Physics 221

Sections 1 and 2

S. Yost
September 11, 2009

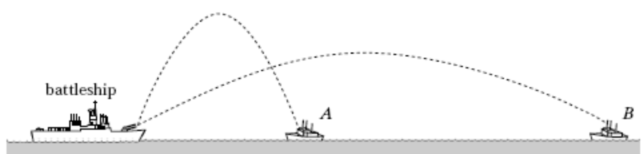
Two-Dimensional Motion: Part 2

Announcements

- Set 4 is open and due 7 AM **Monday**. It covers chapter 4 and has 8 problems: 4.6 (modified), 4.11, 4.17, 4.24, 4.31, 3.53, 4.41, and 4.45
- The topic is 2-dimensional motion. Today, sec. 4 – 6 : circular motion (uniform and non-uniform), relative velocity
- Monday – problem discussion and exam review.
- Wednesday: Exam 1, Chapters 2 – 4.

Battleships

A battleship fires shells at two ships A and B simultaneously, with the trajectories shown. Which ship gets hit first?

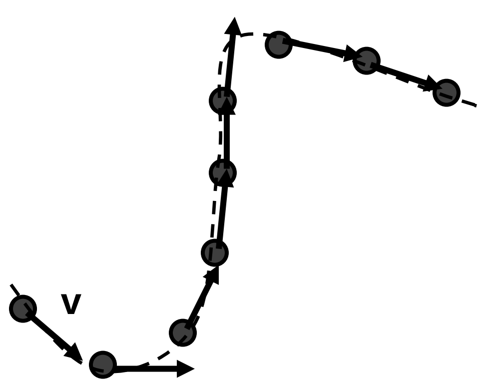


The time of flight is determined by the vertical motion – the higher trajectory will have the longer time of flight. Ship B is hit first.

Acceleration at Constant Speed

Can an object with a constant speed have a nonzero acceleration?

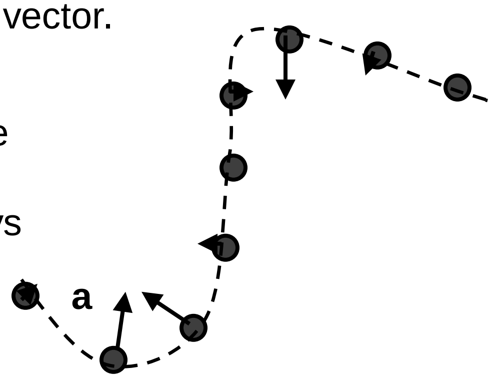
- Yes – if the speed is constant, the magnitude of the velocity is constant, but the direction changes.
- The acceleration is zero only if the velocity **vector** is constant.



Acceleration at Constant Speed

Draw the acceleration vector.

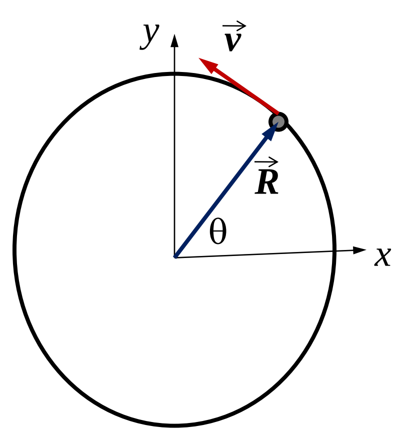
- Since the velocity component along the path is constant, the acceleration is always perpendicular to the path, showing which way it turns.
- The sharper the turn, the bigger the acceleration.



Uniform Circular Motion

A simple example of this is uniform circular motion, where a particle moves in a circle of radius R at constant speed v.

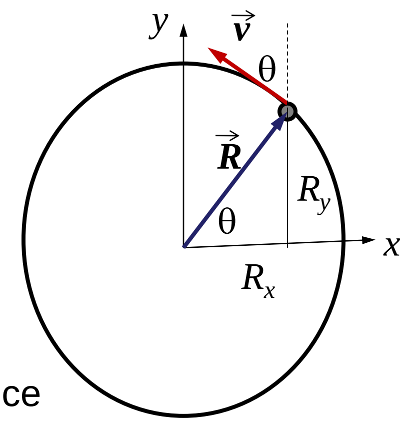
The particle is always accelerating perpendicular to its velocity vector, toward the center of the circle.



Uniform Circular Motion

Position vector:

$R_x = R \cos\theta$, $R_y = R \sin\theta$



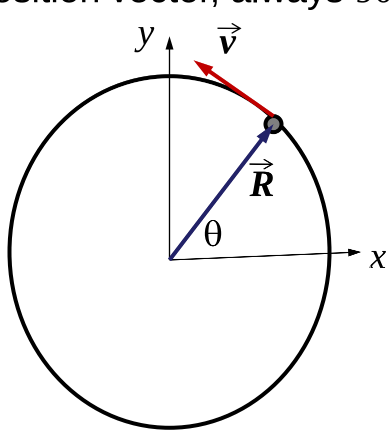
The velocity vector is perpendicular to $\vec{R}$:

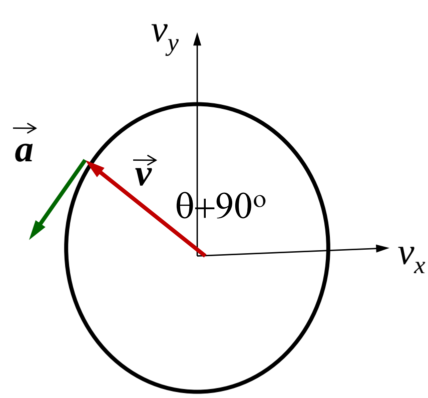
$v_x = -v \sin\theta$, $v_y = v \cos\theta$.

The speed is the circumference divided by the period for one revolution: $v = 2\pi R/T$.

Uniform Circular Motion

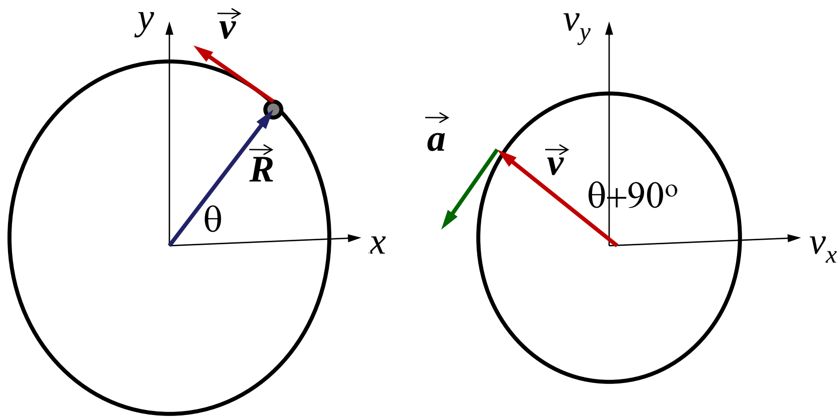
The velocity vector rotates at the same rate as the position vector, always 90^o ahead of it.





Uniform Circular Motion

Note the complete analogy between the position diagram and velocity diagram. $\vec{v}$ is to $\vec{R}$ as $\vec{a}$ is to $\vec{v}$.



Uniform Circular Motion

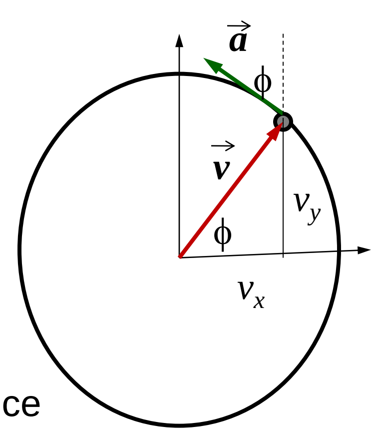
Velocity vector: $\phi = \theta + 90^o$

$v_x = v \cos\phi,\ v_y = v \sin\phi$

The acceleration vector is perpendicular to $\vec{v}$:

$a_x = -a \sin\phi,\ \ a_y = a \cos\phi.$

The acceleration is the distance the tip of the velocity vector travels divided by the period for one revolution: $a = 2\pi v/T$.

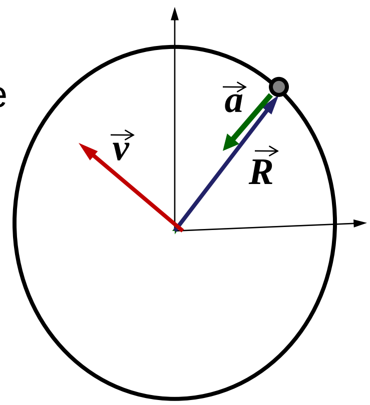


Compare: $v = 2\pi R/T$.

Uniform Circular Motion

The position, velocity, and acceleration vectors all rotate in a circle as shown. Note that $\vec{a}$ is always opposite $\vec{R}$.

From the point of view of the particle, $\vec{a}$ is pointing toward the center, and is therefore called **centripetal acceleration**.



$$a = \frac{2\pi v}{T} = \frac{2\pi v}{2\pi R / v} = \frac{v^2}{R}$$

Example

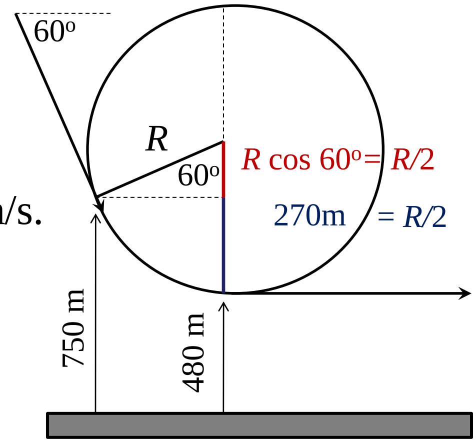
Suppose a plane is headed downward at 125 m/s at an angle of 60^o . At an elevation of 750 m, it begins to follow a circular arc leading into a horizontal path at an elevation of 480 m.

What is the plane's acceleration in the arc if its speed remains constant?

Express the answer in g's: as a multiple of the acceleration of gravity.

Example

Constant speed implies uniform circular motion, so
$a = v^2/R$. But how big is the circle?



$R \cos 60^{o} + 270 = R$

$\cos 60^{o} = ½$

$R = 540$ m. $v = 125$ m/s.

$a = v^2/R = 29.9$ m/s^2

$a/g = 29.9/9.8 = 3.05.$

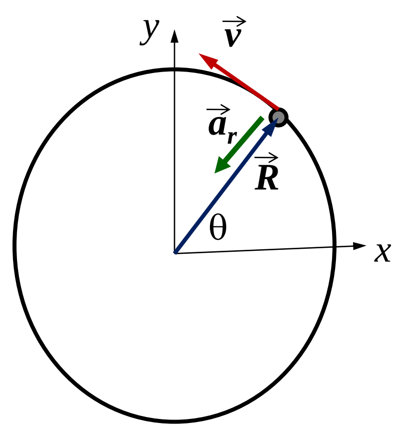
$a = 3.05\ g$'s

Non-Uniform Circular Motion

There is still a **centripetal**, or **radial**, acceleration, causing the direction of the path to change:

$$a_r = v^2/R$$

It is directed perpendicular to the path in the direction the path turns (inward).



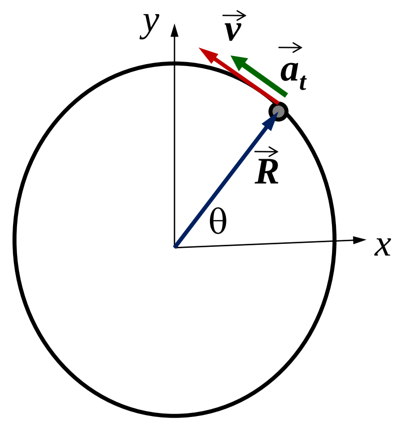
But this is not the whole story.

Non-Uniform Circular Motion

If there is a change in the speed along the path, there is also tangential acceleration, determined by the rate of change of the speed:

$$a_t = dv/dt$$

along the path (forward if speeding up, backward if slowing down).



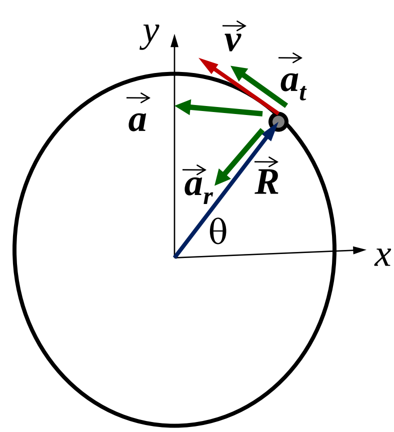
Non-Uniform Circular Motion

The net acceleration is the vector sum of the radial (centripetal) and tangential acceleration:

$$\vec{a} = \vec{a}_r + \vec{a}_t \ .$$

In terms of unit vectors along the radius and velocity,

$$\vec{a} = -\left(\frac{v^2}{R}\right)\hat{R} + \left(\frac{dv}{dt}\right)\hat{v}$$



General Motion

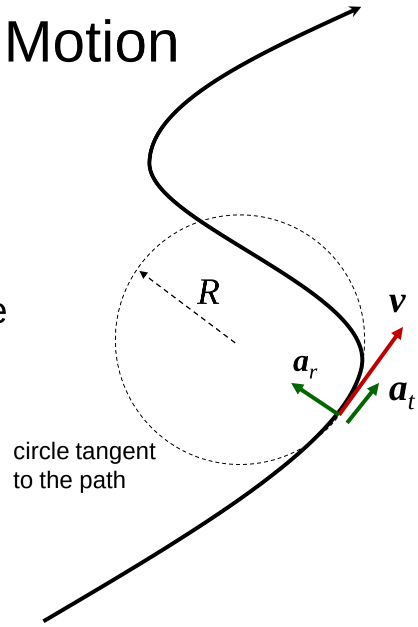
- This can be generalized further: the radius can be the instantaneous radius of curvature of a path.
- If an object is moving on any path at speed $v(t)$, the acceleration can always be separated into a tangential term,

  $a_t = dv/dt,$

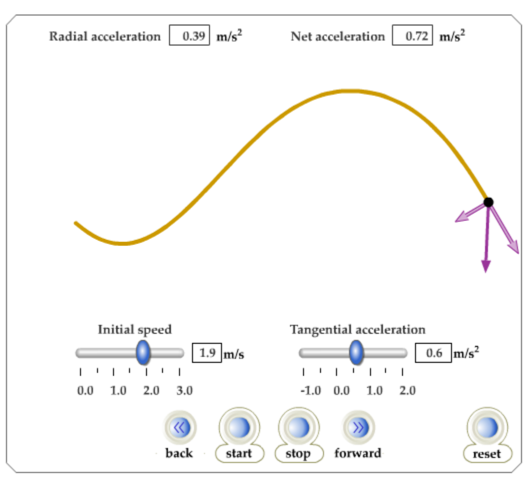
  and a radial term,

  $a_r = v^2/R$

  with $R(t)$ the instantaneous radius of curvature.



Tangential and Radial Acceleration



Example

- A car travelling 25 m/s enters a curve of radius 200 m and slows down to 20 m/s in 2.0 seconds. If the acceleration is uniform in the direction of motion, what is the car's accleration when it is travelling 22 m/s? Give the magnitude and direction relative to the forward direction.

Example

- Radial acceleration:

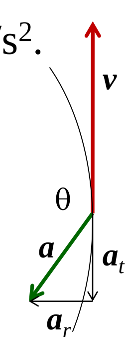
  $a_r = v^2/R = (22\text{ m/s})^2 / 200\text{ m} = 2.42\text{ m/s}^2$.
- Tangential acceleration:

  $a_t = \Delta v/\Delta t = (-5\text{ m/s}) / 2.0\text{ s} = -2.5\text{ m/s}^2$.
- Magnitude: these are perpendicular, so

$$a = \sqrt{a_r^2 + a_t^2} = 3.46\,\text{m/s}^2$$

- Direction: $\cos\theta = -2.5 / 3.46 = -0.723$

$$\theta = 136^{\circ}.$$



Relative Velocity

In general, the velocity of an object depends on the observer's point of view. We've been assuming the observer is at rest, but that is not always the case.

- If object A moves with velocity $\vec{v}_{AB}$ from the point of view of object B, and object B moves with velocity $\vec{v}_{BC}$ from the point of view of object C, then object A moves with velocity $\vec{v}_{AC}$ from the point of view of object C, where $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$.

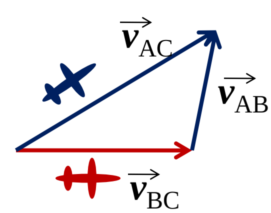
Relative Velocity

Example: Object C could be the ground. You could be in plane A (red), observing plane B (blue).

Your plane's velocity with respect to the ground is $\vec{\boldsymbol{v}}_{BC}$. The other plane's velocity with respect to the ground is $\vec{\boldsymbol{v}}_{AC}$.

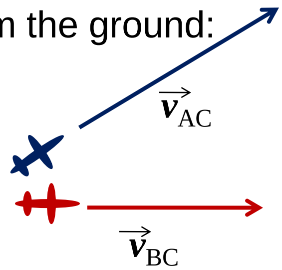
The apparent velocity of the other plane as seen from your plane is $\vec{\boldsymbol{v}}_{AB}$.

$$\vec{\boldsymbol{v}}_{AC} = \vec{\boldsymbol{v}}_{AB} + \vec{\boldsymbol{v}}_{BC}$$

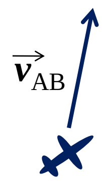


Relative Velocity

From the ground:



From your plane (red):



The plane appears to move in the direction of the vector $\vec{v}_{AB} = \vec{v}_{AC} - \vec{v}_{BC}$ with its nose along $\boldsymbol{v}_{AC}$.

Relative Velocity

Another context:

A plane in the wind: A = plane, B = wind, C = ground.

$\vec{v}_{AC}$ = plane’s ground speed in the direction it actually moves.

$\vec{v}_{AB}$ = plane’s air speed in the direction the nose is pointed (compass heading).

$\vec{v}_{BC}$ = the wind speed in the direction it is blowing.

Or similarly, with a boat in moving water, where v_{AB} is often given as the boat’s speed in still water.