Physics 221

Sections 1 and 2

S. Yost

November 18, 2009

Buoyancy, Hydrodynamics

Announcements

- Problem set 13 due Friday: Chapter 14, numbers 7, 9, 16, 21, 27, 36, 38, 50
- Exams: maybe Friday? Not gauranteed.
- Happy Thanksgiving!!
- After the break: Chapter 15, through 15.5.
- The last WebAssign is due Dec. 4. It will be posted during the break.

Fluid in Equilibrium

Note: The Pascal's Principle example is in the previous lecture.

We found the pressure as a function of depth by considering the force on a small rectangular volume of fluid.

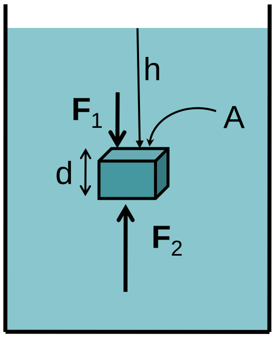
The difference between the force on the top and bottom supports the weight of the "box" of fluid:

$$F_2 - F_1 = mg$$

Pressure difference:

$$P_2 A - P_1 A = \rho d A g.$$

$$P_2 - P_1 = \rho d g.$$



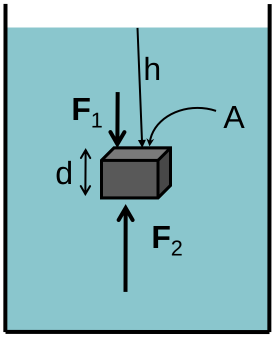
box drawn around some fluid

Fluid in Equilibrium.

The force $\mathbf{F}_2 - \mathbf{F}_1$, due to the fluid's pressure, that supports the weight of the box is called the **buoyant force**.

How would the buoyant force change if we replaced the constructed box of water by a real box of some sort?

Not at all: The box's presence doesn't change the pressure around it, so the buoyant force $\mathbf{F}_2 - \mathbf{F}_1$ is the same.



a real submerged box

Archimedes' Principle

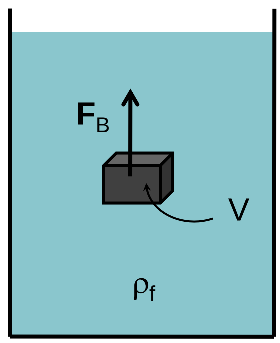
The net upward force F_B of the pressure on the box is then given by the weight of the water that would occupy the same space as the box.

This does not depend in any way on the shape of the box.

Archimedes' Principle:

The buoyant force is always equal to the weight of the fluid displaced by an object.

If the object has volume V, the buoyant force is $F_B = \rho_f g V$.

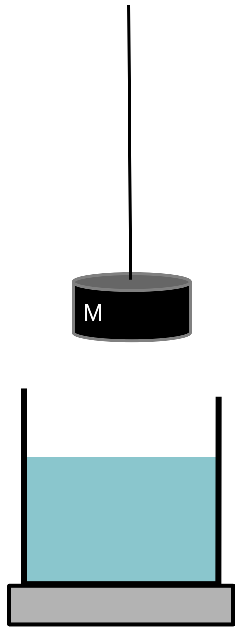


a real submerged box

Example

An 4.0 kg iron weight is held on a string and lowered into a pan of water weighing 8.0 kg.

What would a scale under the pan read, if the weight is submerged but not touching the bottom?



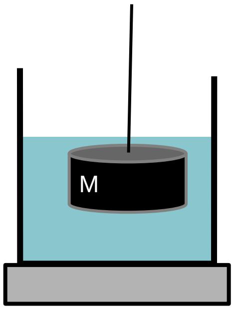
Example

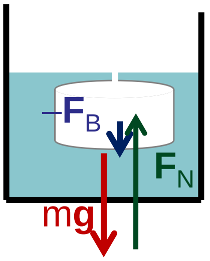
There is an upward buoyant force $F_B = \rho_w g V$, where V is the volume of the weight.

Its reaction force pushes down on the water.

The net downward force on the scale balances the normal force: $F_N = (8\text{ kg} + \rho_w V)g$

The scale would read $8\text{ kg} + \rho_w V$. $\rho_w = 1\text{ g/cm}^3$.





Example

The scale would read $8\text{ kg} + \rho_w V$. $\rho_w = 1\text{ g/cm}^3$.

What is the volume V?

$V = m/\rho$ for iron: $\rho \approx 7.87\text{ g/cm}^3$.

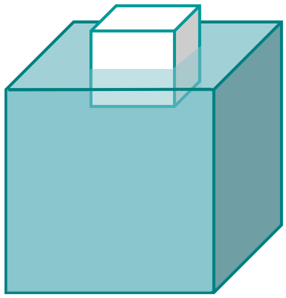
$\rho_w V = m\,\rho_w / \rho = 0.508\text{ kg}$

Scale reads 8.51 kg.

Floating Example

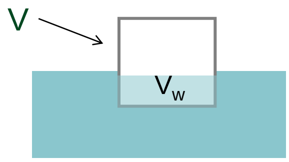
An object floats when the buoyant force balances its weight.

A block with with density 300 kg/m^3 floats in water. What fraction of the block's volume is under water?



Floating Example

If the volume of the block is V and the volume under water is V_w, we need to find V_w/V.



The buoyant force balances the weight:

$F_B = mg$

$\rho_w V_w g = \rho V g.$

$V_w/V = \rho/\rho_w$

$= (300\ \text{kg/m}^3) / (1000\ \text{kg/m}^3) = 0.30$

30% of the block is under water.

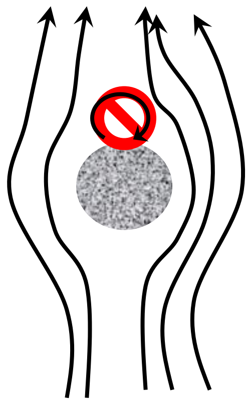
Fluid Motion

We will limit our considerations to **ideal fluids** with **laminar** flow (no turbulence).

We will also assume r is constant: the fluid is **incompressible**.

And – The fluid is **irrotational**. It doesn't form vortices (like in a flushing toilet), but always follows **streamlines**, as shown here.

Finally, we will ignore **viscosity:** no friction in the fluid.



Volume Rate of Flow

The **volume rate of flow** is defined to be the volume of fluid that flows past a point in a pipe per unit time.

$$Q = \Delta V / \Delta t \, .$$

$$\Delta V = A \Delta L = Av \, \Delta t$$

$$Q = Av$$

The book doesn't use a symbol for this, so I introduced Q to give it a name.

where A is the area, v the flow velocity.

Volume Rate of Flow

The volume rate of flow is measured in m^3/s.

Another common unit is liters/second (L/s)

$$1 \text{ liter} = 1000 \text{ mL} = 1000 \text{ cm}^3$$

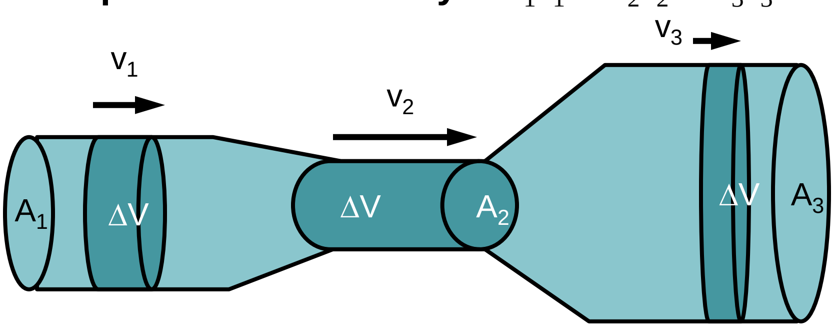
$$1 \text{ m}^3 = 1000 \text{ L}.$$

Other terms: volume flux, flow rate.

The flow rate of an incompressible fluid is the same throughout a pipe.

Volume Flux

$Q = Av$ is fixed: the fluid's velocity is inversely proportional to the cross-sectional area. The fact that Q is constant is called the **equation of continuity:** $A_1v_1 = A_2v_2 = A_3v_3$.

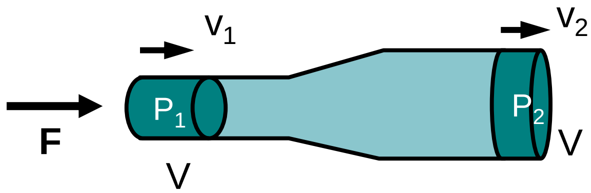


Bernoulli Principle

- Suppose a volume V of incompressible fluid is pushed into a pipe with a pressure difference. The same volume of fluid will be pushed out the other end.
- The force F pushing the fluid in does work, since it pushes the fluid a distance $L = V/A_1$:

$$W_1 = FL = FV/A_1 = P_1V. \qquad (P_1 = F/A_1)$$

- The fluid leaving the pipe **does** work, so the work done **on** the system when it leaves is $W_2 = -P_2V$.



Bernoulli Principle

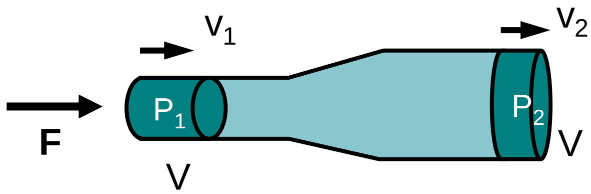
The net work done on this volume of fluid must change its kinetic energy. It mass is ρV, so

$$\tfrac{1}{2}(\rho V)\, v_2^2 - \tfrac{1}{2}(\rho V)\, v_1^2 = P_1V - P_2V.$$

Therefore, dividing by the volume,

$$\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$$

This is **Bernoulli's Principle**: the work-energy theorem for ideal fluids.

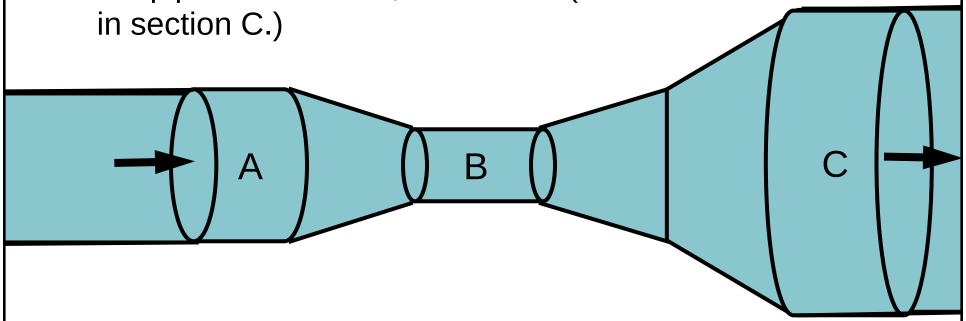


Question

1. A fluid flows through the pipe shown. In which section is the flow velocity the greatest?

A (B) C The same

The volume flux $Q = vA$ is constant for an incompressible fluid. The fluid moves fastest where the pipe is narrowest, section B. (It moves slowest in section C.)

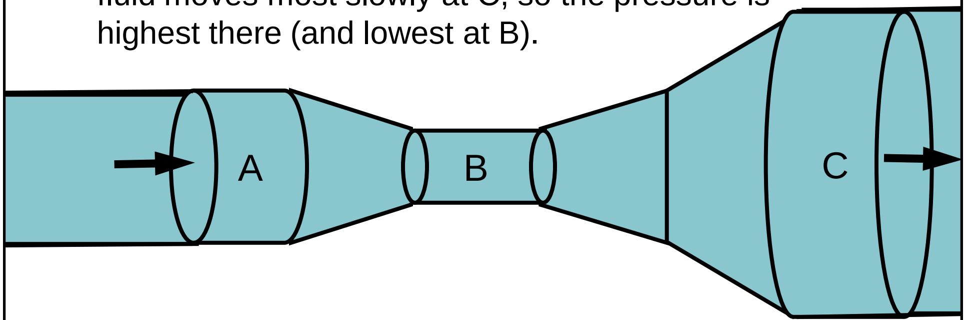


Question

1. In which section is the pressure the greatest?

A B (C) The same

- Bernoulli's principle: The pressure in a fluid decreases when the flow velocity increases. The fluid moves most slowly at C, so the pressure is highest there (and lowest at B).

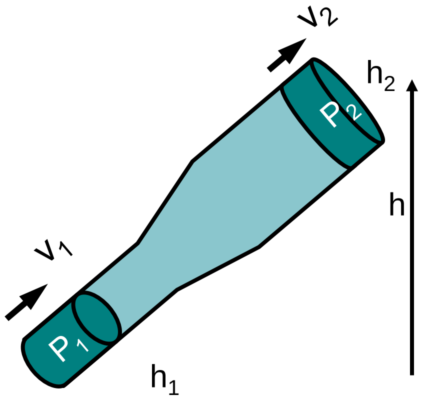


Bernoulli Principle

- For a pipe of changing elevation, the energy is $\frac{1}{2} mv^2 + mgh$, so a PE term must be added to Bernoulli's Equation:

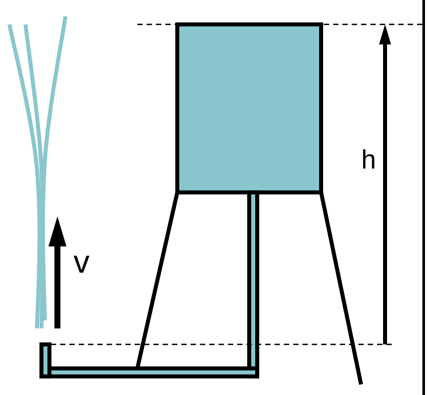
$\frac{1}{2} \rho v^2 + \rho g h + P = \text{constant}.$

P can be either absolute or gauge pressure – but be consistent.



Water Tower and Fountain

- A water tower feeds a fountain, which shoots water straight up in the air.
- How fast does the water leave the fountain?
- Assume the top of the water is a height $h = 55$ m above the fountain.



Water Tower and Fountain

We'll assume the tank is big, so the top of the water stays fixed:

$h_1 = 55$ m,

$v_1 = 0$,

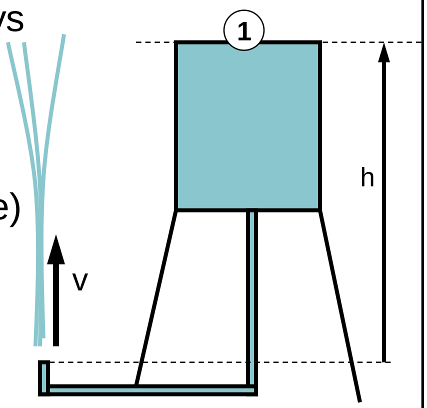
$P_1 = 0$ (gauge pressure)

At the fountain,

$h_2 = 0$

$v_2 = v$

What is P_2?



Water Tower and Fountain

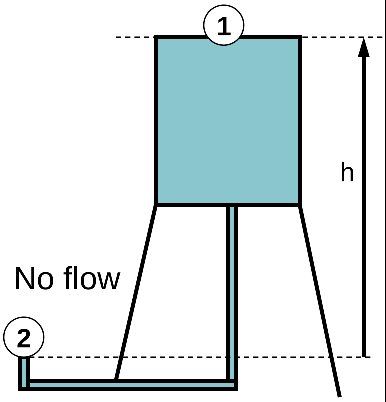
- Careful!

This is not hydrostatics.

If the fountain were **turned off**, the gauge pressure would be

$P_2 = \rho g h = 1000\ \mathrm{kg/m^3} \times 9.8\ \mathrm{m/s^2} \times 55\ \mathrm{m} = 5.4 \times 10^5\ \mathrm{N/m^2}$.

This does not apply when the fountain is flowing.



Water Tower and Fountain

The pressure just outside the pipe is $P_2 = 0$, normal atmospheric pressure.

When water flows freely, the pressure at the end of a pipe is always zero!

The velocity is given by Bernoulli's equation with

$P_1 = P_2 = 0$; $h_1 = h$, $h_2 = 0$; $v_2 = 0$, $v_1 = v$

The only terms remaining are $\frac{1}{2} \rho v^2 = \rho g h$.

The result is the same as if the water had fallen from the top of the tower:

$$v = \sqrt{2gh} = 33 \text{ m/s}.$$

Water Tower and Fountain

What is the volume flux if the pipe has diameter 1 cm?

$Q = Av$

$A = \pi\ (0.5 \text{ cm})^2 = 0.785 \text{ cm}^2$

$v = 33 \text{ m/s} = 3300 \text{ cm/s}$

$Q = 2600 \text{ cm}^3\text{/s} = 2600 \text{ mL/s} = 2.6 \text{ L/s}$

Water Tower and Fountain

How high does the water rise from the fountain?

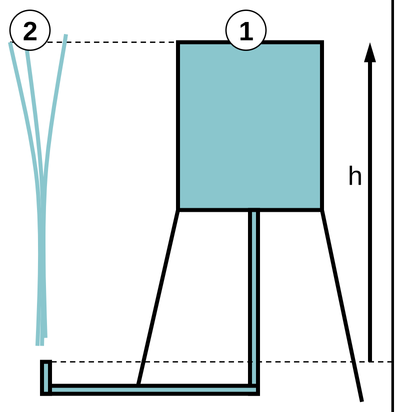
Bernoulli's equation between points 1 and 2:

$P_1 = P_2 = 0$,

$v_1 = v_2 = 0$

implies $\rho g h_1 = \rho g h_2$.

This is what you should have expected, going back to energy conservation – the basis of Bernoulli's Equation.



Water Tower and Fountain

The water rises to the height of the tower.

This assumes energy conservation:

No friction (viscosity or air resistance)

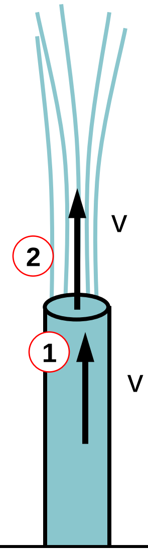
Also, the flow must remain laminar throughout the system: no turbulence or break-up of the stream flow is permitted.

These are clearly idealizations – how likely is it that the flow could reach 33 m/s in the pipe while remaining laminar, with no viscosity?

Note: viscosity is proportional to speed!

Water Tower and Fountain

- What is the pressure inside the pipe feeding the fountain?
- If there is no nozzle constricting the flow, then v is the same in both places, so the pressure must be zero both inside and outside the pipe.



Water Tower and Fountain

Putting a nozzle on the hose would change this.

Energy conservation still gives

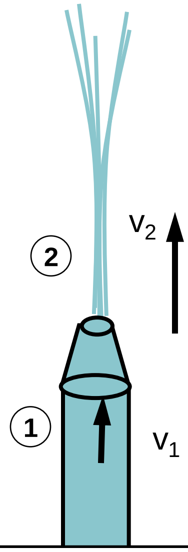
$$v_2 = \sqrt{2gh} = 33\,\text{m/s}$$

but v_1 is much slower.

If the nozzle is $1/8\ \text{cm}$ across, what is the pressure inside the pipe (with diameter $1\ \text{cm}$)?

If the height is about the same,

$$P_1 + ½\,\rho v_1^2 = P_2 + ½\,\rho v_2^2.$$



Water Tower and Fountain

The gauge pressure in the hose is

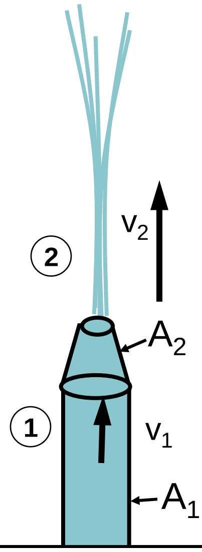
$P_1 - P_2 = P_1 = ½ \rho(v_2^2 - v_1^2)$.

Eliminate v_1 using the equation of continuity: $A_1 v_1 = A_2 v_2$.

$P_1 = ½ \rho v_2^2 (1 - A_2^2/A_1^2)$

$A_2 = (1/8)^2 A_1$ so $A_2^2/A_1^2 = (1/8)^4 = 1/4096$.

$P_1 = 0.9998 \times ½ \rho v_2^2 = 5.4 \times 10^5$ Pa



Water Tower and Fountain

Note that $P_1 \approx \rho g h = 5.4 \times 10^5$ Pa

The pressure inside the pipe is nearly the same as when the pipe is shut off entirely!

Either way, the water shoots as high! What good is the nozzle?

That's only true for an idealized flow.

The flow speed in the pipe is now much slower –

$v_1 = A_2 v_2 / A_1 = 33$ m/s / 64 = 0.52 m/s.

Less viscosity, more ideal behavior.

Air Flow

- Bernoulli's principle can also be applied to air flow, although the idealizations aren't as good. Air easily becomes turbulent, and is very compressible.

- Air blowing over a roof can lift it off due to the pressure difference. In a region where wind is blowing, Bernoulli's equation says that

$$P + \tfrac{1}{2}\rho v^2 = 1 \text{ atm}.$$

- P decreases when v increases, causing low pressure in a hurricane or tornado. The pressure difference can **lift** (not blow) the roof off a house.

Air: $\rho = 1.29$ kg/m^3