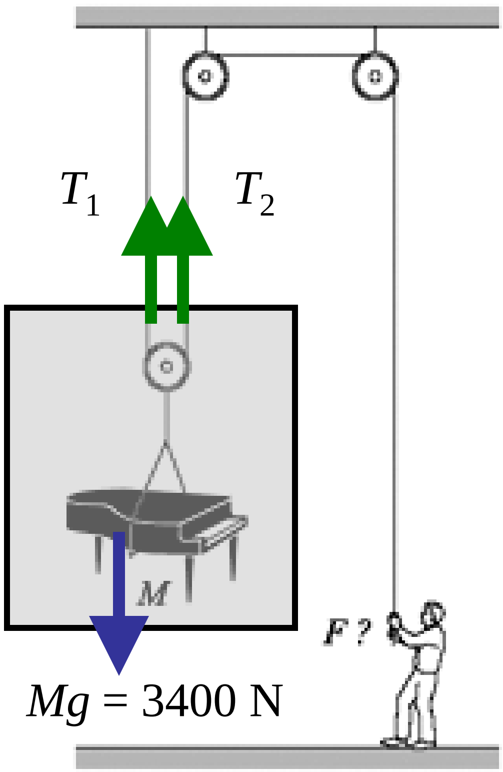
- How is T_1 related to T_2?

  The ropes are connected by a pulley free to rotate, so the tensions are the same: $T_1 = T_2$.

- How is T_2 related to F ?

  Again, the ropes are connected by pulleys, which just redirect the force, so

  $T_2 = F$.



Balance forces:

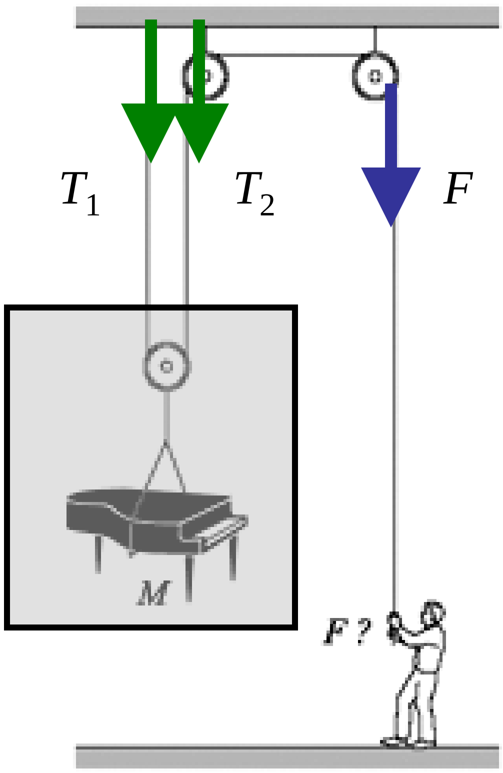
$$2F = 3400\ \text{N}$$
$$F = 1700\ \text{N}$$

Pulleys

- How much total force is on the roof?

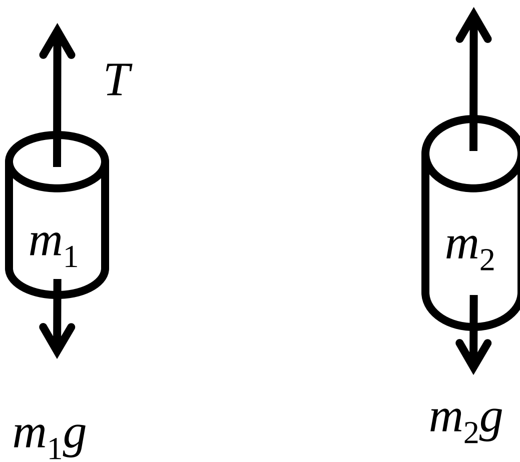
The three downward forces on the pulleys also pull down on the roof.

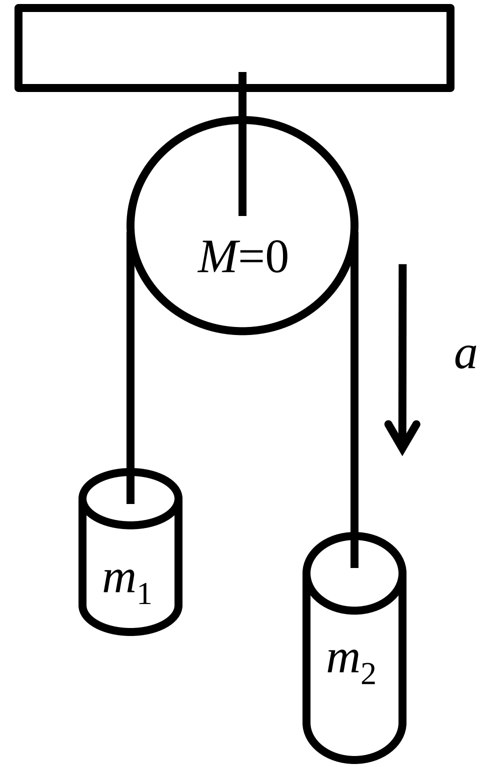
$$T_1 + T_2 + F$$
$$= 3F = 5100 \text{ N}$$



Atwood Machine

- What is the acceleration?
- Free body diagrams:





$$m_1\, a = T - m_1\, g$$

$$m_2\, a = m_2\, g - T$$

Add:

$$(m_1 + m_2)\, a = (m_2 - m_1)\, g$$

$$a = \frac{m_2 - m_1}{m_2 + m_1}\, g$$

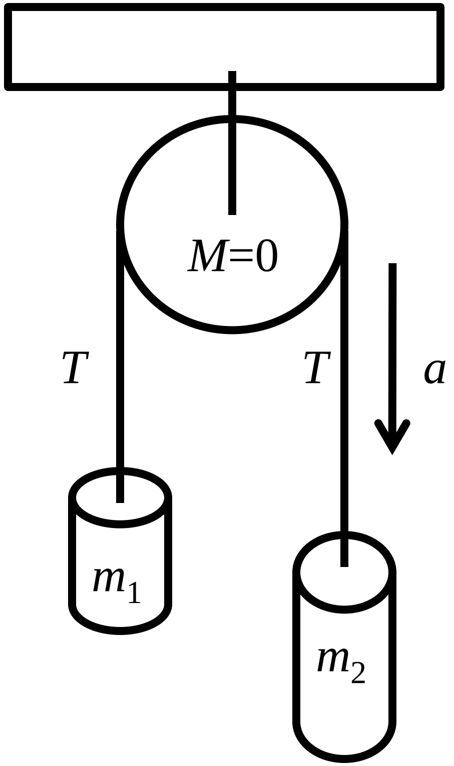
Atwood Machine

- What is the tension in the cord?

$$m_1\, a = T - m_1\, g$$

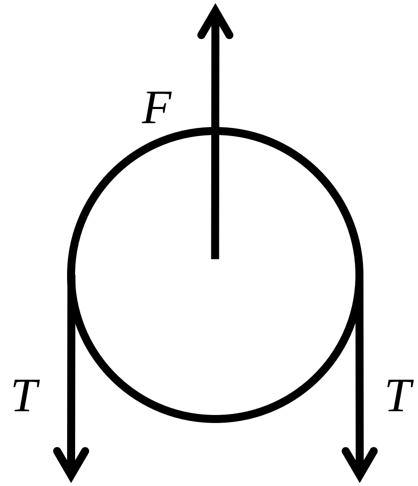
$$T = m_1(a+g) = m_1\left(\frac{m_2 - m_1}{m_2 + m_1} + \frac{m_2 + m_1}{m_2 + m_1}\right)g$$

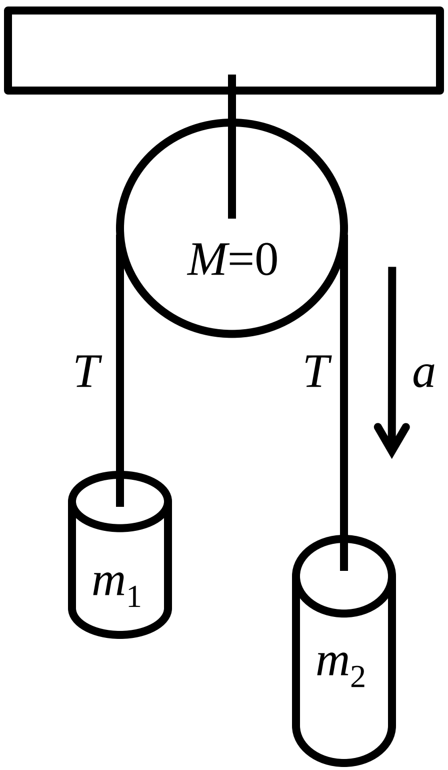
$$= \frac{2m_1 m_2 g}{m_2 + m_1}$$



Atwood Machine

- What is the force on the ceiling?

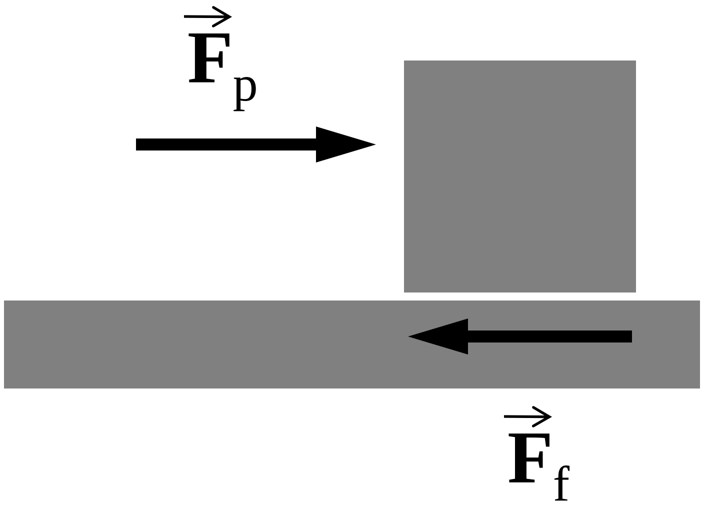




$$F = 2T.$$

Friction

- If I push on a block and it doesn't move, then my force must be balanced by another force: friction.
- The force of friction equals the force I push with, in the opposite direction, up to a limit.



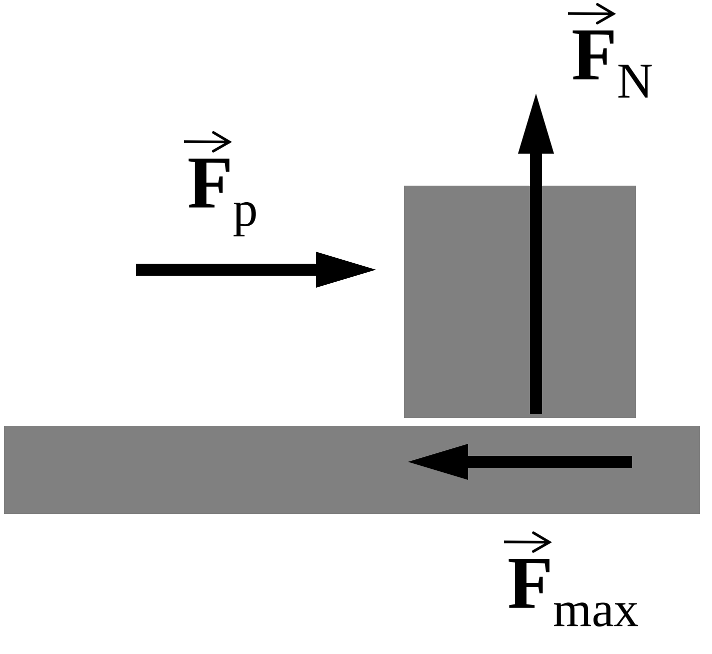
Friction.

The force it takes to overcome friction depends on the normal force.

The maximum force of friction before the box starts sliding is

$$F_{\mathrm{max}} = \mu_s F_N,$$

μ_s = coefficient of static friction.

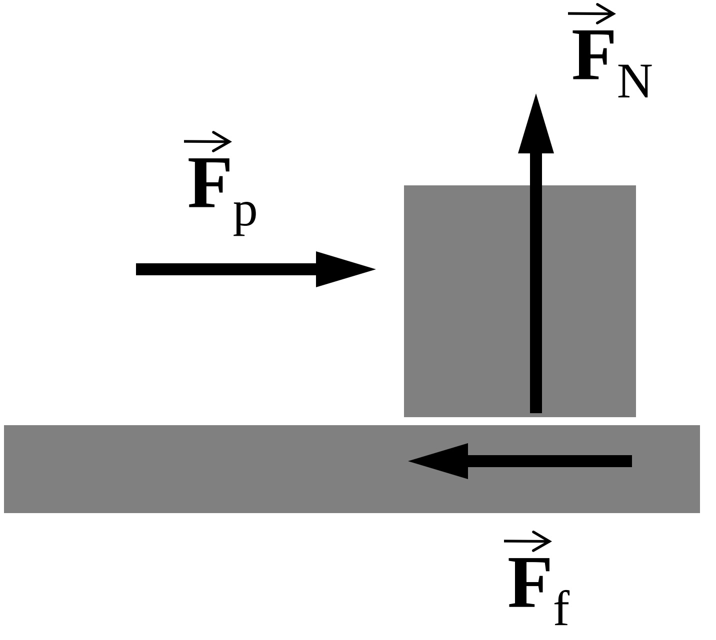


Friction.

When the box is sliding, the frictional force is proportional to the normal force, but somewhat less than the force needed to get the motion started.

$$F_f = \mu_k F_N,$$

μ_k = coefficient of kinetic friction.

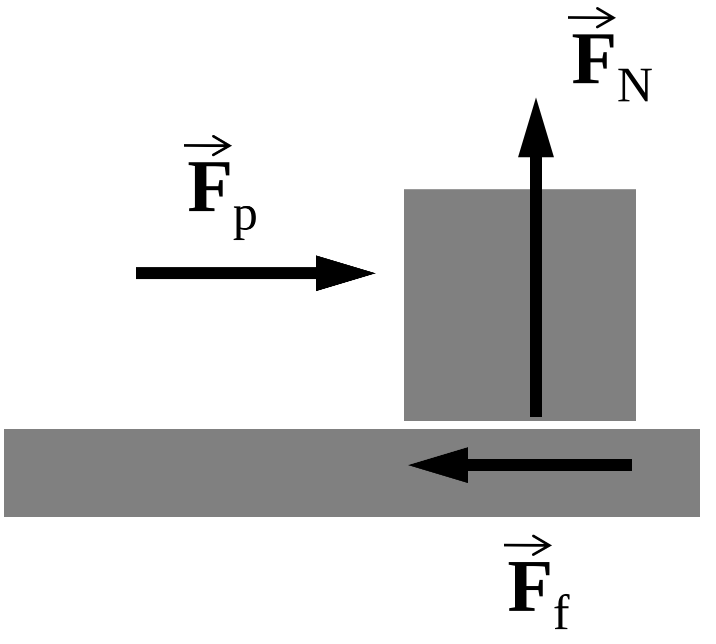


Static vs Kinetic Friction

Suppose $\mu_s = 0.75$, $\mu_k = 0.50$ and the box weighs 100 N.

(a) What is the frictional force if I push with a force of 40 N?

(b) What is the frictional force if I push with a force of 80 N?



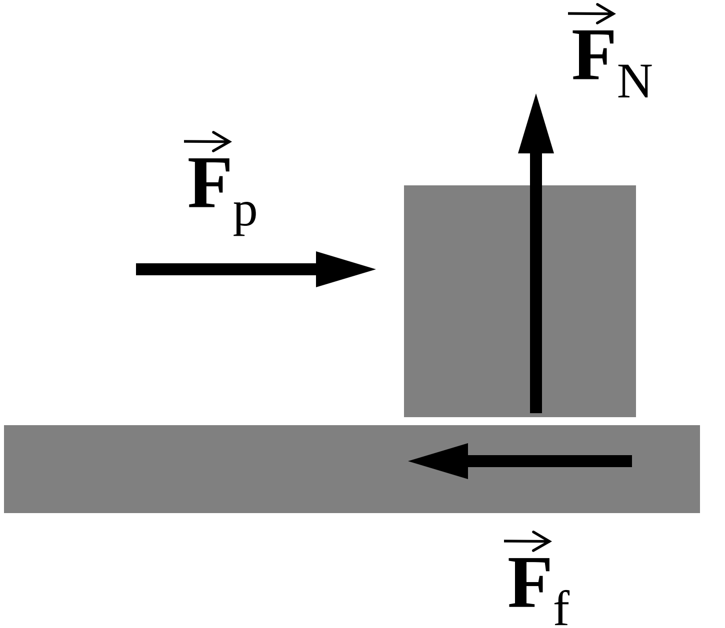
Static vs Kinetic Friction

(a) The force needed to overcome static friction is

$$\mu_s F_N = 75 \text{ N}.$$

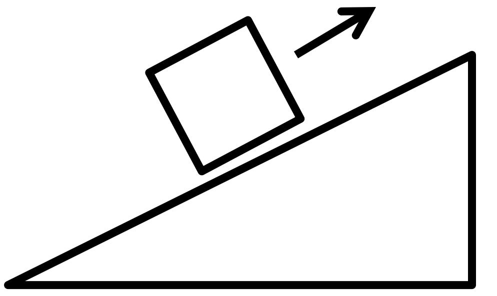
40 N is not enough to overcome static friction, so the box will push back with a frictional force of $F_f = 40$ N.

(b) 80 N is enough for the box to be moving, so $F_f = \mu_k F_N = 50$ N



Sliding Box

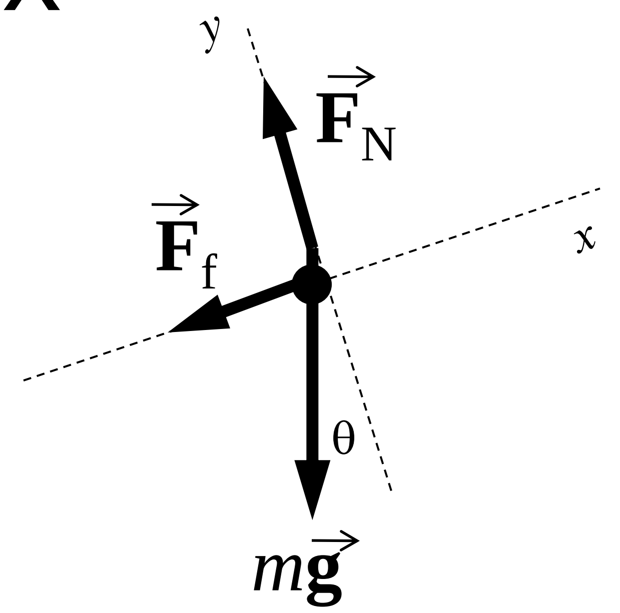
- A box of mass 10 kg is given an initial push at an initial speed of 2.5 m/s up a plane with coefficient of friction 0.10 and angle 30°.



- How far up the plane does it go?
- If the acceleration is a (< 0),

$$v_f^2 - v_0^2 = 0 - v_0^2 = 2ax.$$

Sliding Box



Start with a free body diagram.

Let the x axis run up the plane,

The y axis perpendicular to it.

y: $F_N = mg \cos \theta$.

Friction is $F_f = \mu_k F_N = \mu_k mg \cos \theta$.

x: Net force: $F_f + mg \sin \theta$ downhill.

$$ma = -F_f - mg \sin \theta$$

$$= -mg(\mu_k \cos \theta + \sin \theta).$$

$$a = -(\mu_k \cos \theta + \sin \theta)g = -0.5866\, g$$

The masses cancel: no need to know m.

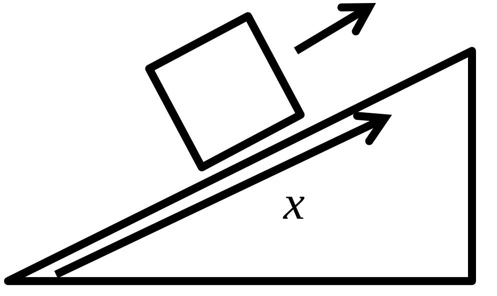
Sliding Box

$v_0^2 = -2ax.$

$(2.5 \text{ m/s})^2 = 2(0.5866\ g)\ x$

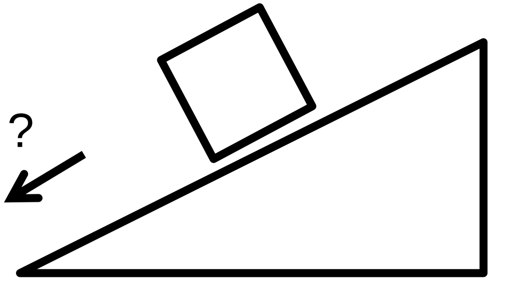
$6.25 \text{ m}^2/\text{s}^2 = (11.50 \text{ m/s}^2)\ x$

$x = 54.4 \text{ cm.}$

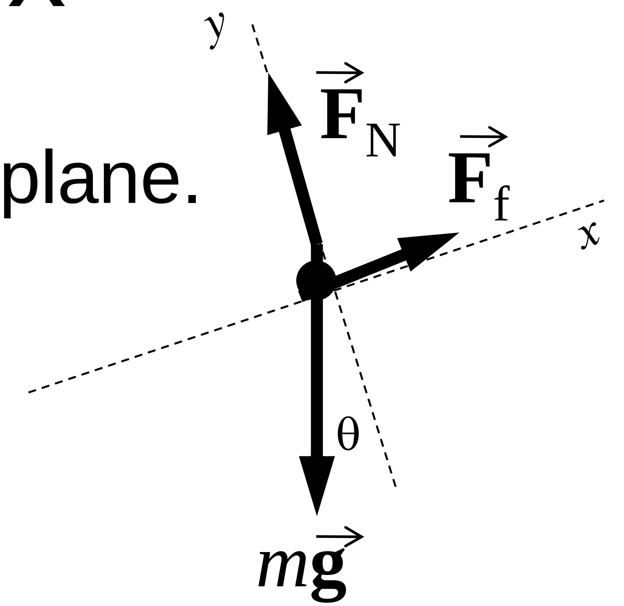


After it stops, will it slide down the hill again?

Assume $\mu_s = 0.20.$



Sliding Box



Now friction acts up the inclined plane.

The box will slide down again if

$mg \sin\theta > F_f = \mu_s F_N$

$= \mu_s mg \cos\theta.$

This means $\tan\theta > \mu_s$.

$\tan 30^o = 0.577$.

If $\mu_s < 0.577$, the box will slide back down the hill. The given value was $\mu_s = 0.2$, so the box will slide back down.