

# Physics 221

## Sections 1 and 2

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## Vectors

## Announcements

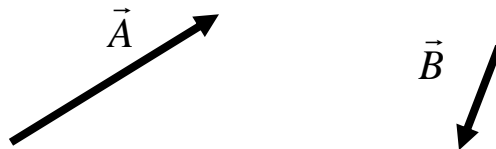
- Set 3 is open and due 7 AM **Monday**. It covers chapter 3 and has 5 problems: 2, 4, 9, 22, and 23.
- The topic is Vectors, which are used to describe motion in more than 1 dimension, and to model many other things in physics, such as forces.
- We will apply vectors to motion in 2 dimensions in Chapter 4, which we will begin Wednesday.
- I also plan to ask you to work an in-class problem from Chapter 2 or 3 on Monday. Equations will be provided, as on an exam.

## Vectors

- To talk about motion, we first must be able to give a precise mathematical representation of the location of objects, and the displacements between them.
- Specifying locations requires talking about both distances and direction.
- In physics, quantities which require a direction for a complete specification are called **vectors**.

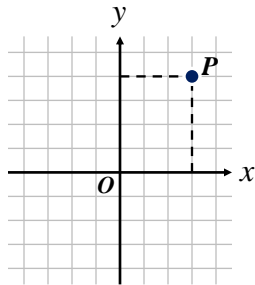
## Vectors

The magnitude (length) and direction and can be shown geometrically by an arrow of the appropriate length, pointing in the direction of the vector.



## Specifying Positions using Coordinates

The position of a point  $P$  can be described by measuring distances along perpendicular  $x$  and  $y$  axes, relative to an origin at point  $O$  with coordinates  $(0,0)$ .

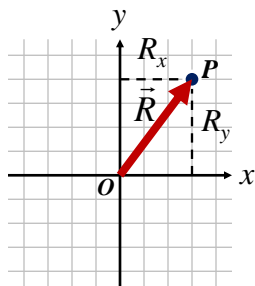


- The numbers specifying the distances to the point along the axes are called its **Cartesian Coordinates**.

Here,  $P$  has coordinates  $(x, y) = (3, 4)$ .

## Specifying Positions using Vectors

The position of a point  $P$  can also be described by a vector  $\vec{R}$  from  $O$  to  $P$ :

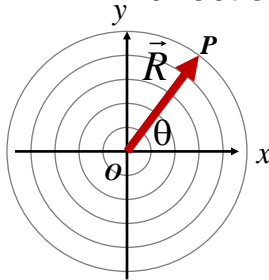
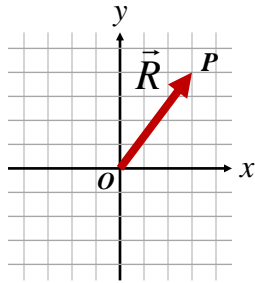


- The  $x$  and  $y$  coordinates of  $P$  are called the  $x$  and  $y$  **components** of the vector  $\vec{R}$ .
- The components are denoted by  $R_x$  and  $R_y$ .

Here,  $R_x = 3$  and  $R_y = 4$ .

# Polar Coordinates

We can alternatively specify the magnitude  $R$  and direction  $\theta$  of the vector  $\vec{R}$  to define it.



- $R$  and  $\theta$  are called the **polar coordinates** of the point  $P$ .

$$\text{Here, } R = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \text{atan}(4/3) = 53^\circ$$

# Trigonometry

The Pythagorean theorem gives the hypotenuse in terms of the other sides of a right triangle.

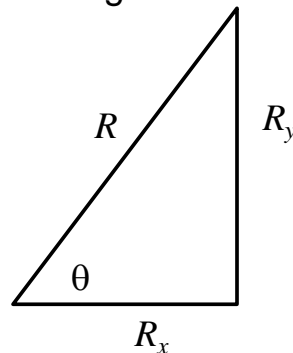
$$R = \sqrt{R_x^2 + R_y^2}$$

Trigonometric functions relate the ratios of sides to the angle  $\theta$ .

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{R_y}{R}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{R_x}{R}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta} = \frac{R_y}{R_x}$$

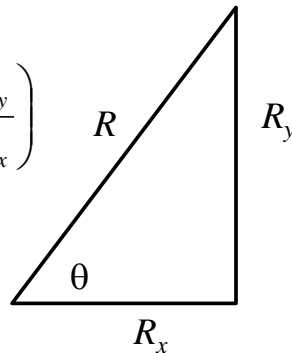


# Trigonometry

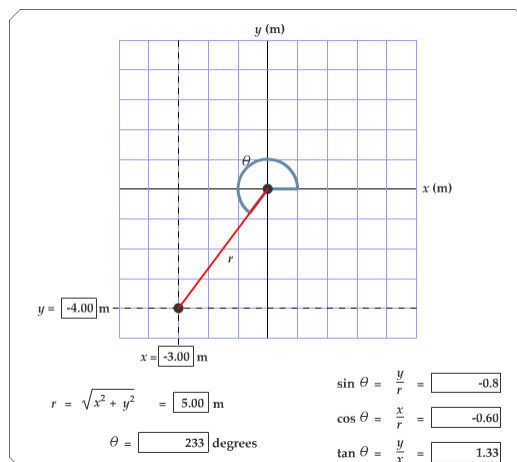
The inverse trigonometric functions give the angles if you know the ratios of sides.

$$\theta = \text{asin}\left(\frac{R_y}{R}\right) = \text{acos}\left(\frac{R_x}{R}\right) = \text{atan}\left(\frac{R_y}{R_x}\right)$$

These are called the arc-sine, arc-cosine, and arc-tangent. Sometimes you will see them written as  $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ .

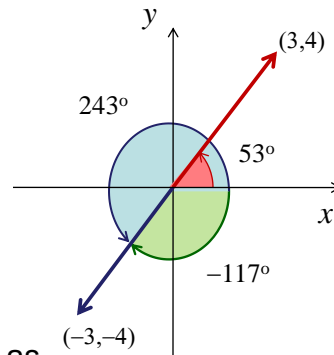


# Cartesian and Polar Coordinates



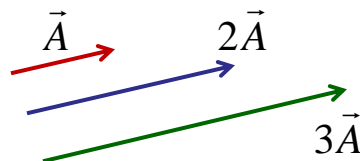
## Pitfalls

- Be careful when calculating an angle using the inverse tangent.
- There is more than one angle that gives the same tangent:  
 $\tan(\theta) = \tan(\theta + 180^\circ)$ .
- $(x,y) = (3,4)$  has  $\theta = 53^\circ$   
 $(x,y) = (-3,-4)$  has  $\theta = 243^\circ$   
Both have  $\tan \theta = 4/3$ .
- And – you can measure the angle clockwise instead:  $243^\circ$  is the same as  $-117^\circ$ .
- Be aware of these possible ambiguities when answering WebAssign questions.



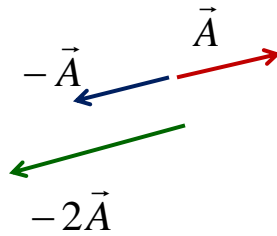
## Vector Algebra

- We can apply two operations to vectors: multiplying them by a number, and adding them.
- Multiplying a vector by a positive number changes the magnitude but not the direction.  
 $c\vec{A}$  has magnitude  $cA$  in the direction of  $\vec{A}$ .

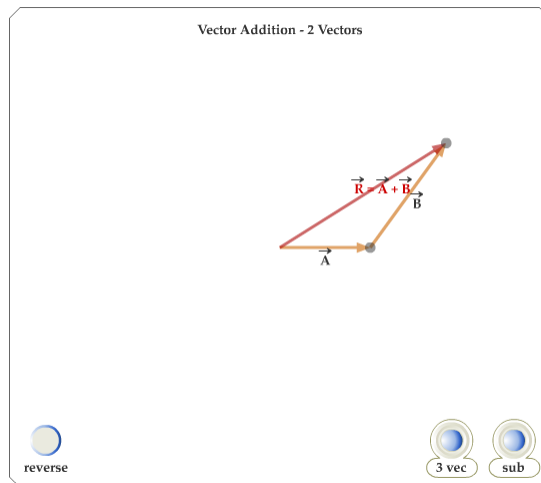


# Vector Algebra

- Multiplying a vector by a negative number changes the magnitude and reverses the direction.



# Vector Addition



## Vector Addition: Components

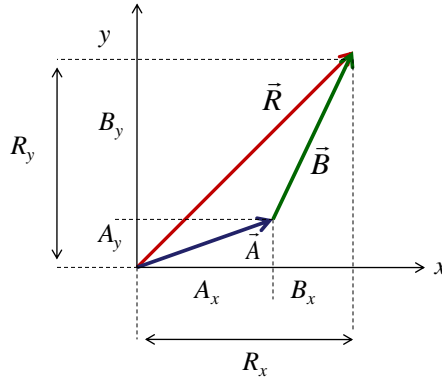
Vectors add component by component.

The vector sum is also called the **resultant**.

$$\vec{R} = \vec{A} + \vec{B}$$

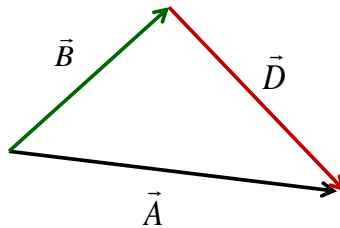
$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$



## Displacement Vector

- The difference between two position vectors is the **displacement vector** from one to the other.
- The length of the displacement vector is the distance between the two points.



$$\vec{D} = \vec{A} - \vec{B}$$

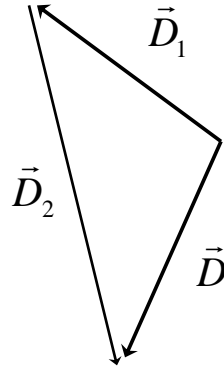
## Example

Suppose I walk 75 m in a direction  $55^\circ$  W of N.

Then I walk 118 m in a direction  $164^\circ$  E of N.

What is my net displacement from the starting point, in terms of magnitude and direction?

Use vector addition:  $\vec{D} = \vec{D}_1 + \vec{D}_2$



## Example

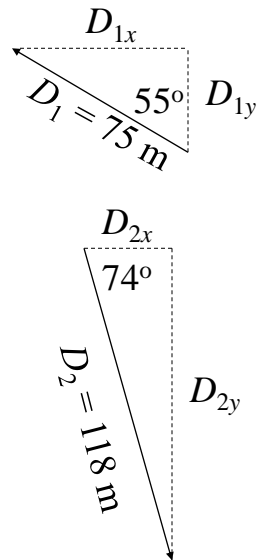
Strategy: add the components of the two vectors, then find the magnitude and direction.

$$D_{1x} = -D_1 \sin 55^\circ = -61.4 \text{ m}$$

$$D_{1y} = D_1 \cos 55^\circ = 43.0 \text{ m}$$

$$D_{2x} = D_2 \cos 74^\circ = 32.5 \text{ m}$$

$$D_{2y} = -D_2 \sin 74^\circ = -113.4 \text{ m}$$

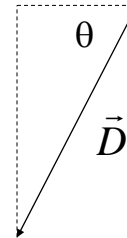


## Example

The components of the vector sum are then

$$D_x = D_{1x} + D_{2x} = -28.9 \text{ m}$$

$$D_y = D_{1y} + D_{2y} = -70.4 \text{ m}$$



The distance from the starting point is then  $D = \sqrt{D_x^2 + D_y^2} = 76.1 \text{ m}$ .

The angle  $\theta$  is  $\text{atan}(70.4/28.9) = 67.7^\circ \text{ S of W}$ .

## Example

### Alternative solution:

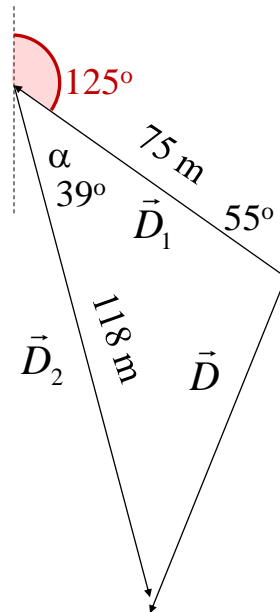
Use triangle geometry to find the angle  $\alpha$ . Then find  $D$  using the Law of Cosines.

The red angle at the top must be  $125^\circ$ , since it is **supplementary** to the  $55^\circ$  angle, and

$$125^\circ + 55^\circ = 180^\circ$$

Also,  $125^\circ + \alpha = 164^\circ$

$$\alpha = 39^\circ$$



## Example

The **Law of Cosines** gives the side opposite an angle if the other two sides are known:

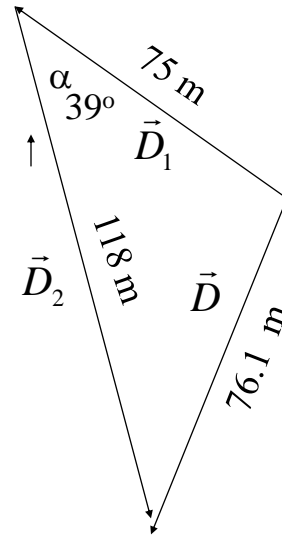
$$D^2 = D_1^2 + D_2^2 - 2 D_1 D_2 \cos \alpha$$

$$= 5794 \text{ m}^2,$$

so that

$$D = 76.1 \text{ m.}$$

as before.



## Example

How do we get the angle? Knowing  $\phi$  would give the desired angle.

If we can find  $\beta$ , then

$$\phi = 180^\circ - 55^\circ - \beta.$$

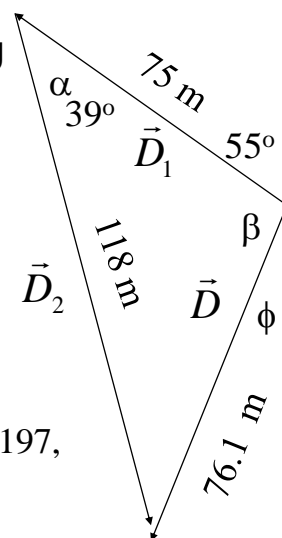
Using the Law of Cosines again,

$$D_2^2 = D_1^2 + D^2 - 2 D_1 D \cos \beta$$

which implies

$$\cos \beta = \frac{D_1^2 + D^2 - D_2^2}{2 D_1 D} = \frac{-2508}{11415} = -0.2197,$$

$$\beta = 102.7^\circ, \quad \phi = 125^\circ - \beta = 22.3^\circ$$



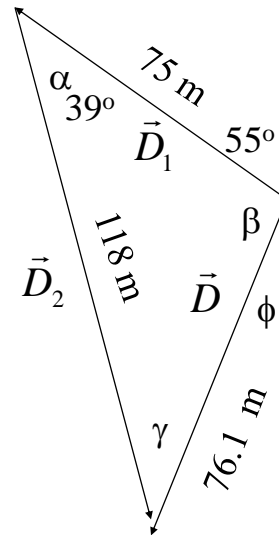
## Example

You can also use the **Law of Sines**, which says that the ratio of the sines of each angle to the opposite side is the same for all angles in a triangle:

$$\frac{\sin \alpha}{D} = \frac{\sin \beta}{D_2} = \frac{\sin \gamma}{D_1}$$

$\sin \beta / 118 = \sin 39^\circ / 76.1$  gives

$\beta = 77.4^\circ$  **or**  $102.6^\circ$ . Unfortunately,  $\beta$  and  $180^\circ - \beta$  have the same sine.  
**Which is correct?**



## Example

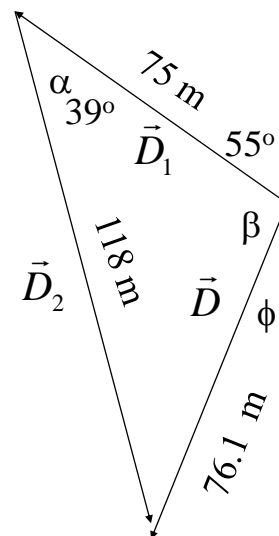
If you have an accurate picture, you can tell  $\beta$  is obtuse, so  **$102.6^\circ$**  is correct.

You could also note that if  $D_1 = 75\text{m}$ ,  $\alpha = 39^\circ$ , and  $\beta = 90^\circ$ , then we would have

$$D_2 = 75 \text{ m} / \cos 39^\circ = 96.5 \text{ m.}$$

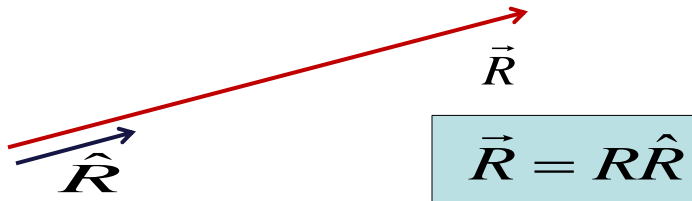
Since  $D_2$  is actually longer than this,  $\beta$  must be greater than  $90^\circ$ .

The **Law of Cosines** is unambiguous, so it is **safer**.



## Unit Vectors

- A vector of length 1 is called a unit vector.
- The unit vector parallel to any given vector  $\vec{R}$  points in the same direction and has length 1.
- This vector is written  $\hat{R}$



## Basis Unit Vectors

- Unit vectors pointing along the coordinate axes have special names:  $\hat{i}$ ,  $\hat{j}$ . These can be used to write any 2-dimensional vector.

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

- $\hat{k}$  is used if there is a  $z$  direction also.

