Physics 221

Department of Physics
The Citadel

Lecture Notes

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November 30

Harmonic Oscillator

Announcements

- Problem Set 14 on Oscillation is due Friday night.
- It includes problems 2, 9, 11, 17, 25, 44 in Ch. 15.
- This will be the last full lecture, though we may finish up some of it Wednesday.
- I expect to use part of Wednesday for review, after briefly discussing the pendulum.
- You should prepare your own one-page (front and back) summary of equations for the final.

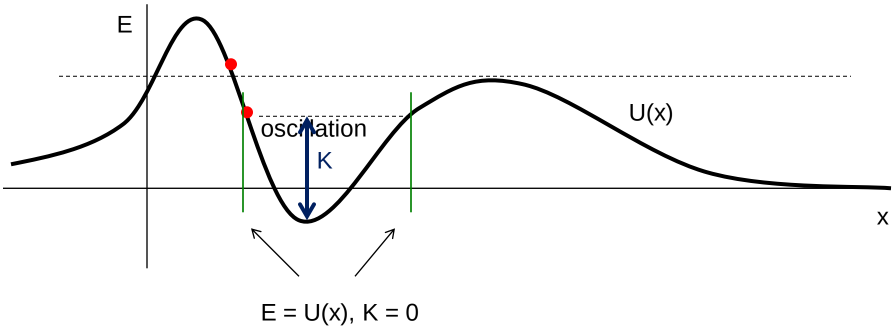
Oscillatory Motion

Oscillation happens about a minimum of the potential energy. Total energy is conserved.

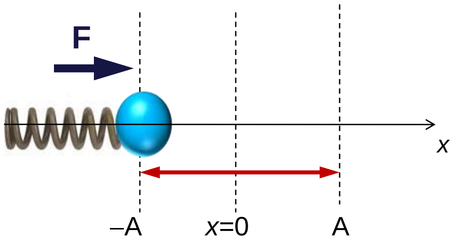
For what range of motion can oscillation occur?

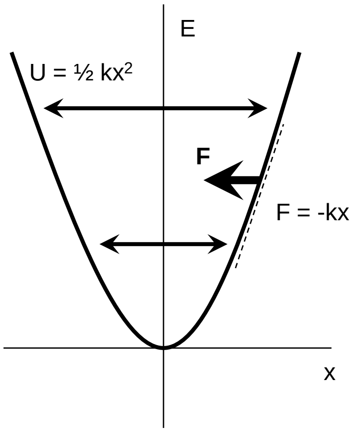
Where is the motion fastest?

Where are the turning points?



Simple Harmonic Motion





Consider a spring: (Hooke's Law)

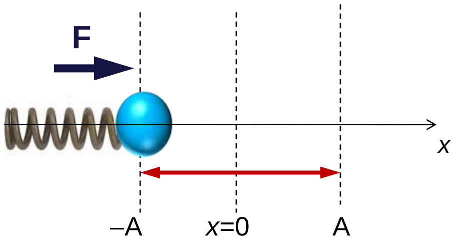
$F = -kx$, $U = ½ kx^2$

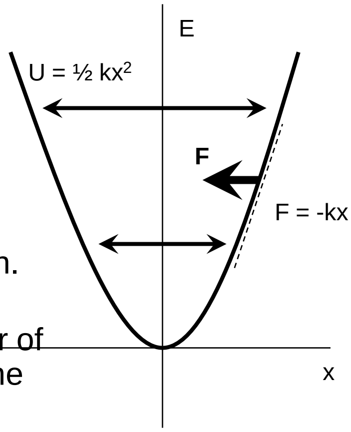
The particle moves back and forth between turning points

$x = A$ and $x = -A$,

where A is called the **amplitude**.

Simple Harmonic Motion





This is called Simple Harmonic Motion.

The time to go back and forth once is called the period T, and the number of oscillations per unit time is called the frequency f. We have $f = 1/T$.

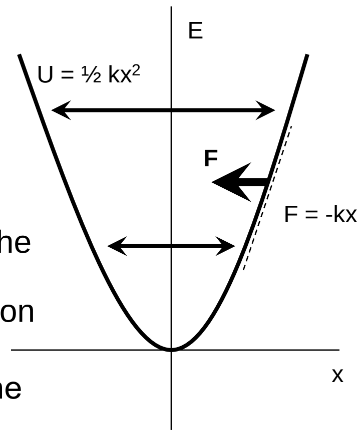
At the turning points, $U = E$ so

$$E = \tfrac{1}{2} kA^2.$$

Simple Harmonic Motion

If the period doesn't depend on the amplitude, the motion is called **simple harmonic**.

A quadratic P.E. or linear restoring force produces SHM.



Note: a bell oscillates when struck. The oscillations produce the sound we hear, and the frequency of oscillation determines the pitch.

If the frequency doesn't depend on the amplitude, hitting the bell at any strength should produce the same note. This is why it's called harmonic.

Small Oscillations

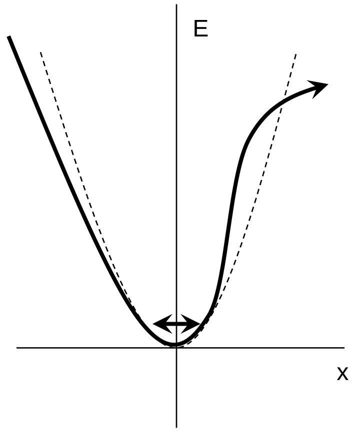
Even if the PE is not perfectly parabolic, but is smooth, $F = 0$ at the minimum, and is approximately $F = -kx$ near the minimum.

Minimum: $dU/dx = 0$.

Near minimum,

$F = -kx$

$k = dF/dx = d^2U/dx^2$.



This makes harmonic oscillators ubiquitous throughout physics!

Simple Harmonic Motion

Oscillation happens about a minimum of the potential energy. $U + K = E$.

The fastest motion occurs at the equilibrium position.

Motion of a mass on a spring…

Let $x = 0$ be the equilibrium position.

$F = -kx$

$a = -(k/m)\, x$

$U = ½\, kx^2$

$½\, mv^2 + ½\, kx^2 = E = ½\, kA^2$.

How to find the motion $x(t)$? There is a very useful trick that avoids calculus…

Projected Circular Motion

Remember Uniform Circular Motion:

$a = v^2/r = r\omega^2$ inward.

$\mathbf{a} = -\omega^2\, \mathbf{r}$ as a vector.

force:

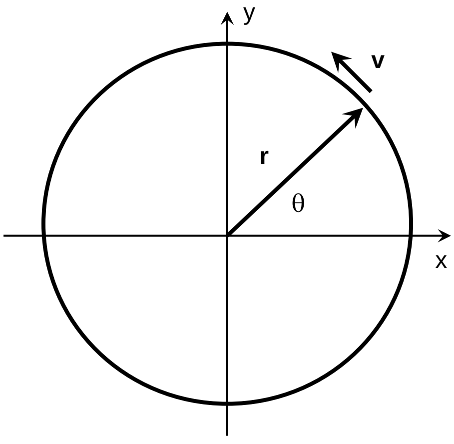
$\mathbf{F} = -m\omega^2\, \mathbf{r}.$

$F_x = -kx,\ \ F_y = -ky$

with $k = m\omega^2$.

Both the x and y motion obey Hooke's Law with this k!

The amplitude is $A = r$.



Projected Circular Motion

acceleration:

$\mathbf{a} = -\omega^2\, \mathbf{r}.$

$a_x = -(k/m)x$

with $k = m\omega^2$.

The x motion can be identified with SHM:

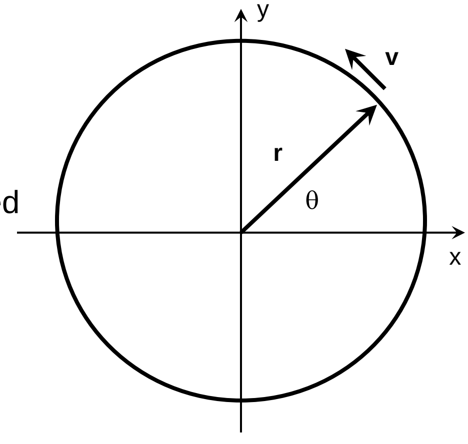
$$a = -\,\omega^2\, x.$$

position:

$x = r\cos\theta \qquad \theta = \theta_0 + \omega t$

Oscillator:

$x = A\cos(\theta_0 + \omega t)$



Projected Circular Motion

velocity:

$v_x = -v \sin\theta = -vy/r$

with $y = \pm(r^2 - x^2)^{1/2}$, $v = r\omega$

Translate to SHM:

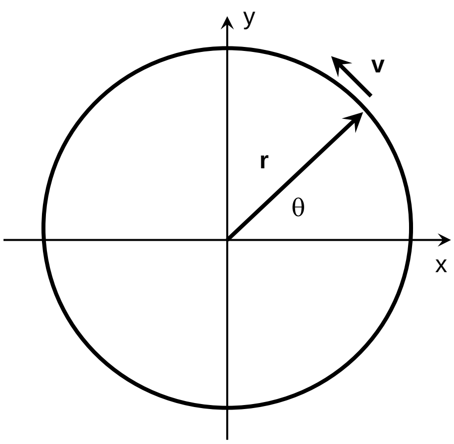
$v = \pm\omega(A^2 - x^2)^{1/2}$

Compare result from energy conservation:

$\frac{1}{2} mv^2 = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$.

$v = \pm(k/m)^{1/2} (A^2 - x^2)^{1/2}$

Period: $T = 2\pi/\omega = 2\pi\ (m/k)^{1/2}$



Equation for SHM

If I push a block of mass m on a spring with spring constant k, giving it initial speed v_0 at position x_0, what is the equation of motion?

Remember: $x = A \cos\theta$ where the angle is $\theta = \theta_0 + \omega t$ with frequency $\omega = (k/m)^{1/2}$.

A, θ_0 must be determined from the initial conditions.

Equation for SHM

$x_0 = A \cos \theta_0$.
$v = -A\omega \sin \theta$ (previous slide)
$v_0 = -A\omega \sin\theta_0$ so $v_0/\omega = -A \sin \theta$

$A^2 = A^2 (\sin^2\theta_0 + \cos^2\theta_0) = (v_0/\omega)^2 + x_0^2$.
$\tan \theta_0 = -v_0/\omega x_0$.

- Note the similarity to finding the magnitude and angle of a vector. As usual – it's not the result that's significant, but the way we got it.
- Also note that you don't really need calculus to find v(t), since you can use the circular motion:

  The facts that $v = r\omega$ and $\mathbf{v}$ is perpendicular to **r** imply that $v = -r\omega \sin \theta$ when $x = r \cos \theta$. (Replace r by A.)