# Physics 221

## Sections 1 and 2

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## Hydrostatics

# Announcements

- Exam 3 will be held Monday: Ch. 9 (except 9.7 and 9.8), 10, 11, 12 (except 12.4).
- Due to the short time remaining in the semester, we will skip section 12.4. It is about elastic solids (stress, strain, …).
- We will also skip Chapter 13 on Universal Gravitation, but may come back to it next semester, since it there are analogies with electrostatics.
- Next – Chapter 14, fluids. Sec. 1-3 (4) today.

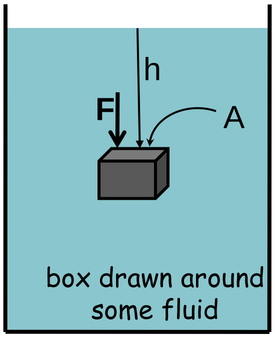
# Pressure in a Fluid

Pressure is force per unit area on a surface:

$$P = F/A$$

where $A$ is the surface area, and only the force perpendicular to the area counts.

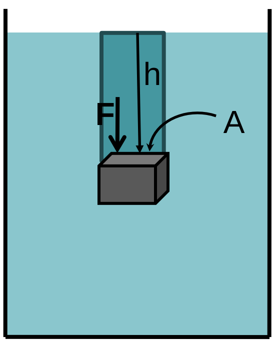
If I put draw a box enclosing a bit of fluid in equilibrium, there is a downward force on the top of the box, $F = PA$. What is this force?




# Pressure in a Fluid

The force is the weight of all the water on top (plus the force due to the atmosphere, but for now, assume the water is in a vacuum).

The weight of the water is $F = \rho ghA$, so the pressure is $P = F/A = \rho gh$ where ρ is the density of water, 1000 kg/m$^3$.

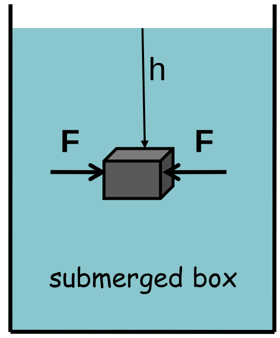


## Pressure in a Fluid

The pressure is the same at any point at the same depth. Otherwise, there would be different forces on the sides of the box – it wouldn't be in equilibrium.

This result generalizes to any point in the fluid:

$$P = \rho g h.$$




## Pressure in a Fluid

For example…

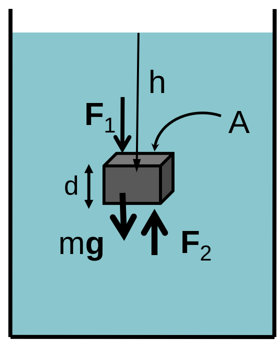
The pressure on the bottom of the box is the weight needed to hold up a column of water of height $d + h$.

Therefore, $F_1 = \rho g h A$ on top, and $F_2 = \rho g(h+d)A$ on the bottom.

The pressure difference between the bottom and top of the box of water is

$$P_2 - P_1 = \rho g d.$$

This pressure difference supports the weight of the box of water, which is $mg = \rho g A d$.



# Pressure in a Fluid

This implies that when you descend a distance $\Delta y = -d$, the pressure increases by

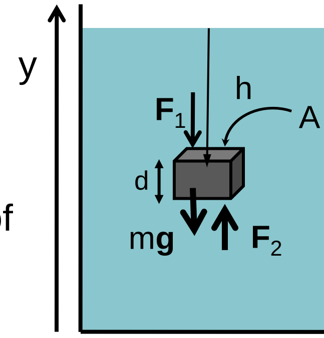
$\Delta P = \rho g d = -\rho g \Delta y$, or, in calculus terms,

$$dP/dy = -\rho g.$$

If the density is independent of the depth (incompressible fluid), this gives

$$P = \rho g h.$$

Sign convention:
y is positive **upward** on this slide.




# Water behind a Dam

- Suppose I have two reservoirs, both of the same depth and width, but one holding a lake 2 miles long, and the other holding a lake 20 miles long.
- Which dam has more force on it?

The depths are the same.

## Water Behind a Dam

It doesn't matter: The pressure on the dam at depth $h$ is $\rho gh$ in either case. This means the force on the dam is the same.

The same would be true if the dam just held back **an inch** of water!

## Different shaped Vessels

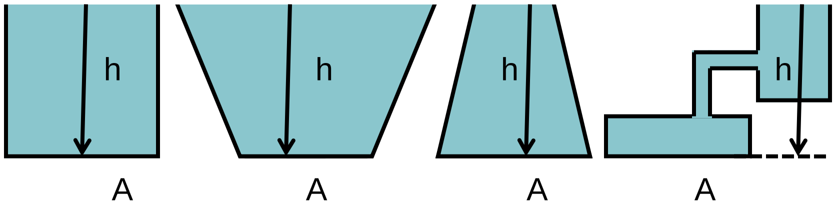
- Which of these vessels, filled with the same depth of water and with the same base area, has the greater force on the base?

Vessels with bottoms of equal area.

# Different shaped Vessels

- The force on the base is the same in each case, even though the mass of water in each container is different. The pressure only depends on the depth of the water.

$F = \rho g h A.$




# Atmospheric Pressure

The weight of the air above us produces atmospheric pressure at sea level equal to

$$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2.$$

Pressure is also measured in **Pascals**:

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

Pressure gauges are normally set to zero when only atmospheric pressure is present.

**Gauge pressure** is defined as the additional pressure beyond that due to the atmosphere.

The total pressure including atmospheric pressure is called **the absolute pressure.**

# Swimming Pool

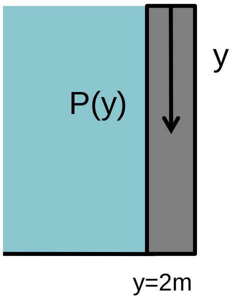
For example, at the bottom of a 2m deep swimming pool, the gauge pressure is

$$P_{\text{gauge}} = \rho g h$$
$$= (1000 \text{ kg/m})(9.8 \text{ m/s}^2)(2\text{m})$$
$$= 1.96\text{x}10^4 \text{ Pa.}$$

The absolute pressure is

$$P_{\text{abs}} = P_{\text{gauge}} + 1 \text{ atm}$$
$$= 1.227\text{x}10^5 \text{ Pa.}$$

# Swimming Pool




What is the total force of the water on the side of the pool, if it is 10 m wide and 2 m deep?

The water's pressure $P(y) = \rho g y$ increases linearly with depth, so the average pressure on the wall is

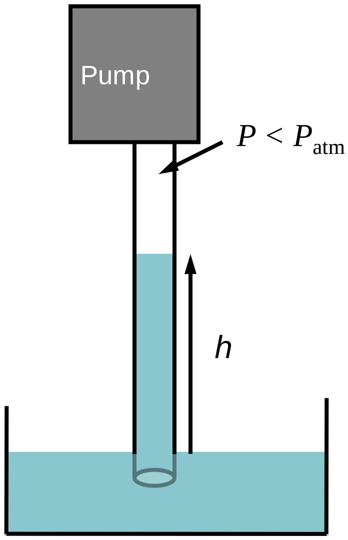
$P_{\text{avg}} = ½\, P_{\text{bottom}} = 9800$ Pa, and the total force on the wall is

$$F = P_{\text{avg}} A = 9800 \text{ Pa x } 20 \text{ m}^2$$
$$= 1.96\text{x}10^5 \text{ N.}$$

## Suction

A negative gauge pressure corresponds to suction.

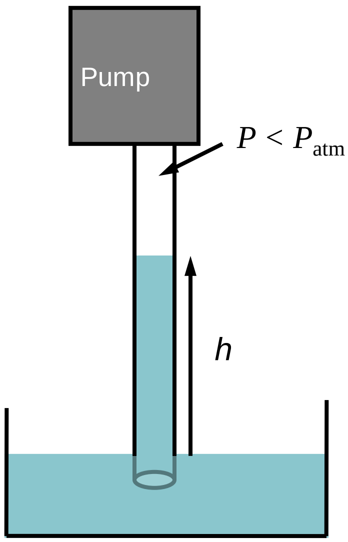
If we produce a negative gauge pressure on a straw, water will be "sucked" up the straw.




## Suction

What is the highest a suction pump can draw water up a tube?

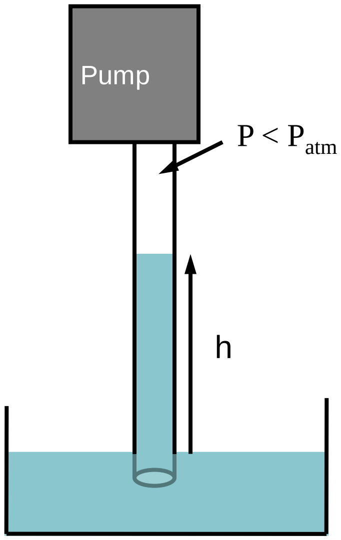
You will have to think about what force is actually causing the water to go up the tube.



# Suction

The absolute pressure in the tube can't be reduced to less than 0, so the maximum pressure difference between the bottom and top is 1 atm.

It is important to realize that it is the atmosphere's pressure that is pushing liquid up a tube when suction is applied. The pump isn't "pulling" on the fluid.



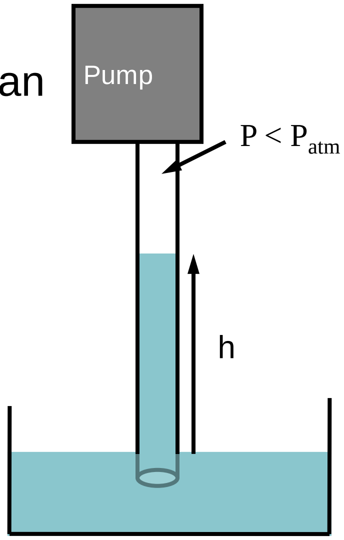

# Suction

The maximum height water can be pumped with a suction pump is then given by

$\rho g h$ = 1 atm = 1.013 x $10^5$ N/m$^2$

with $\rho$ = 1000 kg/m$^3$ and $g$ = 9.8 m/s$^2$, giving

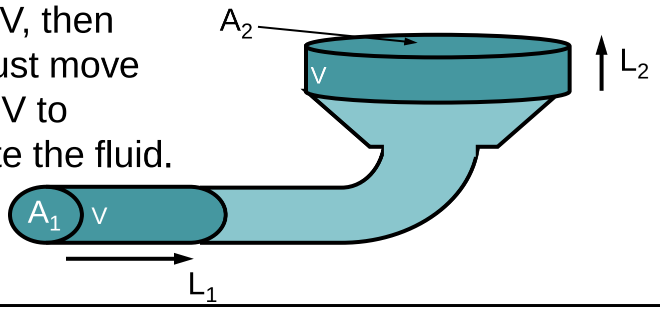
$h$ = 10.3 m.



# Hydraulic Lift

A hydraulic lift is a simple machine which uses the fact that any fluid pushed into a cylinder on one end must come out the other.

If cylinder 1 is compressed by a volume V, then cylinder 2 must move up a volume V to accommodate the fluid.



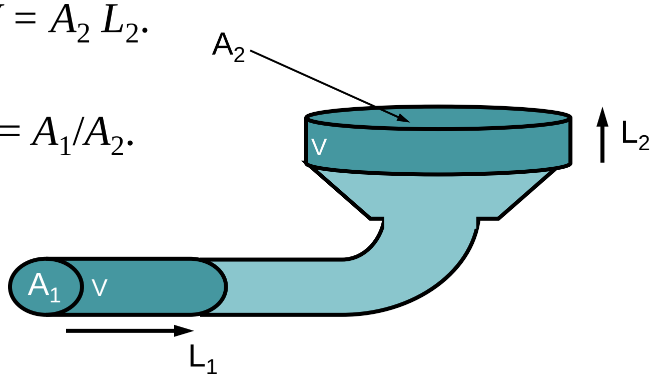

# Hydraulic Lift

Since the volumes are the same, the distances the cylinders move is related to their areas:

$A_1\, L_1 = V = A_2\, L_2$.

or

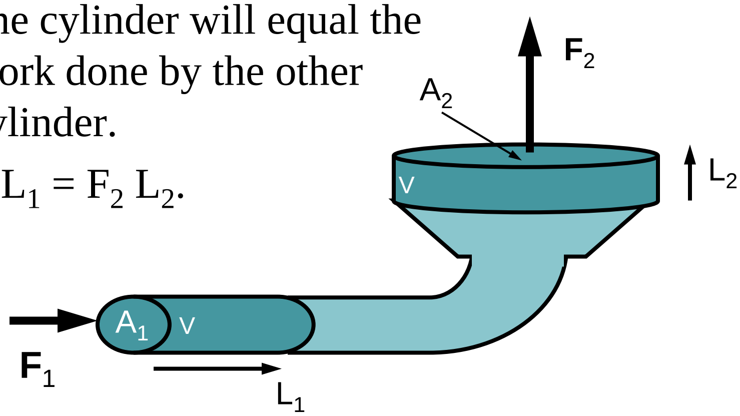
$L_2/L_1 = A_1/A_2$.



## Hydraulic Lift

The work done by a force on one cylinder will equal the work done by the other cylinder.

$F_1\ L_1 = F_2\ L_2$.




## Hydraulic Lift

The ratio of forces is

$F_2 / F_1 = L_1 / L_2 = A_2 / A_1$.

This can also be written as

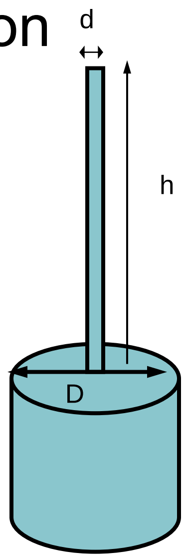
$F_2/A_2 = F_1/A_1$ or, since the force per unit area is the added pressure on the system,

$$\Delta P_2 = \Delta P_1.$$

This is **Pascal's Principle**. In words…

When an force is applied to a closed vessel, the pressure increases by the same amount throughout the vessel.

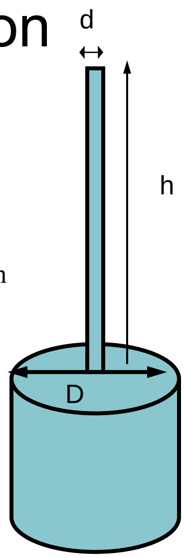
## Pascal's Demonstration




Pascal demonstrated his principle by inserting a long thin tube of diameter $d = 6$ mm into a wine barrel of diameter 40 cm.

He found that filling the tube to a height of 12 m caused the barrel to burst.

## Pascal's Demonstration




(a) What was the weight of the water in the tube?

$m = \rho\,(\pi r^2)\,h$

$= 1000\ \text{kg/m}^3$

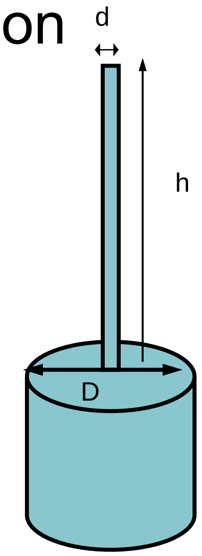
$\times\ (2.8 \times 10^{-5}\ \text{m}^2) \times 12\ \text{m}$

$= 0.340$ kg.

$mg = 0.340\ \text{kg} \times 9.8\ \text{m/s}^2$

$= 3.33\ \text{N} = 0.75$ lb.

# Pascal's Demonstration




What is the pressure at the bottom of the tube?

$F_1 = mg = 0.75$ lb on area

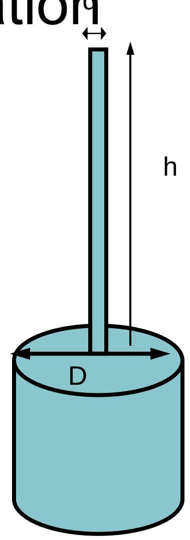
$A_1 = \pi d^2/4 = 2.83 \text{ x } 10^{-5} \text{ m}^2$

Gives pressure

$P = F_1/A_1 = 2.65 \text{ x } 10^4 \text{ lb/m}^2$

(Mixed units – but you probably have a better feeling for pounds, so I'll keep it that way.)

# Pascal's Demonstration




The force on the lid of the barrel is

$F_2 = PA_2$ with $A_2 = \pi D^2/4 = 0.126 \text{ m}^2$, giving

$$F_2 = 3340 \text{ lb!}$$

The ¾ pound of force of the water is multiplied by the ratio $(D/d)^2 = 4444$.

# Fluid in Equilibrium

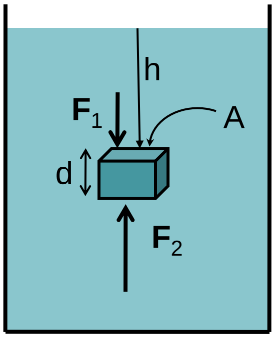
We found the pressure as a function of depth by considering the force on a small rectangular volume of fluid.

The difference between the force on the top and bottom supports the weight of the "box" of fluid:

$F_2 - F_1 = mg$

$P_2 A - P_1 A = \rho d A g.$

$P_2 - P_1 = \rho d g.$



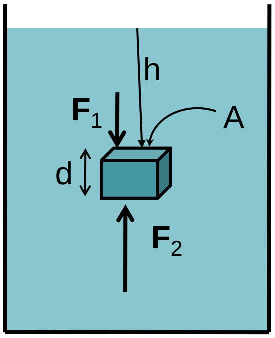
box drawn around some fluid

# Fluid in Equilibrium.

The force $\mathbf{F}_2 - \mathbf{F}_1$, due to the fluid's pressure, that supports the weight of the box is called the **buoyant force**.

How would the buoyant force change if we replaced the constructed box of water by a real box of some sort?

Not at all: The box's presence doesn't change the pressure around it, so the buoyant force $\mathbf{F}_2 - \mathbf{F}_1$ is the same.



a real submerged box