Physics 221

Sections 1 and 2

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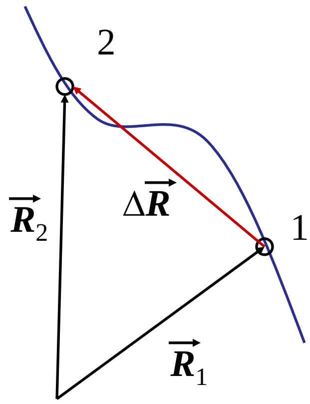
September 9, 2009

Two-Dimensional Motion: Part 1

Announcements

- Set 4 is open and due 7 AM **Monday**. It covers chapter 4 and has 8 problems: 4.6 (modified), 4.11, 4.17, 4.24, 4.31, 3.53, 4.41, and 4.45
- The topic is 2-dimensional motion. Today, sec. 1 – 3: vector velocity and acceleration, projectile motion.
- Friday: sec. 4 – 6: circular motion and relative velocity.
- Next Wednesday: Exam 1, Chapters 2 – 4.
- See last year's exam and equation sheet on my web site under Exams.

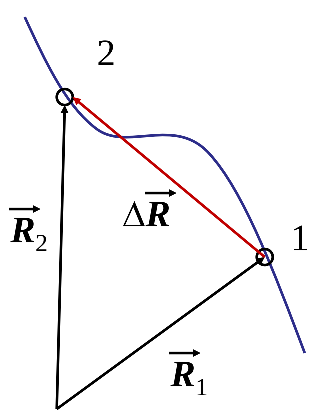
Displacement Vector



If object moves from point 1 to point 2 in time Δt, the displacement vector is $\Delta\vec{R}$.

We will define velocity to be the rate of change of the position vector, by dividing the displacement by the time.

Average Velocity

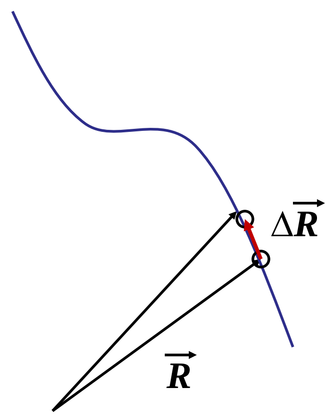


If object moves from point 1 to point 2 in time Δt, the average velocity is

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{R}}{\Delta t}$$

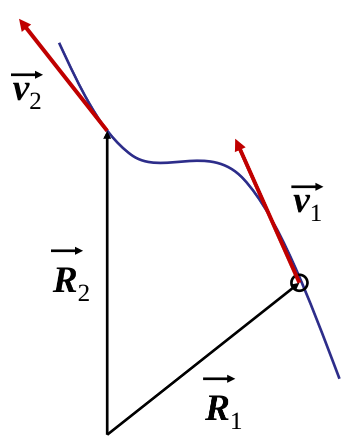
To get instantaneous Velocity, we must take Δt very short.

Average Velocity



As Δt becomes shorter, so does $\Delta\vec{R}$, which eventually becomes a displacement along a straight line. It then points in the direction of motion, and the ratio $|\Delta R|/\Delta t$ is the **instantaneous speed** v (**not** a vector)

Average Velocity



This limit defines the **instantaneous velocity**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{R}}{\Delta t} = \frac{d\vec{R}}{dt}$$

which points in the direction of motion and has magnitude equal to the speed.

Average Velocity

The derivative of a vector can be taken one component at a time, so

$$\vec{v} = \frac{d\vec{R}}{dt}$$

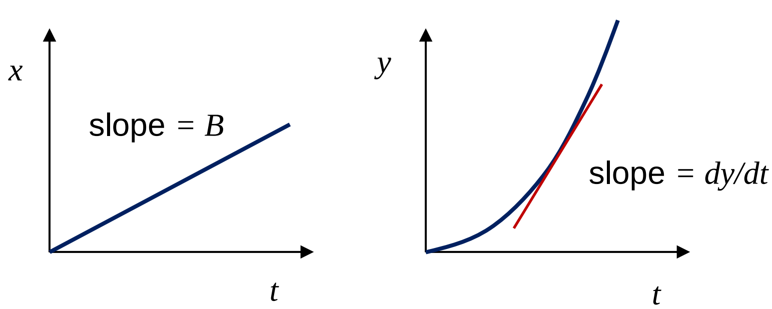
is really two equations, one for each component (each just as in 1 dimension):

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

Velocity Components

Suppose $x(t) = Bt$ and $y(t) = Ct^3$. In other words, the position vector is $\vec{\boldsymbol{R}}(t) = Bt\,\hat{\boldsymbol{i}} + Ct^3\,\hat{\boldsymbol{j}}$.

- Find $\vec{\boldsymbol{v}}(t)$.

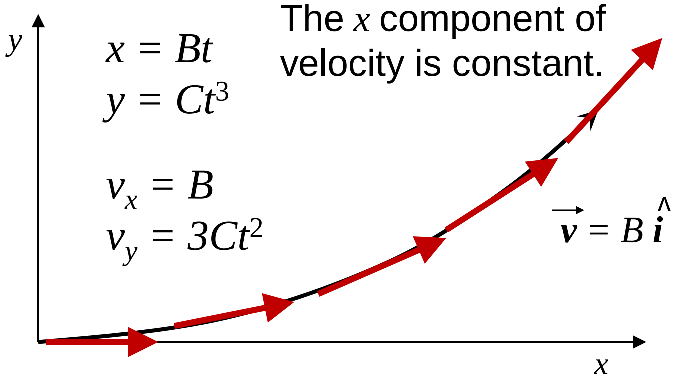


Velocity Vector

$x = Bt$
$y = Ct^3$

$v_x = B$
$v_y = 3Ct^2$

The x component of velocity is constant.

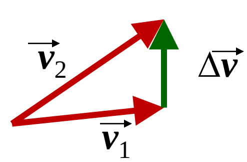


$\vec{v} = B\,\hat{i} + 3Ct^2\,\hat{j}$

At each point, the velocity has magnitude equal to the speed and direction along the path.

Acceleration Vector

If we look at the velocity vector as a function of time, its rate of change is the acceleration vector. In the previous example, the velocity vector at two times separated by Δt could be as shown.



The average acceleration for this time interval is defined to be

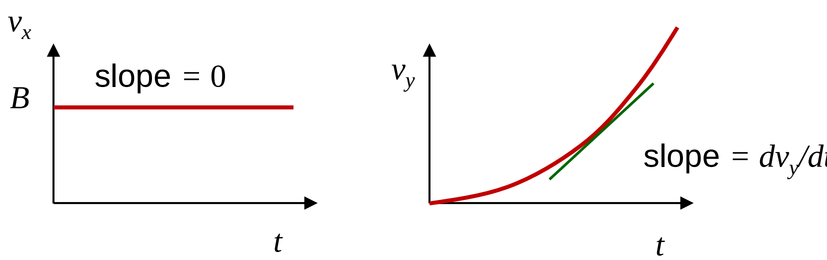
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Acceleration Vector

The instantaneous acceleration is defined by the time derivative of the instantaneous velocity:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

In components, $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$

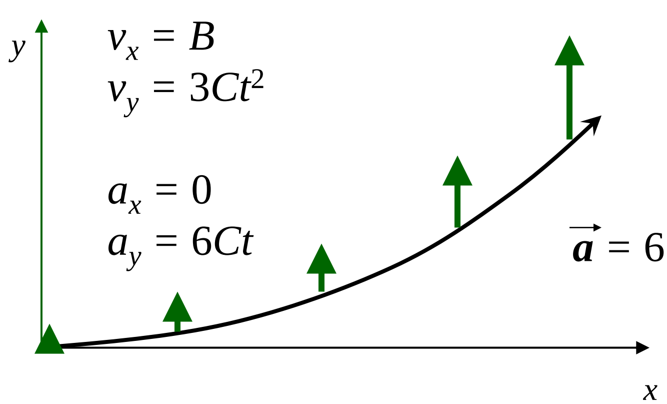


Acceleration Vector

$v_x = B$
$v_y = 3Ct^2$

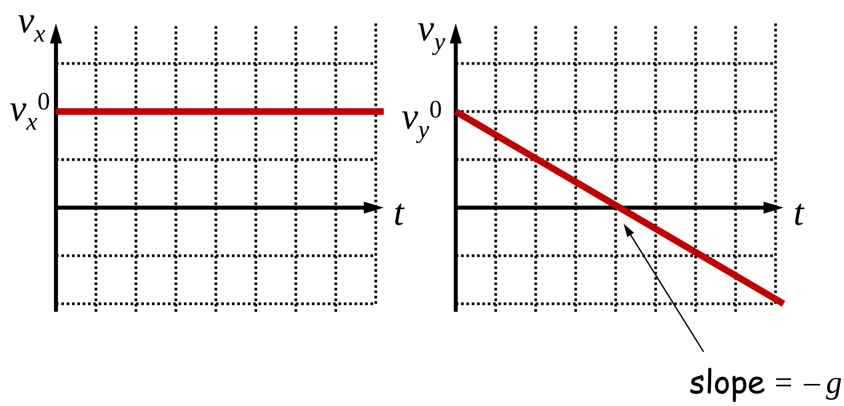
$a_x = 0$
$a_y = 6Ct$

$\vec{a} = 6Ct\,\hat{j}$



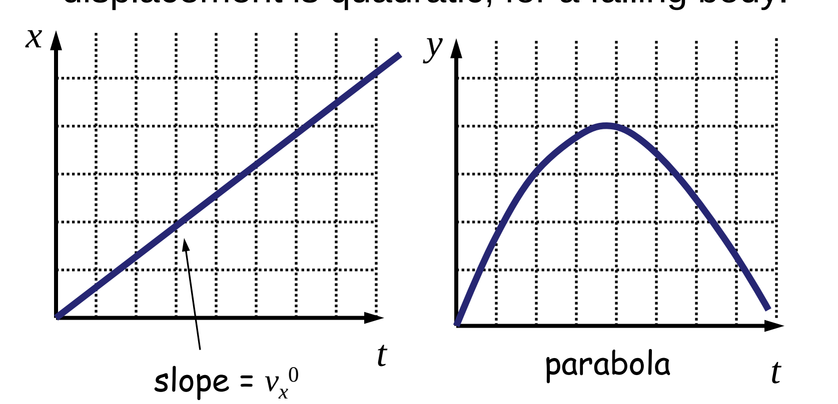
Projectile Motion

Projectile Motion is defined by constant acceleration in the y direction and constant velocity in the x direction.



Projectile Motion

- The x displacement is linear in time, and the y displacement is quadratic, for a falling body.



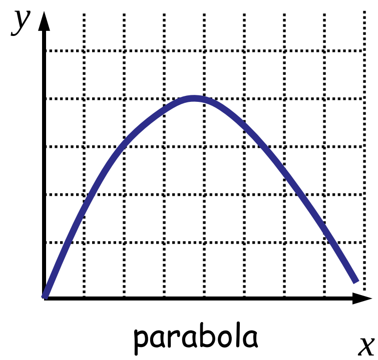
Projectile Motion

- The spatial motion of the projectile is also parabolic.

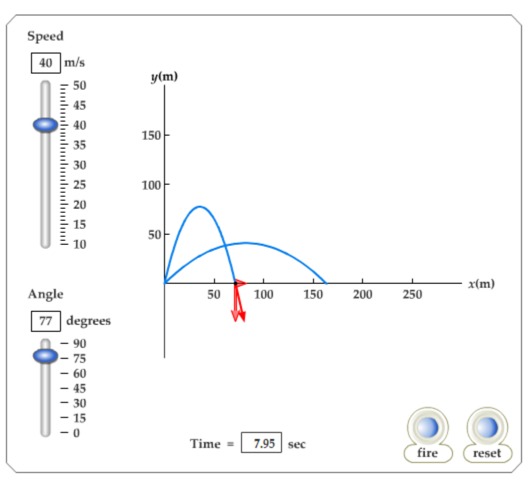
$$y = v_0^y\, t - ½\, g\, t^2$$

$$x = v_0^x\, t$$

$$y = \left(\frac{v_0^y}{v_0^x}\right)x - \frac{gx^2}{2\left(v_0^x\right)^2}$$



Projectile Motion



Projectile Range

The distance the projectile travels is

$$x = v_0^x\, t = v_0\, (\cos\theta)\, t\ .$$

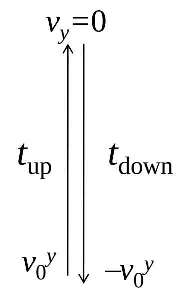
The travel time is determined by the vertical motion: $v_{0y} = gt_{\text{up}} = gt_{\text{down}}$,

$$t = t_{\text{up}} + t_{\text{down}} = 2v_0^y/g = 2(v_0\,/g)\sin\theta.$$

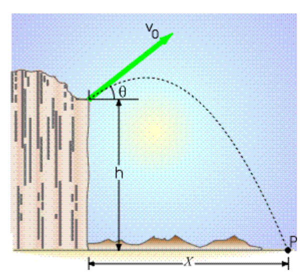
$$x = (2v_0^2/g)\sin\theta\cos\theta = (v_0^2/g)\sin 2\theta.$$

identity: $\sin 2\theta = 2\sin\theta\cos\theta$

x is maximum if $\theta = 45^{\circ}$. The maximum range on flat ground is $x = v_0^2/g$.



Example



A projectile is shot from the edge of a cliff as shown.

Given:

$h = 125$ m

$v_0 = 65.0$ m/s

$\theta = 37.0^{\circ}$

a) At what velocity does it hit the ground? (magnitude and direction)

Example

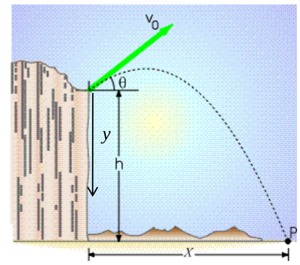
Strategy: Find the final velocity components.

x direction: constant velocity, $v_x = v_0^x$.

y direction: constant acceleration, known distance.

$v_y^2 = (v_0^y)^2 + 2a\,\Delta y$

$= (v_0^y)^2 + 2gh$.



The origin is at the edge of the ciff.

I am measuring y positive downward!

Example

$v_x = v_0 \cos 37^o$ always

$= 51.9$ m/s

$v_y^2 = (v_0 \sin 37^o)^2 + 2gh$

$= (39.1\text{ m/s})^2 + 2450\text{ m}^2/\text{s}^2$

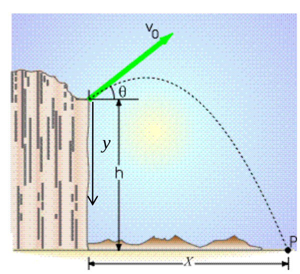
$= 3979\text{ m}^2/\text{s}^2$.

$v_y = 63.1$ m/s

$v = (v_x^2 + v_y^2)^{1/2} = 81.7$ m/s

Angle: $\tan\theta_f = v_y/v_x = 63.1/51.9$

$\theta_f = 50.6$ downward



Given:
$h = 125$ m
$v_0 = 65.0$ m/s
$\theta = 37.0^o$

Question

- Would the landing speed change if the projectile is fired in a different direction and still lands at the bottom of the hill at the same height?

No! Solve for v^2 as a function of the angle:

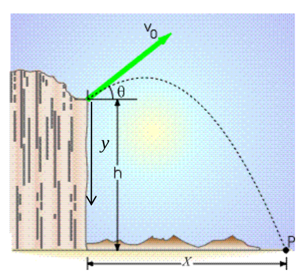
Components: $v_x = v_0 \cos\theta$, $v_y{}^2 = (v_0 \sin\theta)^2 + 2gh$

$$v^2 = v_0^2 \cos^2\theta + v_0^2 \sin^2\theta + 2gh$$

Identity: $\sin^2\theta + \cos^2\theta = 1.$

$v^2 = v_0^2 + 2gh$ independent of the angle.

Example



Given:
$h = 125$ m
$v_0 = 65.0$ m/s
$\theta = 37.0^{o}$

c) How long does it take to hit the ground?

Use the vertical motion.

$v_0{}^y = –39.1$ m/s. $v_y = 63.1$ m/s.

$\Delta v_y = 102.2$ m/s $= gt$

$g = 9.8$ m/s^2, $t = 10.4$ s.

d) How far away does it land (x)?

$x = v_x t = (540$ m/s$)$ $(10.4$ s$)$

$= 5.61$ km.