Physics 221

Department of Physics
The Citadel

Lecture Notes

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December 2

Harmonic Oscillator

Part 2

Announcements

- Problem Set 14 on Oscillation is due Friday night.
- It includes problems 2, 9, 11, 17, 25, 44 in Ch. 15.
- Today we will do the course evaluation first.
- Then we will finish the parts of Chapter 15 we are discussing: examples and the pendulum.
- A review problem set with one problem from each of chapters 4 – 12, 14, and 15 is now open.

SHM Example

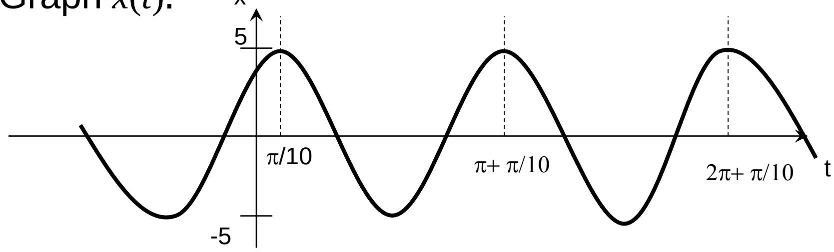
Let the position be in m, the time in s.

Suppose $x = 5\cos(2t - \pi/5)$.

- What are the amplitude and period?

  $A = 5\text{m},\ \omega = 2\text{s}^{-1}\ \ T = 2\pi/\omega = \pi$ s.

- Graph $x(t)$.



SHM Example

- What is the initial velocity?

  $\omega = 2\text{s}^{-1}$, $r = 5$ m, $v = r\omega = 10$ m/s tangential

  $v_x(\theta = -\pi/5) = 10\sin(\pi/5)$ (m/s)

- What is the maximum velocity?

  $v_{\text{max}} = v = 10$ m/s.

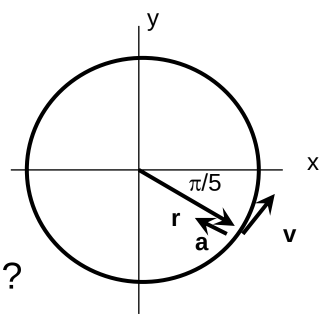
- What is the initial acceleration?

  $a = r\omega^2 = \omega v = 20\ \text{m/s}^2$ inward

  $a_x(\theta = -\pi/5) = -20\cos(\pi/5)$

- What is the maximum acceleration?

  $a_{\text{max}} = a = 20$ m/s.



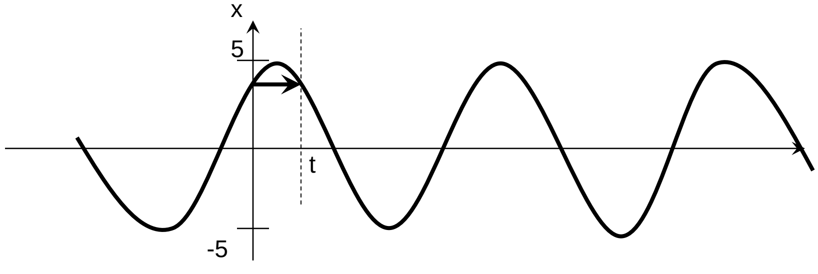
SHM Example

The velocity and acceleration can also be found using calculus:

$$v = dx/dt = -10\sin(2t - \pi/5),$$
$$v(t = 0) = 10\sin(\pi/5) \quad \text{(m/s)}$$
$$a = dv/dt = -20\cos(2t - \pi/5).$$
$$a(t = 0) = -20\cos(\pi/5) \quad \text{(m/s}^2\text{)}$$

SHM Example

How long does it take to get back to its initial position the first time?



SHM Example

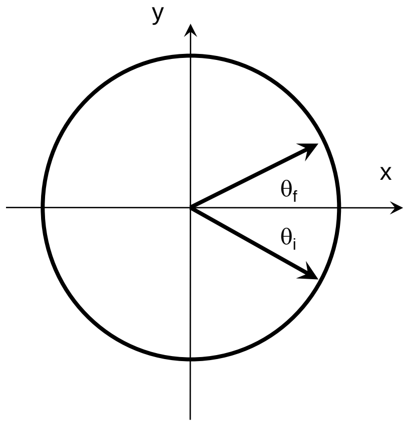
At $t = 0$, $x = 5\cos(-\pi/5)$,

$\theta_i = -\pi/5$.

Think in terms of the circle. It gets back to the same x when

$\theta_f = +\pi/5$

$T = (\theta_f - \theta_i)/\omega = \pi/5$ s



Car and Bump

- If you push down on a car, it vibrates up and down with a period of 0.80 s.
- Four people of mass 75 kg get into a car and it sinks down 5 cm.
- What is the mass of the empty car?

$k = mg/x = 4\text{x}75 \text{ kg} \times 9.8 \text{ m/s}^2 / 0.03 \text{ m}$

$= 98 \text{ kN/m}$

$\omega = 2\pi/0.8\text{s} = 7.85 \text{ s}^{-1}$. $\omega^2 = k/m$

$m = k/\omega^2 = 1590 \text{ kg}$

Car and Bump

As the car with the 4 people drives down the road, it goes over a bump 2 cm high.

Are the people thrown from their seats?

The people and car together have mass $M = 1890$ kg. Their oscillation frequency together is determined by $\omega^2 = k/M$.

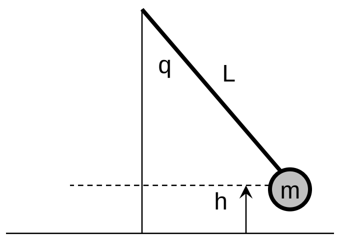
The maximum acceleration is $\omega^2 A$, $A = 2\ cm$.

To stay in their seats, they need $a = \omega^2 A < g$.

The boundary is

$$A_c = g/\omega^2 = Mg/\mathrm{k} = xM/m = 1.19 \text{ x } 3 \text{ cm} = 3.6 \text{ cm}.$$

Pendulum



$U = mgh = mg(L - L \cos \theta)$

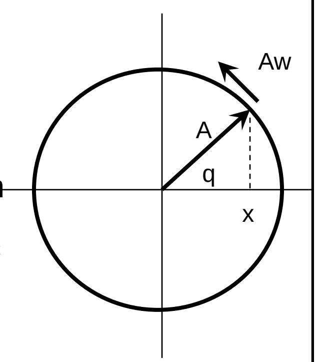
$K = ½\, mv^2 = E - U = U(\theta_0) - U(\theta)$

$v^2 = (2gL)(\cos\theta - \cos\theta_0)$

Approximate form for small angles:

$\cos \theta \approx 1 - ½\ \theta^2$

$v^2 \approx (gL)(\theta_0^2 - \theta^2)$



This looks like the relation between position and velocity for a harmonic oscillator. To make it clearer, write it in terms of arc length $s = L\,\theta$:

$$v^2 = (g/L)(s_0^2 - s^2).$$

Recall that $v^2 = \omega^2 (A^2 - x^2)$ for SHM.

Then $\omega = (g/L)^{1/2}$ for the pendulum.

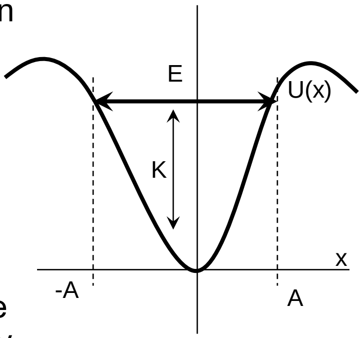
Pendulum

The exact integral for the period can be obtained from

$v^2 = L^2 (d\theta/dt)^2 = (2gL)(\cos\theta - \cos\theta_0)$

$dt/d\theta = (2g/L)^{-1/2}(\cos\theta - \cos\theta_0)^{-1/2}$

$T = 4(L/2g)^{1/2} \int_0^{\theta_0} (\cos\theta - \cos\theta_0)^{-1/2}\, d\theta$



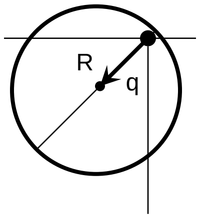
This is an elliptic integral. It can't be expressed in terms of elementary functions.

For $\theta_0 << 1$ rad (57^o), the small amplitude approximation gives

$T \approx 4(L/g)^{1/2} \int_0^{\theta_0} d\theta/(\theta_0^2 - \theta^2)^{1/2} = \boxed{2\pi(L/g)^{1/2}}.$

Hoop on a Nail

- What is the period of a hoop of radius R swinging (without slipping) on a nail? Assume a small amplitude of oscillation.



$U = mgR(1 - \cos\theta) \approx \frac{1}{2} mgR\theta^2.$

$K = \frac{1}{2} I_{nail}\omega^2$

$= mR^2 (d\theta/dt)^2.$ $I_{nail} = 2mR^2$ (parallel axis theorem)

$(d\theta/dt)^2 = (g/2R)(\theta_0^2 - \theta^2)$

Recognize hramonic oscillator: $\omega^2 = g/2R$.